Sovereign Debt, Domestic Banks and the Provision of Public Liquidity*

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Abstract. This paper explores two mechanisms through which a sovereign default can disrupt the domestic economy via its banking system. First, a default creates a negative balance-sheet effect on banks, which prevents the flow of resources to productive investments. Second, default undermines internal liquidity as banks replace government securities with less productive investments. A quantitative analysis of the model shows that these mechanisms generate a deep and persistent fall in output post-default, which accounts for the government's commitment necessary to explain observed levels of external public debt. The model is used to study policies that address the government's lack of commitment.

Keywords: Sovereign default, public debt, banks, liquidity.

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1. Introduction

Sovereign governments borrow not only from international investors but also from domestic residents. When domestic financial institutions buy bonds issued by their own government, they expose themselves to sovereign risk.[1] A sovereign default will thus deteriorate the defaulting country's financial system. In this context, it becomes important to analyze how a sovereign default can affect the domestic economy and how domestic holdings of public debt can shape the government's incentives to default.

This paper proposes a theory to explore two mechanisms through which a sovereign default can disrupt the domestic economy via its financial system and affect the government's repayment incentives, using a model of endogenous default enriched with a financial sector. A first mechanism is related to banks' balance-sheet exposure to public debt. As argued by existing research, a sovereign default has a negative impact on banks' wealth, which reduces their ability to raise funds and prevents the flow of resources to productive investments. A second and novel effect is related to the liquidity value of public debt. Banks that do not have good investment opportunities invest in public debt to transfer their wealth across time. After a default the domestic supply of public debt is scarce and these banks substitute away from the use of government securities to investments in their less productive projects. A quantitative analysis of the model for Argentina shows that these mechanisms can generate a deep and persistent fall in output. Additionally, the presence of this endogenous output cost of default generates repayment incentives for the government that are strong enough to explain observed levels of external public debt.

The theoretical framework features an economy with heterogenous banks and a government that can issue external and domestic public debt and choose to default on it ex-post. Banks can finance projects with idiosyncratic productivity, lend to the government or lend to other banks. The joint analysis of domestic and external debt gives rise to a new insight that is the dual role of sovereign debt. First, public debt is a security that allows the government to transfer aggregate resources across time when the holders of this security are foreign investors. Second, it provides liquidity to the domestic financial system given the presence of financial frictions that prevent the banking sector from satisfying its demand for liquidity with privately issued securities.

[1] More than half of the total public debt in emerging and advanced economies is held by domestic residents. Additionally, more than 10% of banks' net assets are claims on their own government.

A negative *liquidity effect* arises following a sovereign default as a consequence of the shortage of public debt. Consider a bank with low-productivity investment projects that finds profitable to invest in public debt. After a default the aggregate supply of public debt is low and so is its return; therefore, this bank will now prefer to finance its low-productivity projects. These projects demand labor, which is now allocated to projects that are, on average, of lower productivity. This in turn, translates into a lower level of aggregate output. The *balance-sheet effect* of default arises due to the presence of a borrowing constraint for banks that links the maximum amount that banks can borrow from each other to their wealth. Consider now a bank that is invested in public debt and currently has the opportunity to finance high-productivity projects. A sovereign default reduces the wealth of that bank, which in turn reduces the amount of credit it can obtain from other banks to finance its projects. This lowers the amount of labor demanded for these projects, thereby reducing the aggregate demand for labor and equilibrium wages. The fall in wages increases the expected return on projects and induces banks with lower-productivity-projects to invest in them. As a result, there is a drop in the average productivity of the economy through a less efficient allocation of labor.

The presence of these effects gives rise to an internal cost of default that the government takes into account when making repayment decisions. The optimal repayment decision entails a trade-off. On the one hand, a default precipitates an endogenous output cost, as well as an exogenous cost of a temporary exclusion from external financial markets. On the other hand, by defaulting, the government saves resources from being paid back to foreign investors. The attractiveness of default thus depends on the residence composition of the government's creditors.

The model is quantified using aggregate macroeconomic and banking data for Argentina for the 1994.Q1-2012.Q4 period. The model is able to explain several salient features of emerging markets' business cycles such as the high variability of consumption and the counter-cyclicality of the trade balance and interest rate spreads. Additionally, the simulated output dynamics around episodes of sovereign default matches the observed behavior of output in Argentina during the 2001 default, both in terms of the magnitude of the recession and the dynamics of the recovery.

The model is used to perform counterfactual exercises designed to assess the relevance of the balance-sheet effect and the liquidity effect in determining post-default output dynamics and government's commitment to repay debt. Data on aggregate exposure of banks to public debt and on banks' liquidity management allows me to identify and disentangle the strength of each mechanism. The counterfactual exercises indicate that although both channels are economically relevant, the balance-sheet effect is more important as it accounts for 65% of the output cost of default while the remaining 35% is due to the liquidity effect. Additionally, the balance-sheet effect explains most of the government commitment. Without the balance-sheet effect, the average levels of external public debt would be 66% lower. Without the liquidity effect the average levels of external public debt would be 37% lower. These two effects contribute differently to the output cost of default over different horizons. While the depth of the output cost of default is directly related to the balance-sheet effect, its persistence is more linked to the liquidity effect. The liquidity effect, while less important, makes the slump last longer.

A higher exposure of the domestic financial system to public debt leads to a stronger balance-sheet effect. This implies that sovereign risk is negatively related to the stock of domestic debt according to the model's predictions. I test this prediction together with other testable implications of the model regarding the conditional co-movements of sovereign spreads with economic activity and public debt levels. To do so, I use quarterly data on GDP, external and domestic public debt and sovereign bond spreads for a panel of fifteen emerging economies for the 1994.Q1-2012.Q4 period. Consistent with the model's predictions, I find that: *(i)* sovereign spreads covary negatively with the level of economic activity, *(ii)* spreads covary positively with the level of external public debt, *(iii)* spreads covary negatively with the level of domestic public debt. While the first two findings are consistent with previous empirical studies of sovereign spreads,[2] the last result has not been previously analyzed as it is motivated from this particular model of endogenous default with external and domestic public debt.

The model is also used to study the effects of domestic policies that are targeted to address the government's lack of commitment problem. First, I study the welfare effects of allowing for a post-default bailout of the banking system. A post-default bailout of the banking system consists of a tax to households for an amount equivalent to the aggregate exposure of the banking system to public debt that is then reimbursed to banks as lump sum transfers. It is designed to eradicate the balance-sheet effect of defaults as banks' wealth is no longer affected by these episodes. The flip side of eliminating a source of internal costs of default is the associated weakening of the government's ex-ante commitment to repay debt. Results indicate that post-default bailouts of the banking system can be desirable ex-post, once the government is heavily indebted. However, the desirability of this policy is subject to time inconsistency as there are significant welfare gains from committing ex-ante not to implement post-default

[2]See, for example, Edwards (1984) and Uribe and Yue (2006).

bailouts when the levels of external debt are low. The reason for this last result is that by prohibiting bailouts, the government enhances its commitment and is therefore able to increase its level of external debt, which allows households to enjoy the benefits of larger consumption front-loading.

Second, I study policies that are targeted at increasing the banks' exposure to public debt. These types of policies can have a positive effect on welfare given the presence of a positive externality generated by banks' holdings of public debt. When individual banks solve their portfolio problem, they do not take into account the fact that by investing in public debt they increase the cost of default through a stronger balance-sheet effect and enhance the government's commitment to repay its debt. This in turn allows the government to credibly issue higher levels of external debt in equilibrium. I consider the implementation of a minimum requirement of public debt holdings in every bank. This policy entails a trade-off between higher government commitment and lower levels of output due to a crowding-out of high-productivity investments. I find that welfare is maximized with a minimum public debt requirement of 55% of a bank's net worth which is equivalent to 6% of its total assets.

Related Literature

This paper builds upon the literature on sovereign debt as well as the vast literature on financial frictions. It is most closely related to a rising theoretical and quantitative literature that studies the internal costs of sovereign defaults.

Following the original framework of sovereign defaultable debt developed in Eaton and Gersovitz (1981), a recent body of literature has studied the quantitative dynamics of sovereign debt and sovereign defaults. Arellano (2008) and Aguiar and Gopinath (2006) analyze sovereign debt and business cycle properties in emerging economies. Several studies have extended the framework to study different aspects related to sovereign debt.[3] These papers find that the presence of reputational costs in the form of exclusion from financial markets cannot quantitatively account for observed levels of external borrowing.[4] In particular, they argue that

[3]For example, Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) analyze the effects of introducing long-term debt into the standard framework whereas Arellano and Ramanarayanan (2012) focus on optimal debt maturity structure. Yue (2010), Benjamin and Wright (2009) and D'Erasmo (2011) study post-default debt renegotiation. Cuadra and Sapriza (2008) analyze the effects of political uncertainty on sovereign debt and spreads. Du and Schreger (2015) analyze an inflation-default trade-off when debt is denominated in local currency. Na et al. (2015) study the joint occurrence of defaults and devaluations.

[4]Several papers analyze the role of reputational costs in generating commitment to repay debt. Bulow and Rogoff (1989) show that under autarky costs, no debt can be sustained in equilibrium if countries are allowed

the presence of a domestic cost of default is necessary to reconcile observed levels of external debt with low frequencies of default. This paper sheds light into the nature of those costs by studying the effects of a default on the financial system. Recent theoretical studies depart from the assumption of a representative agent and study the government's incentives to repay when heterogeneous agents hold sovereign debt (for example, Broner and Ventura (2011) and Guembel and Sussman (2009)). As in these papers, the composition of debt by residence of the creditors is important for the governments incentives to repay. This paper contributes to this strand of the literature by providing empirical support to this prediction by analyzing how spreads co-move with the stock of domestic and external public debt using data for a panel of emerging countries.

The paper also relates to the literature that studies the economic effects of financial frictions. The modeling of the financial sector on this paper builds on the quantitative framework developed in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) where financial intermediaries are constrained on the amount they can borrow by their level of wealth. This friction makes the wealth of the aggregate banking sector a relevant variable that determines the efficiency of the aggregate economy.[5] In this paper, the presence of this friction, coupled with banks' exposure to public debt gives rise to the balance-sheet effect of default. In a recent paper, Bocola (2014) explores a similar effect to study the pass-through of sovereign risk to economic activity. This paper departs from Bocola (2014) by introducing an optimizing government that chooses public debt issuance and repayment and analyzing the effect of the cost of default on government's commitment.

The presence of financial frictions also determines the role of sovereign debt as public liquidity. Woodford (1990) and Holmström and Tirole (1998) show that there is room for an active management of public liquidity through the issuance of government securities whenever there is a lack of commitment problem in the private sector that prevents it from satisfying its demand for liquidity with privately issued securities. A strand of the literature has studied different

to save after default. This result motivated subsequent research on mechanisms that offset this result. Aguiar and Amador (2014) provide a survey of recent advances in the literature.

[5]This is a feature that is present in several papers that study the macroeconomic effects of financial frictions that stem from limited commitment or moral hazard problems. Some classic references include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1999).

aspects related to the provision of public liquidity.[6] A novel insight of this paper is that the provision of public liquidity can be undermined after a sovereign default and this in turn serves as a commitment device to repay for the government.

This paper also contributes to the theoretical literature on the internal costs of default. Gennaioli et al. (2014), Basu (2009) and Mengus (2013) provide a theoretical analysis of how a sovereign default can weaken the balance-sheet of banks and explore its effect on government's commitment. Brutti (2011) studies the effect of a sovereign default in preventing firms from refinancing investment projects. The contribution of the paper to this literature is the proposal of a new source of internal costs of default that is given by the liquidity effect, as well as the welfare analysis of different government policies.

Finally, the paper is closely related to quantitative studies of sovereign default with effects on the domestic economy. Mendoza and Yue (2012) analyze internal costs of default in the context of a quantitative model of endogenous default. In their model, a sovereign default is assumed to restrict external credit for firms which forces them to substitute imported inputs for domestic ones that are imperfect substitutes, creating a decline in output. This paper focuses on a different aspect that is the effect of default on the banking system. However, its analysis is complementary to theirs as it sheds light into what are the mechanisms that can trigger a decline in credit following a sovereign default. Lastly, Sosa Padilla (2012) considers a closed economy framework to study how a sovereign default can affect domestic credit through the balance-sheet effect. This paper complements his analysis by considering both the balance-sheet effect and the liquidity effect and disentangling their relevance in a model in which public debt can be held domestically and abroad.

Layout

The remaining of the paper is organized as follows. Section 2 presents the model setup and characterizes equilibrium. Section 3 discusses the balance-sheet effect and the liquidity effect and analyzes the government's optimal repayment decisions. Section 4 presents cross-country evidence on spreads and debt that is consistent with the model's predictions. Section 5 analyzes the model's calibration, its business cycle properties and provides counterfactual exercises designed to disentangle the relevance of the balance-sheet and liquidity effect. Section

[6]Kiyotaki and Moore (2005) discuss the role of public liquidity and studies its effect over asset prices. Aiyagari and McGrattan (1998) study how public debt can alleviate financial frictions and crowd-out capital. Krishnamurthy and Vissing-Jorgensen (2012) provide a quantification of the liquidity value of public debt for the case of US Treasuries.

6 studies domestic policies aimed at addressing the government's lack of commitment problem. Finally, section 7 concludes.

2. A Model of Sovereign Debt and a Financial Sector

In this section I formulate a dynamic stochastic general equilibrium model of a small open economy enriched with a financial sector (along the lines of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)) and a sovereign government that lacks commitment and has access to debt markets (as in Eaton and Gersovitz (1981)).

Households

Each household is composed of a continuum of members that includes bankers and workers. Workers supply a fixed amount of labor in a competitive labor market and return their labor income to the household. Bankers manage a bank and transfer non-negative dividends to the households. Within the household there is perfect consumption insurance. Households are risk averse and their preferences are defined over an infinite stream of non-storable consumption

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right]$$

where $\beta \in (0,1)$ is the discount factor, C_t is consumption in period t and $u(\cdot)$ is increasing and concave. Household members are hand-to-mouth consumers and do not make any savings decision. Let w_t be the wage paid to workers in period t, π_t the dividend payments from bankers and τ_t the lump sum taxes paid to the government, the household budget constraint is given by

$$C_t = w_t + \pi_t - \tau_t \tag{1}$$

where the aggregate labor supply is normalized to one.

Banks

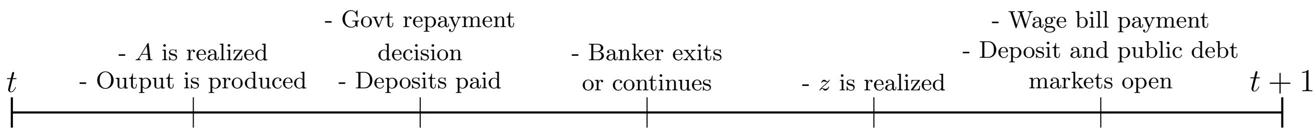
There is a continuum of banks that have access to a constant-returns-to-scale production technology. The technology is stochastic and uses labor l_{t+1} chosen in period t to deliver

$$A_{t+1} z_t l_{t+1}$$

units of consumption in period $t+1$, where A_{t+1} is an aggregate productivity shock and z_t is an idiosyncratic productivity shock. The aggregate shock is subject to trend shocks

$$A_t = \exp(g_t) A_{t-1}$$

Figure 1. Timing of events for a banker



where g_t follows a Markov process with transition probability $f(g_{t+1}, g_t)$ with bounded support. The idiosyncratic shock z_t is known to each banker at period t, and is iid with cummulative distribution function $G(z)$. Since idiosyncratic shocks are independent across bankers and there is a continumm of bankers, $G(z)$ is also the realized fraction of bankers with idiosyncratic shock below z.

In order to hire labor, banks need to pay the wage bill $w_t l_{t+1}$ in period t before production takes place. This assumption about the timing gives rise to a need for banks of obtaining credit to produce.

Bankers exit their business with probability $1-\sigma$ each period. When they exit they distribute their accumulated wealth, or net worth, as dividends to the households. The bankers objective is to maximize the expected value of dividends paid to households

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Lambda_{0,t+1} \sigma^t (1-\sigma) n_{t+1} \right] \tag{2}$$

where n_t is the bank's net worth in period t and $\Lambda_{t,s} \equiv \beta^s u'(C_s)/u'(C_t)$ is the household stochastic discount factor.

In addition to the production technology, bankers have access to two asset markets: the public debt market and the interbank market. Public debt is a risky one-period security that pays one unit of consumption in the following period if the government repays and zero if the government defaults. Interbank deposits are also risky one-period securities that pay one unit of consumption in the following period, except in those states where there is sovereign default, in which they pay zero. In summary, banks can lend to or borrow from other banks, invest in their production technology by hiring labor and buy public debt. The timeline of events for an individual banker within a period is depicted in Figure (1).

Let $\{l_t, b_t^d, d_t\}$ be the claims on labor, the stock of public debt and the stock of interbank deposits with which a banker comes into period t. Then the amount of consumption goods a banker obtains in a period (net worth) is given by the net repayments on these claims

$$n_t = A_t z_{t-1} l_t + \iota_t (b_t^d + d_t) \tag{3}$$

where $\iota_t \in \{0, 1\}$ indicates whether the government defaults or repays its debt in period t, respectively. The net worth that a banker brings into a period, plus the goods he borrows from other banks (if any), can be used to invest in their productive technology, buy public debt or lend to other banks. Let q_t^b, q_t^d be the price of public debt and interbank deposits, respectively, then the banker's balansce-sheet is given by

$$n_t = w_t l_{t+1} + q_t^b b_{t+1}^d + q_t^d d_{t+1}. \tag{4}$$

Note that $d_{t+1} \leq 0$ indicates borrowing from other banks.

The interbank credit market is subject to a financial friction. I assume that the amount of borrowing that any banker can raise through interbank loans is capped by a multiple of its own net worth[7]

$$q_t^d d_{t+1} \geq -\kappa n_t. \tag{5}$$

This type of financial friction is commonly used in quantitative models of credit markets. It can be micro-founded by an agency problem in which the banker has the ability to run away with a fraction of his assets and transfer them to their own household.[8] Finally, I also assume that banks cannot take short positions on public debt

$$b_{t+1}(z) \geq 0. \tag{6}$$

The banker's problem is then to choose a sequence $\{l_t, b_t^d, d_t\}_{t=1}^{\infty}$ that maximizes (2), subject to (3)-(6), given an initial level of net worth n_0 and idiosyncratic productivity z_0.

Government

The sovereign government issues one-period non-state-contingent bonds that pay one unit of consumption next period. These securities can be purchased by domestic banks and/or foreign investors. The government is the only agent that has access to foreign borrowing from external investors. Foreign investors are risk-neutral and can borrow and lend at a constant risk-free interest rate R.

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[7]This assumption alone does not guarantee that the banker will always have enough consumption goods to pay back its deposits. However, it can be shown that by imposing a parametric assumption that bounds the lowest realization of the aggregate productivity $\left(\text{i.e. } \frac{A_{min}}{\mathbb{E}[A]} > \frac{\kappa}{1+\kappa}\right)$ ensures that any banker that borrows will always have enough goods to repay its debt.

[8]For a micro-foundation of this type of financial frictions (and similar variants of it) that stem from agency problems and its role as an accelerator of macroeconomic shocks see, for example, Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

The government lacks commitment to repay its debt and can ex-post choose to default on its entire stock of public debt. Let B_t the stock of total public debt due at period t. The government budget constraint in states in which it has access to the public debt market is given by

$$q_t^b B_{t+1} + \tau_t = \iota_t B_t. \tag{7}$$

The government is benevolent and its objective is to maximize expected lifetime utility of the representative household. To do so it chooses the total stock of public debt, lump sum taxes to households and repayment decisions.

If the government chooses to default on its debt it faces an exogenous cost of exclusion from external financial markets for a stochastic number of periods. In particular, if the government defaults it immediately losses access to the market for external credit. Once in financial autarky the government regains access to the external credit market with probability ϕ and, when it does so, it starts with zero external public debt.

While in external financial autarky the government can still issue domestic public debt that can be held by banks. It does so following a suboptimal exogenous policy of aggregate supply of risk-less domestic public debt such that its equilibrium price is given by $q_t^b = 1/\zeta$ with $\zeta < R$. In other words, during autarky I assume that there will be scarcity of public debt that is reflected in a return of ζ which is assumed to be lower than the international risk-free interest rate. Parametrizing the domestic debt policy during periods of external financial autarky gives enough flexibility to consider different cases. For example, the case of zero domestic debt issuance (which would be consistent with a particularly low value of ζ) would correspond to the extreme case of complete financial autarky from both domestic and external debt markets. Another particular case is the issuance of non-interest bearing securities (i.e. cash), in which case ζ would be given by the inverse of the gross inflation rate.

Discussion of Assumptions

This section discusses the assumptions that underlie the setup. Households are agents that do not make active decisions. In particular, they are assumed not to make savings decisions. This assumption is made since the government, through an active management of lump-sum taxes to households, indirectly makes the inter-temporal savings decisions for the households.

Banks are assumed to have access to a production technology. The banks in this economy represent a consolidation of the financial and productive sector of the economy. This assumption assigns a direct role of banks in the productive process. The production technology is subject to idiosyncratic productivity shocks and therefore banks face an idiosyncratic risk that is not insurable. These uninsurable shocks can represent geographic components or specific knowledge of bankers on certain types of industries that are subject to idiosyncratic shocks.[9] This formulation of banks, together with the timing assumption that wages need to be prepaid before production takes place, embeds the idea that domestic credit is important to realize productive projects (as in Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Brunnermeier and Sannikov (2014)).

The characterization of aggregate productivity shocks as trend shocks -rather than transitory fluctuations around a stable trend- is consistent with recent empirical findings. Aguiar and Gopinath (2007) find that shocks to trend growth are the primary source of fluctuations in emerging markets. Additionally, as shown in Aguiar and Gopinath (2006), the presence of trend shocks in quantitative models of sovereign default help explain high sovereign spreads observed in the data.

The assumption that interbank deposits are not repaid in the state of a sovereign default is done for simplicity. A more standard assumption of risk-less interbank deposits could be adopted and the main theoretical and quantitative results would still carry through.

Implicit in the writing of the government budget constraint (7) is the assumption that the government is not allowed to default selectively on only one type on debt. This assumption is important since, as will become clear later, the government has ex-post incentives to default on its external debt and repay its domestic debt. In practice sovereign governments often contain cross-default clauses (see, for example, IMF (2002) and Hatchondo et al. (2012)). These clauses state that a default in any government obligation constitutes a default in the contract containing that clause. Another relevant assumption is the inability of the government to make transfers to the banks. If the government could make transfers to banks, it would be able to replicate a selective default on external debt by defaulting on the total public debt and bailing out banks. In practice, imperfect government bailouts of the banking system are occasionally observed in emerging economies. I relax this assumption in Section 6 and analyze the impact of allowing governments to bail out the banking system after a sovereign default.

Finally, two assumptions are made regarding public debt issuance after default. The exclusion from external financial markets for a stochastic number of periods can be thought of as a reduced

[9]It can be shown that this setup is equivalent to a model in which there are perfectly competitive firms that operate the production technology in different islands (that face idiosyncratic shocks) and banks can buy claims on firms of a particular island.

form of a punishment from foreign investors in the context of a dynamic game. This exclusion cost of default, common among Eaton-Gersovitz models, is in line with the empirical evidence from recent emerging market default episodes (see Gelos et al. (2011) and Dias and Richmond (2008)). A less stringent assumption is made regarding domestic public debt issuance in periods of external financial autarky. In these states the stock of public debt is scarce and this is reflected in its return which is assumed to be lower than the risk-free international interest rate. This assumption reflects a restriction in the amount of public debt that domestic banks are willing to buy due to a potential punishment or loss of confidence in the government's credibility after the default. In this case, a more flexible approach is adopted to obtain a better fit of the data. In particular, the parameter ζ is disciplined by the data in the calibration section. However, all the theoretical mechanisms of the model do not rely on this assumption and would still hold under the symmetric case of exclusion from both external and domestic debt markets.

Recursive Equilibrium

I focus on Markov equilibrium in which agents' strategies depend on payoff relevant states. Equilibrium is defined in two steps. First I define a *competitive equilibrium* for a given government policy. Second I define a *Markov perfect equilibrium* as the competitive equilibrium associated to the government policies that are chosen optimally given its time inconsistency problem.

I focus in equilibria in which banks follow cutoff rules to determine their portfolio choices and later argue that the unique solution to the banks' problem is of this type. In particular, denote $\underline{z}$ a threshold level of productivity above which banks invest in their own technology. Additionally, let A_{-1} indicate the level of aggregate productivity in the previous period, B^d the aggregate stock of domestic public debt (public debt held by banks) and B^x the stock of external public debt (debt held by foreign investors).[10] The aggregate state of the economy is $\mathbf{s} = (s, h)$ where $s = \left(A_{-1}, g, \underline{z}, B^d, B^x\right)$ and $h \in \{m, a\}$ indicates whether the government has access to external financial markets ($h = m$) or whether it is in external financial autarky ($h = a$). Since I define equilibrium in two steps, the relevant state for the private allocations is the augmented state $(\mathbf{s}, B', \iota)$ that includes the current government policies.

[10]For any variable x of an individual banker define its aggregate counterpart as

$$X \equiv \int x(n, z) d\mathcal{G}(n, z)$$

where $\mathcal{G}(n, z)$ is the endogenous distribution of net-worth and idiosyncratic productivity. $\underline{z}, B^d$ are two sufficient statistics that characterize the solution to the bankers' problem.

The bank's problem admits a recursive representation (that can be found in Appendix A). This problem depends on future government policy functions $(\mathcal{B}'(\mathbf{s}), \mathcal{I}(\mathbf{s}))$ and on the law of motion of the aggregate state $\Gamma(\mathbf{s'}, \mathbf{s}, B', \iota)$ which denotes the density function of state $\mathbf{s'}$ conditional on $(\mathbf{s}, B', \iota)$. Denote $v(n, z; \mathbf{s}, B', \iota)$ the value of an individual bank with net worth n, idiosyncratic productivity z in augmented aggregate state $(\mathbf{s}, B', \iota)$ that solves the bank's problem in recursive form.

DEFINITION **1.** *Given the augmented aggregate state $\tilde{\boldsymbol{s}} = (\boldsymbol{s}, B', \iota)$ and future government policies $\{\iota(\boldsymbol{s}), B'(\boldsymbol{s})\}$, a competitive equilibrium are household consumption $\{C(\tilde{\boldsymbol{s}})\}$, bank allocations $\{l'(n, z; \tilde{\boldsymbol{s}}), b^{d\prime}(n, z; \tilde{\boldsymbol{s}}), d'(n, z; \tilde{\boldsymbol{s}})\}$ and value functions $v(n, z; \tilde{\boldsymbol{s}})$ for all z, dividend payments $\pi(\tilde{\boldsymbol{s}})$, lump-sum taxes $\tau(\tilde{\boldsymbol{s}})$, prices $\{q^d(\tilde{\boldsymbol{s}}), q^b(\tilde{\boldsymbol{s}}), w(\tilde{\boldsymbol{s}})\}$, the distribution of bankers $\mathcal{G}(n, z; \tilde{\boldsymbol{s}})$ and the law of motion of the aggregate state $\Gamma(\boldsymbol{s'}, \boldsymbol{s}, B', \iota)$ such that:*

(1) Government policies and taxes satisfy the government budget constraint (7)

(2) Given taxes, wages and dividend payments, household consumption is consistent with its budget constraint (1)

(3) Given prices, bank allocations and value functions solve the recursive representation of banks' problem (2)-(6)

(4) The labor market and the interbank deposit market clear

$$\int l'(z, n, \tilde{\boldsymbol{s}}) d\mathcal{G}(n, z; \tilde{\boldsymbol{s}}) = 1 \tag{8}$$

$$\int d'(z, n, \tilde{\boldsymbol{s}}) d\mathcal{G}(n, z; \tilde{\boldsymbol{s}}) = 0 \tag{9}$$

(5) The public debt market clears

for $h = m$:

$$\int b^{d\prime}(z, n, \tilde{\boldsymbol{s}}) d\mathcal{G}(n, z; \tilde{\boldsymbol{s}}) \leq B' \tag{10}$$

$$q^b(s, B') \geq \frac{\mathbb{E}\left[\iota(\boldsymbol{s}') | \tilde{\boldsymbol{s}}\right]}{R} \tag{11}$$

$$\left(\int b^{d\prime}(z, n, \tilde{\boldsymbol{s}}) d\mathcal{G}(n, z; \tilde{\boldsymbol{s}}) - B'\right) \left(q^b(s, B') - \frac{\mathbb{E}\left[\iota(\boldsymbol{s}') | \tilde{\boldsymbol{s}}\right]}{R}\right) = 0 \tag{12}$$

for $h = a$:

$$q^b(s) = \frac{1}{\zeta} \tag{13}$$

(6) The joint distribution of net-worth and productivity evolves according to

$$\mathcal{G}'(n', z'; \tilde{\boldsymbol{s}}') = \iint_{(n,z):n'=\eta(n,z;\tilde{\boldsymbol{s}},\boldsymbol{s}')} \mathcal{G}(n, z; \tilde{\boldsymbol{s}}) g(z') dndz$$

where $\eta(\cdot)$ is consistent with the evolution of idiosyncratic net worth given by the bank's allocations and the law of motion of the aggregate state.

(7) *The law of motion of the aggregate state is consistent with current government policies and private allocations, i.e.*

- h' *evolves according to the transition probability*

$$\Pr(h' = m) = \begin{cases} 1 & \text{if } h = m, \iota = 1 \\ 0 & \text{if } h = m, \iota = 0 \\ \phi & \text{if } h = a \end{cases}$$

- $A = A_{-1}\exp(g)$ *and* g' *evolves according to the conditional density* $f(g', g)$
- $B^{d'}(\tilde{\mathbf{s}}) = \int b'^{b}(z, n, \tilde{\mathbf{s}})d\mathcal{G}(n, z; \tilde{\mathbf{s}})$, $B^{x'}(\tilde{\mathbf{s}}) = B' - B^{d'}(\tilde{\mathbf{s}})$ *and the cutoff productivity* $\underline{z}'(\tilde{\mathbf{s}})$ *is given by the minimum productivity of a bank that chooses to invest in his own technology*

The way the public debt market clears is nontrivial. For states in which the government is in financial autarky ($h = a$), the government follows an exogenous policy by suppliying the necessary securities to satisfy the domestic demand for debt at a price of $1/\zeta$. For states in which the government has access to credit markets ($h = m$), there are two possibilities, as indicated by equations (10)-(12). One possibility is that there is no external debt (equation (10) holds with equality). In this case the equilibrium price of public debt should clear the market domestically and also be such that foreign investors are not willing (or at least indifferent) to buy public debt (inequality (11)). The second case is that there is a positive amount of external public debt. In this case public debt is priced by foreign investors (equation (11) holds with equality) and the amount of external public debt is determined as the residual between the total stock of public debt and the domestic public debt demanded by banks at that price.

We can anticipate that in equilibrium equation (11) will hold with equality as the government has incentives to issue domestic public debt so that its equilibrium return increases and output increases.[11] Therefore, even if the government does not want to issue any external debt it will prefer to issue public debt up until the point in which the foreign investor is indifferent between buying or not buying that public debt. It follows that there is no loss in generality to assume that, when issuing debt the government can anticipate what is the equilibrium demand for domestic debt at any given price and is thereby ultimately choosing the stock of external debt. Henceforth government policy functions will be denoted $\{\iota(\mathbf{s}), B^{x'}(\mathbf{s})\}$. Note also that, as is commonly assumed in Eaton-Gersovitz models, we assume the government faces a pricing curve

[11]See section 3 for a discussion on the optimal issuance of public debt for liquidity purposes.

of public debt for any potential level of external public debt $q^b(s, B^{x\prime})$ and chooses optimally in what point of the curve to issue debt.[12]

In this economy the joint distribution of net worth and idiosyncratic productivity follows and endogenous law of motion. However, since idiosyncratic shocks are assumed to be iid we need not keep track of the entire distribution of net worth across banks but only of the aggregate level of domestic public debt B^d and the threshold productivity $\underline{z}$. Aggregating across banks and using the market clearing condition for deposits we get the evolution of aggregate net worth and dividend payments to households

$$N(\tilde{\mathbf{s}}) = \sigma \left(A\mathbb{E}\left[z|z > \underline{z}\right] + \iota B^d \right) \tag{14}$$

$$\pi(\tilde{\mathbf{s}}) = (1 - \sigma) \left(A\mathbb{E}\left[z|z > \underline{z}\right] + \iota B^d \right). \tag{15}$$

We can now characterize the competitive equilibrium. Given that the discount factor of the representative household is not affected by the portfolio choices of an individual bank, the individual bank's problem is linear in net worth and its solution involves corners. Additionally, given that the payoffs to interbank deposits and public debt are the same in every state it must be the case that $q^b(\tilde{\mathbf{s}}) = q^d(\tilde{\mathbf{s}})$ for all states.[13]

The individual banks' optimal portfolio choice depends on their idiosyncratic productivity z and on wages and the price of public debt and deposits. Banks with high productivity choose to borrow in the interbank market up to their constraint and invest the amount borrowed plus all their net worth in their production technology by hiring labor. Banks with low productivity are indifferent between lending to other banks and investing in public debt. An illustration of the solution to the banks portfolio problem is depicted in Figure 2.

A formal characterization of the solution is stated in the following proposition. Denote $R^x(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')$ the realized return of asset x, $\nu(z; \tilde{\mathbf{s}})$ the marginal value of one unit of net worth and

[12]The presence of a pricing schedule from which the government can choose is consistent with a sequential borrowing game in which the government announces how many bonds it wants to issue and then each lender offers the government a price at which he is willing to buy the bonds the government is issuing. For a formal discussion of this argument and an analysis of how differences in timing assumptions can lead to multiple equilibria see Lorenzoni and Werning (2013) and Ayres et al. (2015). Calvo (1988) and Cole and Kehoe (2000) also study the existence of multiple equilibria and self-fulfilling crises in the context of sovereign debt models.

[13]If it is strictly lower all banks would want to borrow from other banks and investing in public debt, but then the interbank market of deposits would not clear. If it is strictly higher then no bank would buy public debt. But no domestic debt in banks is a suboptimal debt issuance policy for the government as it is argued in the following section.

$\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') = \Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')(1-\sigma+\sigma\mathbb{E}_{z'}\left[\nu(z;\tilde{\mathbf{s}}')|\tilde{\mathbf{s}}'\right])$ the augmented stochastic discount factor.[14] Also let $\underline{z}'(\tilde{\mathbf{s}})$ be a threshold productivity level such that the risk-adjusted expected return of investing in the production technology is the same as the risk-adjusted expected return of lending to other banks, i.e. $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^l(\underline{z}(\tilde{\mathbf{s}}); \tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]$.

Proposition 1. *For states in which $q^b(\tilde{\boldsymbol{s}}) = q^d(\tilde{\boldsymbol{s}})$:*

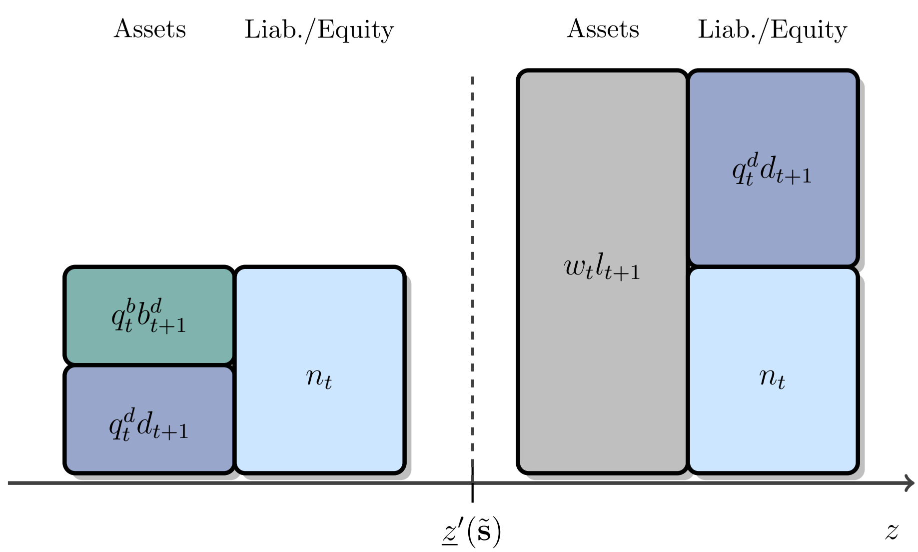
- *Banks with $z > \underline{z}'(\tilde{\boldsymbol{s}})$ prefer to borrow up to their constraint $q^d(\tilde{\boldsymbol{s}})d' = -\kappa n$, invest everything in the productive technology $w(\tilde{\boldsymbol{s}})l' = (\kappa+1)n$ and not buy any public debt $b^{d\prime} = 0$.*
- *Banks with $z \leq \underline{z}'(\tilde{\boldsymbol{s}})$ are indifferent between borrowing to other banks and investing in public debt $q^d(\tilde{\boldsymbol{s}})d' = x \in [0, n]$, and $q^b(\tilde{\boldsymbol{s}})b^{d\prime} = n - x$ and do not invest in labor $l' = 0$.*

Additionally, the value function of bankers is linear in net worth $v(n, z; \tilde{\boldsymbol{s}}) = \nu(z; \tilde{\boldsymbol{s}})n$ where

$$\nu(z;\tilde{\boldsymbol{s}}) = \mathbb{E}\left[\Lambda(\tilde{\boldsymbol{s}}, \tilde{\boldsymbol{s}}')\left(1-\sigma+\sigma\nu(z', \tilde{\boldsymbol{s}}')\right)R^d(\tilde{\boldsymbol{s}}, \tilde{\boldsymbol{s}}')\left[1 + (\kappa+1)\left(\max\left\{\frac{R^l(z;\tilde{\boldsymbol{s}}, \tilde{\boldsymbol{s}}')}{R^d(\tilde{\boldsymbol{s}}, \tilde{\boldsymbol{s}}')} - 1, 0\right\}\right)\right]\right] \tag{16}$$

All proofs can be found in Appendix A.

Figure 2. Solution to Banks' Portfolio Problem



[14] Abusing notation, I refer to next period's augmented aggregate state as $\tilde{\mathbf{s}}' = (\mathbf{s}', \mathcal{B}'(\mathbf{s}'), \mathcal{I}(\mathbf{s}'))$ where $(\mathcal{B}'(\mathbf{s}), \mathcal{I}(\mathbf{s}))$ refer to future government policy functions. Additionally, $\mathbb{E}_z[\cdot]$ refers to the expectation with respect to the random variable z'.

The threshold productivity of the bank that is indifferent between investing in his production technology and investing in public debt (or lending to other banks) is determined by the risk-adjusted expected return on public debt

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')A'\right]\frac{\underline{z}'(\tilde{\mathbf{s}})}{w(\tilde{\mathbf{s}})}=\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]. \tag{17}$$

Higher wages, everything else equal, increase the threshold productivity since it is costlier to hire labor and therefore less profitable to invest in their own technology. The aggregate stock of domestic public debt is determined as a residual of the net worth of those banks with low productivity that did not lend to other banks[15]

$$q^b(\tilde{\mathbf{s}})B^{d'}(\tilde{\mathbf{s}})=N(\tilde{\mathbf{s}})\left(G(\underline{z}'(\tilde{\mathbf{s}}))(1+\kappa)-\kappa\right) \tag{18}$$

Finally, the labor market clearing condition is given by

$$(\kappa+1)N(\tilde{\mathbf{s}})\left[1-G\left(\underline{z}'(\tilde{\mathbf{s}})\right)\right]=w(\tilde{\mathbf{s}}). \tag{19}$$

The demand for labor depends positively on the aggregate level of banks net worth (which ultimately determines the volume of interbank lending) and negatively on the fraction of bankers that choose not to invest in their production technology $G\left(\underline{z}(\tilde{\mathbf{s}})\right)$.

The following proposition formalizes the above-mentioned characterization of prices and aggregate allocations in a competitive equilibrium. Let $\overline{q}(\tilde{\mathbf{s}})$ and its associated return a $\overline{R}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')=\mathcal{I}(\tilde{\mathbf{s}}')/\overline{q}(\tilde{\mathbf{s}})$ be the price of debt such that its risk-adjusted expected return is the same as the risk-adjusted expected return of interbank deposits in an economy without public debt. Formally, $\overline{R}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')$ satisfies

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\overline{R}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]=\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')A'\right]\frac{G^{-1}\left(\frac{\kappa}{\kappa+1}\right)}{N(\tilde{\mathbf{s}})}$$

Proposition **2.** *For any state equilibrium wages solve (19). For states in which the price of debt is $q^b(\tilde{\mathbf{s}})<\overline{q}(\tilde{\mathbf{s}})$, the price of deposits is $q^d(\tilde{\mathbf{s}})=q^b(\tilde{\mathbf{s}})$ and the law of motion for the threshold productivity and aggregate level of domestic debt solve (17)-(18).*

Given the characterization of the dynamics of aggregate variables in the domestic economy we can define the government's problem. Since the government is unable to commit to future policy rules, it chooses its policy rules at any given period taking as given the policy rules that represent future governments' decisions, and a Markov perfect equilibrium is characterized by a

[15]Note that for the stock of domestic public debt to be non-negative we must have $G(\underline{z}'(\tilde{\mathbf{s}}))\geq\frac{\kappa}{\kappa+1}$. If this condition does not hold, then the equilibrium is with $B^{d'}(\tilde{\mathbf{s}})=0$ and $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]>\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$. This case does not happen in equilibrium in the simulations of the model.

fixed point in these policy rules. At this fixed point, the government does not have the incentive to deviate from other government's policy rules, thereby making these rules time-consistent.

Denote $\underline{z}'\left(s,h;B^{x\prime},\iota\right)$, $B^{d\prime}\left(s,h;B^{x\prime},\iota\right)$ and $C\left(s,h;B^{x\prime},\iota\right)$ be the competitive equilibrium allocations associated to *current government policies* $\{B^{x\prime},\iota\}$ and *future government policies* $\{\mathcal{B}^{x\prime}(\mathbf{s}),\mathcal{I}(\mathbf{s})\}$. These allocations satisfy the conditions stated in Proposition 2.

Given its time inconsistency problem the government optimally chooses current period repayment and external debt issuance to maximize the value function of the representative households given that foreign investors and domestic agents expect future government policies to be $\{\mathcal{B}^{x\prime}(\mathbf{s}),\mathcal{I}(\mathbf{s})\}$. The value for the government associated to an optimal one-period deviation solves

$$W(A_{-1},g,\underline{z},B^d,B^x)=\max_{\iota\in\{0,1\}}\iota W^m(A_{-1},g,\underline{z},B^d,B^x)+(1-\iota)W^a(A_{-1},g,\underline{z}) \tag{20}$$

Where the value of repaying and maintaining access to financial markets is given by

$$W^m(A_{-1},g,\underline{z},B^d,B^x)=\max_{B^{x\prime}}u(C(s,m,B^{x\prime},1))+\beta\mathbb{E}\left[W(A,g',\underline{z}',B^{d\prime},B^{x\prime})|s\right] \tag{21}$$

subject to

$$\begin{aligned}C(s,m,B^{x\prime},1)&=A\mathbb{E}\left[z|z>\underline{z}\right]-B^x+q^b(s,B^{x\prime})B^{x\prime}\\ \underline{z}'&=\underline{z}'\left(s,m;B^{x\prime},1\right)\\ B^{d\prime}&=B^{d\prime}\left(s,m;B^{x\prime},1\right)\end{aligned}$$

The consumption equation is given by the resource constraint. The value of defaulting and losing access to financial markets is given by

$$W^a(A_{-1},g,\underline{z})=u(C(s,a))+\beta\mathbb{E}\left[\phi W(A,g',\underline{z}',B^{d\prime},0)+(1-\phi)W^a(A,g',\underline{z}',B^{d\prime})\right] \tag{22}$$

where

$$\begin{aligned}C(s,a)&=A\mathbb{E}\left[z|z>\underline{z}\right]\\ \underline{z}'&=\underline{z}'\left(s,a;B^{x\prime},0\right)\\ B^{d\prime}&=B^{d\prime}\left(s,a;B^{x\prime},0\right)\end{aligned}$$

Note that future government policies affect the government problem as they affect the laws of motion of the threshold productivity and the domestic public debt. Having defined the government problem I define a Markov perfect equilibrium.

DEFINITION **2.** *A Markov Perfect equilibrium are aggregate private allocations* $\{C(\tilde{\boldsymbol{s}}), \underline{z}'(\tilde{\boldsymbol{s}}), B^{d'}(\tilde{\boldsymbol{s}})\}$*, prices* $\{q^d(\tilde{\boldsymbol{s}}), q^b(\tilde{\boldsymbol{s}}, B'), w(\tilde{\boldsymbol{s}})\}$*, government policy functions* $\{B^{x\prime}(\boldsymbol{s}), \iota(\boldsymbol{s})\}$ *and future government policy functions* $\{\mathcal{B}^{x\prime}(\boldsymbol{s}), \mathcal{I}(\boldsymbol{s})\}$ *such that:*

(1) Given government policies, aggregate private allocations and prices are consistent with a competitive equilibrium

(2) Given private allocations and future policies, the government policies solve the government problem (20)-(22)

(3) Optimal government policies coincide with future policies $\{\mathcal{B}^{x\prime}(\boldsymbol{s}), \mathcal{I}(\boldsymbol{s})\} = \{B^{x\prime}(\boldsymbol{s}), \iota(\boldsymbol{s})\}$

Given the presence of non-stationary aggregate productivity shocks, to solve for equilibrium, I first derive the de-trended version of the banks' and government's problems. I do so by normalizing the relevant variables by $A_{-1}\mu_g$. Further details on the de-trended recursive problems are provided in Appendix B. The model is then solved using a global solution that uses projection methods. The competitive equilibrium given any government policy is solved using Euler equation iteration and the government problem is solved using value function iteration methods. A description of the numerical solution algorithm is also provided in Appendix B.

3. Domestic Banks and the Internal Costs of Sovereign Default

This section describes the mechanisms through which government's repayment decisions impact economic activity via its financial system and how these effects in turn feed into the optimal repayment and debt issuance decisions of the government. A sovereign default hits the domestic financial system and the aggregate economy via two channels: *(i)* a balance-sheet channel and *(ii)* a liquidity channel.

3.1. *Balance-sheet Costs of Default*

The banking sector in this economy is subject to a financial friction that prevents resources from flowing within the banking system. Given this friction, the aggregate net worth of the banking sector is a determinant of how much interbank borrowing can take place. In this context, a sovereign default hits the banking system net worth and prevents credit from flowing to productive investments.[16]

[16] A similar mechanism is analyzed in Gennaioli et al. (2014). Bocola (2014) provides a quantitative analysis of this mechanism. In his paper the negative net worth shock lowers the amount that banks can borrow from households via deposits and ends up reducing investment and labor allocations. In this model the negative

A sovereign default has a negative impact on the net worth of those banks that were exposed to public debt. Some of the banks that received this negative shock obtain a high productivity draw for the following period. With a lower net worth, those banks can raise less resources in the interbank market (compared to what they could have obtained in the case of government repayment) and reduce the levels of investment in their productive technology, thereby reducing aggregate labor demand (equation (19)). A lower aggregate demand for labor lowers wages (given the inelastic labor supply) and this has an impact on the optimal portfolio choices of individual banks. The banks that used to be indifferent between investing in their own productive technology and investing in public debt (or lending to other banks) now prefer the former option as the costs of this investment are lower. This reduces the threshold productivity level above which banks prefer to invest in their productive technology and lowers the average productivity of the aggregate economy generating an output cost. The output cost is due to a composition effect of how labor is allocated to banks with differing productivities.[17]

3.2. *Liquidity Cost of Default*

One of the roles of public debt in this economy is to provide liquidity to the domestic financial system. By liquidity I refer to the availability of instruments that can be used to transfer wealth across periods (Woodford (1990), Holmström and Tirole (1998)). These papers argue that there is room for an active management of public liquidity through the issuance of government securities whenever there is a lack of commitment problem in the private sector that prevents it from satisfying its demand for liquidity with privately issued securities. In those situations the provision of public liquidity leads to a more efficient functioning of the productive sector.

In this economy individual banks view the availability of public debt as an exogenous technology at which they can transfer resources across time at a given (risky) rate of return. This investment vehicle is attractive for banks with low productivity that cannot obtain high returns by hiring labor and investing in their productive technology. From an aggregate perspective, the availability of public debt provides liquidity value to the domestic economy as it allows low-productivity banks to invest their net worth in an asset with an attractive risk adjusted return

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balance-sheet shock comes from a reduction in interbank lending and less efficient allocation of labor that leads to a lower average productivity.

[17]The presence of this cost of default does not rely on labor supply being inelastic. If we assumed an elastic labor supply wages would still decrease and so would aggregate labor, moving further away from the first best level in which the marginal disutility of labor is equated to its expected marginal valuation of the marginal product. In this case the output cost of sovereign default would come from a combination of a composition effect (lower average productivity) and an extensive margin effect (through lower aggregate labor).

while they wait for a high productivity draw in the future. In the absence of public debt as an available asset, these banks would have to either lend to other banks at a lower expected return, or invest in their own low-productivity technology. It follows that the liquidity value of public debt is related to its risk-adjusted return. As its return increases, it provides a higher liquidity value as it screens away low-productivity banks from investing in their own technology which in turn frees up inputs (labor) that are used by high-productivity banks. Why does output increase when low-productivity banks are dropping from production? Because the labor that was employed by these banks is now employed by banks with higher productivities. And how are high-productivity banks able to employ more labor? Due to a reduction in wages through a general equilibrium effect (given that now the aggregate demand for labor is lower since less banks are producing).

Given the efficient screening effect associated to an increase in the return on public debt, the government is willing to issue sufficient amounts of public debt so that its return (and the return of deposits) increase in equilibrium and this precipitates a more efficient allocation of labor.[18] However, there is a limit on how much liquidity the government can provide. In particular, there is a return at which foreign investors are willing to buy public debt. After it reaches this return then issuing public debt in excess would not provide any liquidity domestically since it will be bought by foreign investors with a perfectly elastic demand.[19] Therefore, the optimal policy of the government is to exhaust the provision of liquidity domestically by issuing public debt until the point where foreign investors are willing to buy this security and then issue any excess debt for the purpose of consumption smoothing and front-loading.

The role of public debt as a security that provides liquidity to the domestic economy is due to the presence of financial frictions (i.e. the borrowing constraint) on the banking sector. If financial frictions within the banking system were removed, which would correspond to the particular case of $\kappa = \infty$, then changes in the return of public debt would not affect real allocations. Particularly, in the case of a distribution of idiosyncratic productivities with bounded support, the equilibrium allocations in the friction-less case would correspond to only the bank with the highest productivity investing in his technology and all the other banks lending to this banks and/or investing in public debt. In this context, changes in the return on public

[18]In the case where the distribution of idiosyncratic productivities has bounded support, the government would want to issue public debt up until the most productive bank is the only bank investing in its technology.

[19]In this economy there is a trade-off between the decision of opening the economy. By doing so the government can smooth and front-load consumption for the household. However, it puts a limit in how much liquidity in can provide domestically.

debt may affect the return on the different investments but will not affect the real allocation of labor.

Given the liquidity value that public debt has domestically, a sovereign default has associated a negative internal liquidity effect. After a sovereign default the expected return on public debt is now ζ, determined by the exogenous policy, which is lower than its expected return prior to the default. This also impacts the equilibrium price of interbank deposits which is now higher (and its return lower), and affects the optimal portfolio choices of banks. The banks that used to be indifferent between investing in their own technology and investing in public debt (or lending to other banks) now prefer the former option as both public debt and interbank deposits yield a lower return. This reduces the threshold productivity level above which banks prefer to invest in their technology and generates an output cost since labor is now allocated into banks that are, on average, of lower productivity. Additionally, given that a higher share of bankers now prefer to invest in their productive technology, labor demand increase and so do wages.[20]

3.3. *Output Costs, Repayment Incentives and Sovereign Risk*

The government's decision to repay or default depends on the aggregate state of the economy. In particular, when defaulting not only does the government take into account the level of aggregate output $A\mathbb{E}\left[z|z > \underline{z}\right]$ and the stock of external public debt B^x (as it is in the standard Eaton-Gersovitz model), but also the stock of domestic public debt B^d as it affects the magnitude of the output cost of default.

Figure 3 characterizes the repayment and default set. The default set is increasing in B^x. This follows from the fact that the value of repaying is decreasing in B^x (since it implies less aggregate resources available for consumption) and the value of default is independent of B^x. On the other hand, the default set is decreasing in the aggregate productivity shock g. This result is less obvious since both the value of repayment and the value of default are increasing in g and is due to the market incompleteness. The intuition behind it is that even if the government faced the same roll-over possibilities in the next period, default is more attractive in low-income times given that the marginal utility of consumption in these states is higher. In addition, investors internalize this and offer more stringent roll-over possibilities in low-income times, which reinforces the attractiveness of default in this states (see, for example, Arellano (2008) for further discussion of this result). Combining these two results we get that

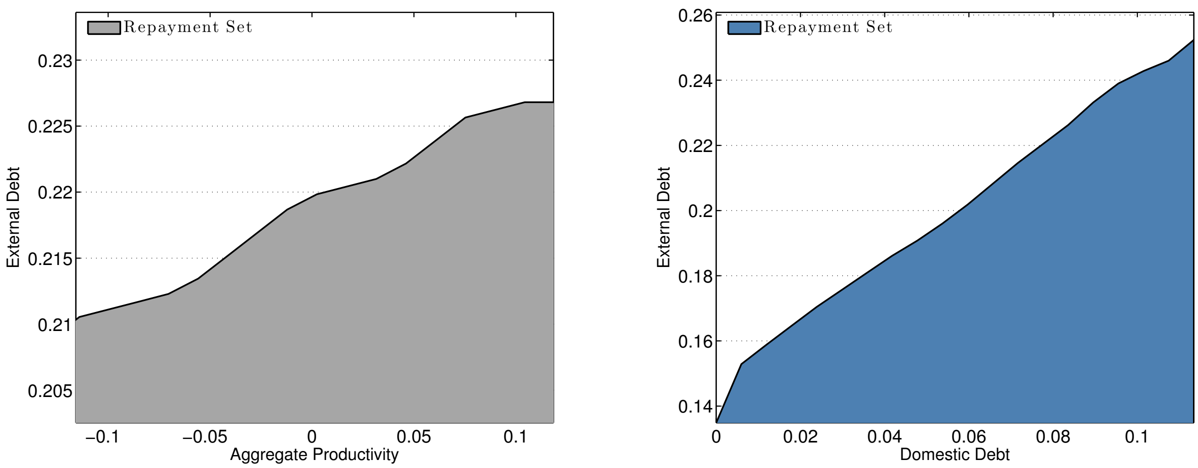
[20]The overall effect of default on wages will depend on the relative strength of the balance-sheet effect and the liquidity effect, which push in opposite directions.

the maximum amount of B^x that is repaid on equilibrium is increasing in the realization of g, as shown in Figure 3a. A corollary of this result is that, given the persistence of the aggregate productivity shock, the government faces a higher price for its debt (for any level of external debt issuance) in a state with a higher realization of g. A high realization of g increases the conditional expectation of next period's shock and therefore increases the repayment probability for any given value of $B^{x\prime}$. These features are shared in most Eaton-Gersovitz class of models and are consistent with the empirical evidence on sovereign defaults (Tomz and Wright (2007)).

Figure 3. Default Decisions as a Function of the States

(a) Aggregate Productivity and External Public Debt

(b) Domestic and External Public Debt



Notes: The parametrization corresponds to the calibration in section 5. The omitted states in the figures are fixed at the average levels of the ergodic set. External and domestic debt are expressed as % of annual GDP.

A more novel result of this model is the role of domestic public debt in affecting the incentives to repay debt and sovereign risk. As shown in Figure 3b, the default set is decreasing in B^d. This follows from the fact that while the value of default is independent of B^d, the value of repaying is increasing in B^d (since higher B^d, everything else equal, implies a higher level of banks net worth in the context of debt repayment, a more efficient allocation of resources within the banking sector and a higher average idiosyncratic productivity). In other words, a sovereign default in the context of higher B^d will have a larger output cost and, for a given benefit of default, this makes it a less attractive option. Given that in equilibrium the policy function $B^{d\prime}(B^d, \cdot)$ is increasing in B^d in repayment states (i.e. with a higher exposure to domestic public debt, the banks' aggregate net worth is higher and this allows banks to buy more domestic public), a high stock of domestic public debt today is an indicator of a high stock of domestic public debt next period and therefore of a lower probability of default. This is reflected in the price that foreign investors ask for a given level of external debt issuance. Given a higher exposure to domestic debt the government faces a higher price for its debt.

4. Cross-Country Evidence on Spreads and Debt

This section conducts an empirical analysis of the testable implications of the model regarding the co-movement of the model's state variables with sovereign risk. I investigate the relationship of sovereign spreads with the stock of domestic and external public debt and the level of economic activity. I collected data on these and other variables for a set of emerging economies. The sample of economies covered in the analysis includes countries that are -or were once included- in J.P. Morgan's Emerging Markets Bond Index Global (EMBIG), subject to the constraint of having sufficient data availability. 15 countries were included in the sample, namely, Argentina, Brazil, Bulgaria, Colombia, Dominican Republic, El Salvador, Hungary, Indonesia, Lithuania, Mexico, Panama, Peru, Philippines, Poland and Uruguay.[21] The empirical analysis is carried out with quarterly data. Data on spreads was collected from Datastream and data on GDP and domestic and external public debt was obtained from the IMF and IDB databases. The time period ranges from 1994.Q1 -when EMBIG spreads are initially available- until 2012.Q2. Spreads are in annualized terms and measured in basis points. The stock of domestic and external public debt are measured as a percentage of annual GDP. Finally, output is measured as percentage deviations from trend output.[22]

To assess the conditional co-movements between sovereign spreads and the level of domestic public debt, external public debt and economic activity, a set of panel regressions were estimated. These regressions estimate sovereign spreads as a linear function of output, the stock of external public debt and the stock of domestic public debt as well as time fixed effects and country fixed effects to control for potential systemic shocks to investors' stochastic discount factor and country-specific risk.

Table 1 reports the regressions estimates. Column 1 shows the results of the baseline estimation that includes both time and country fixed effects. Results indicate that sovereign spreads are negatively related with the level of economic activity. The coefficient on output is negative and significantly different from zero at the 1% level. The coefficient estimate implies that an

[21]The number of countries is relatively small since few countries report data on domestic and external public debt on a quarterly frequency.

[22]Trend output is computed by applying HP filter to the seasonally adjusted output series.

increase of 1% in output gap is associated with a decrease in sovereign spreads of the order of 41 basis points (as a benchmark, the sample average level of sovereign spreads is 410 basis points). The coefficient on external public debt is positive and significantly different from zero at the 1% level. According to the point estimate, an increase in the stock of external public debt from 20% of GDP to 40% of GDP is associated with an increase in spreads of 200 basis points. These results are consistent with the findings of previous empirical studies of sovereign spreads (e.g. Edwards (1984)). A more novel result is the negative relation between sovereign spreads and domestic public debt. The coefficient on the stock of domestic public debt is negative, significantly different from zero at the 1% level and implies that an increase in the stock of domestic public debt from 20% of GDP to 40% of GDP is associated with an decrease in spreads of 150 basis points.

Table 1. Spread Regressions

	(1)	(2)
	Sovereign Spread	Sovereign Spread
Output	-42.63***	-31.95***
	(6.453)	(7.319)
External Public Debt	9.975***	10.35***
	(1.068)	(1.121)
Domestic Public Debt	-7.906***	-5.343**
	(2.156)	(2.172)
Observations	448	448
Average Spread	414.5	414.5
Country & Time Fixed Effects	Yes	Yes
Other Control Variables	No	Yes

Notes: Spreads are measured in annualized basis points. Output is measured in percentage point deviations from trend output (HP filter trend). External and domestic public debt are expressed in percentage points of annual GDP. Other control variables include annual percent variation of nominal exchange rate, international reserves and current account balance (the last two measured percentage points of annual GDP).

Regression 2 estimates a similar specification that also includes other variables that may potentially be related to spreads as additional controls. These variables are the level of international reserves, the exchange rate depreciation and the current account balance.[23] Under this specification, all coefficients remain statistically significant with the same signs as in the baseline regression. The point estimates of the coefficients associated to output and domestic public debt are slightly lower.

In summary, the results presented in this section show that sovereign spreads are affected by economic activity and the level of external and domestic public debt in a way that is consistent with the predictions of the model. It is shown that: *(i)* spreads covary negatively with the level of economic activity, *(ii)* spreads covary positively with the level of external public debt, *(iii)* spreads covary negatively with the level domestic public debt. This last result has not been previously analyzed by the empirical literature that studies sovereign risk.[24]

5. Quantitative Analysis of the Model

This section performs a quantitative analysis of the model by calibrating it to the Argentinean economy for the period 1994-2012. I consider the Argentinean economy to be an interesting case for study for two reasons. First, the period of analysis includes one of the largest sovereign defaults on history. In December 2001 the Argentinean government explicitly defaulted on $95 billion of external debt which represented 37% of its GDP. Additionally, by imposing a unfavorable swaps and the conversion of dollars to pesos of its domestic debt it also implicitly defaulted on the outstanding stock of domestic debt at that time.[25] Finally, throughout the period of analysis the economy exhibited significant levels of external public debt and domestic public debt held in the banking system (23% and 9% of annual GDP on average, respectively), which makes it an appropriate candidate for testing this theory.[26]

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[23]These variables were considered in previous empirical studies of sovereign spreads. The data for these variables was taken from the IMF.

[24]Most of the empirical literature on sovereign spreads investigates the relationship between spreads and the level of public debt or external debt and the level of economic activity. See, for example, Edwards (1984) and Uribe and Yue (2006).

[25]See Sturzenegger and Zettelmeyer (2008) for an analysis of the Argentinean sovereign default.

[26]Historical data on the Argentinean consolidated banking system was obtained from the central bank BCRA. National Accounts data was obtained from finance ministry MECON.

5.1. *Calibration*

One period in the model corresponds to one quarter. The instantaneous utility function is assumed to be

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Additionally, I assume that idiosyncratic productivity shocks z are distributed Pareto with shape parameter λ (i.e. $G(z) = 1 - z^{-\lambda}$) and that the growth rate of the aggregate productivity is approximated with a log-normal AR(1) process with long run mean μ_g and persistence coefficient $|\rho_g| < 1$, i.e.

$$g_t = (1-\rho_a)\left(\ln \mu_a - \frac{1}{2}\frac{\sigma_a^2}{1-\rho_a^2}\right) + \rho_a g_{t-1} + \sigma_a \varepsilon_t \qquad \varepsilon_t \sim N(0,1).$$

The model is parametrized by household specific parameters (β, γ), bank-related parameters $(\sigma, \kappa, \lambda, \mu_a, \rho_a, \sigma_a)$ and government related parameters (R, ϕ, ζ). The model parameter values are summarized in Table 2. The risk aversion coefficient γ is set to 2 and the risk-free interest rate is set to $R = 1.01$, which are standard in quantitative business cycle studies. The reentry probability to external financial markets is set to 0.083 which implies an average period of exclusion of three years which is consistent with the median period of exclusion from international credit markets found in Dias and Richmond (2008) and also in the range of estimates of Gelos et al. (2011).

The value of the shape of the distribution of idiosyncratic productivity shocks is disciplined with estimates of the dispersion of Argentinean firms' productivity during the 2002 crisis from Gopinath and Neiman (2014).[27] I set $\lambda = 3.5$, which generates a standard deviation of productivities of banks that is in line with the cross-sectional dispersion of productivities estimated in their paper. This parameter determines the strength of the output cost of default: a default disrupts the role of the financial sector in reallocating resources and resource reallocation is more important when productivities are dispersed. I perform sensitivity analysis of the main results to this and other key parameters in the model is carried out in Appendix C. The parameters of the exogenous process for aggregate productivity were calibrated to match the standard deviation and autocorrelation of de-trended GDP in the model as well as the average quarterly growth rate. The corresponding estimated values are $\mu_a = 1.01$, $\rho_a = 0.2$ and $\sigma_a = 0.021$.

The parameter κ in the banks' limited commitment constraint is set to 7.5 to match the average leverage ratio of total net worth to total assets in the banking system of 12% during

[27] I thank Brent Neiman and Gita Gopinath for sharing the moments of their data.

TABLE 2. Calibrated Parameters

Parameter		Value	Comments
From Literature			
Risk aversion coefficient	γ	2	Standard RBC value
Risk free interest rate	R	1.01	Standard RBC value
Reentry probability	ϕ	0.083	Dias and Richmond (2008)
Shape of idiosyncratic prod. dist.	λ	3.50	Gopinath and Neiman (2011)
From External Data			
Banks LC constraint	κ	7.50	Argentina Banks data
Public debt return in autarky	ζ	0.98	Argentina Banks data
Average growth rate	μ_a	1.01	Argentina GDP data
Calibrated			*Target from data*
Growth rate autocorrelation	ρ_a	0.20	GDP autocorrelation
Std. deviation of growth shocks	σ_a	0.021	GDP volatility
Discount factor	β	0.90	Frequency of default (0.3% quarterly)
Bankers survival probability	σ	0.787	Domestic public debt in banks (9.3% of GDP)

the sample period. The return ζ on public debt in periods of exclusion from external financial markets is obtained by analyzing the liquidity management of Argentinean banks before and after the sovereign default of December 2001. During the period of 1995-2001 banks held 41% of their liquid assets in government securities. During the period of 2006-2010 after the default, the share of government securities in the banks balance-sheets dropped to 10% of total liquid assets. Most of this reduction was done at the expense of an increase in banks cash holdings from 16% of liquid assets in 1995-2001 to 69% in 2006-2010.[28] Motivated by this fact, I assume that exogenous domestic debt issuance policy after a sovereign default is characterized by a risk-less return on debt that is the same as the return on cash. Therefore, I set $\zeta = 0.98$ which is consistent with an average quarterly rate of inflation of 2.5% observed in the sample period.[29]

[28]I exclude the period 2002-2005 that immediately follows the default since during that period the banks had on their balance-sheet government securities that were granted to them as a compensation for the net worth losses that were due to an asymmetric pesification of banks assets and liabilities.

[29]The average inflation is computed with INDEC data up until 2007. After then the official measure of inflation stops being trustworthy and the average inflation is taken from an average of alternative private measures. See Drenik and Perez (2014) for an explanation of the manipulation of the official CPI figures and for the use of alternative measures of CPI in Argentina for that period.

Finally, the household discount factor β and the exit probability of bankers σ were jointly calibrated to match two moments for the Argentinean economy: a frequency of default of 0.3%, which corresponds to three defaults on external and domestic debt in the past 200 years (Reinhart and Rogoff (2011)) and an average stock of public debt held by banks of 9.3% of annual GDP. The calibrated values were $\beta = 0.9$ and $\sigma = 0.79$. A low discount factor is needed to obtain high interest rate spreads. The calibrated value is in the lines of other models of endogenous sovereign default.[30]

5.2. *Business Cycle Properties of the Model*

t This section assesses the model's quantitative performance by comparing moments from the data with moments from the model's ergodic set. To compute the model's moments I simulate the exogenous productivity process g for 5000 periods and trace the evolution of the endogenous states. The moments are computed by eliminating the first 100 observations. The moments from the data were computed for the sample period 1994-2012, excluding the period 2002-2005 in which the Argentinean government was declared on default.

Table 3 compares the model moments with their data counterparts for those moments that were not targeted in the calibration. The model successfully reproduces the observed average levels of external public debt (23% of annual GDP). This suggests that the presence of an endogenous internal cost of default, together with the exogenous punishment of external financial autarky for a random number of periods, are able to generate enough commitment for the government to explain observed levels of external debt issuance.[31] The model underestimates the median levels of sovereign spreads. The median bond spread in the model's simulations is 94 basis points measured on a quarterly basis, which is below the average spread of 174 basis points observed in the data.

The model reproduces the volatility of the trade balance but underestimates the volatility of public debt and interest rate spreads. Models of endogenous default usually underestimate these

[30]For example, the calibrated value of β is 0.88 in Mendoza and Yue (2012) and 0.8 in Aguiar and Gopinath (2006). Arellano (2008) uses a higher calibrated value for β of 0.953.

[31]Previous quantitative models of endogenous default, calibrated for the Argentinean economy, have generated levels of external debt that are, in some cases, lower than observed levels. For example, Arellano (2008) generates levels of external debt of the order of 6% of quarterly GDP and the average level of debt in the model in Aguiar and Gopinath (2006) is 27% of GDP. More recently, Mendoza and Yue (2012) report an average level of external debt-to-annual GDP ratio of 23% and Chatterjee and Eyigungor (2012) use a model of long-term debt that generates levels of external debt of the order of 70% of quarterly GDP.

Table 3. Business Cycle Statistics

Statistic	Data	Baseline Model
Average		
External Debt (% of annual GDP)	23	22
Interest rate spread (quarterly, in bps)	174	94
Volatility		
Consumption ($\sigma(c)/\sigma(y)$)	1.01	1.58
Trade balance	0.02	0.03
Public debt	7.73	1.88
Interest rate spread	147	60
Correlations		
Output - Consumption	0.93	0.86
Output - Trade balance	-0.27	-0.13
Output - Interest rate spread	-0.41	-0.33
Public debt - Interest rate spread	0.12	0.09

Notes: Data moments are computed with quarterly data for the period of 1994.Q1 - 2012Q4 excluding the the post-default period of 2001.Q4- 2005.Q3. Moments from the model are computed from simulating the economy for 5000 periods. The first moment of spreads correspond to the median.

last two moments.[32] The model correctly predicts the high co-movement between aggregate consumption and output. Additionally, the model predicts excess volatility of consumption with respect to output although higher in magnitude from what is observed in the data. The excess volatility of consumption in the model comes from the presence of shocks to the growth rate of productivity and an endogenous interest rate on debt. A positive shock to the growth rate of productivity implies an increase in current output and a larger increase in future output. Given this permanent shock to productivity, the optimal reaction of agents (under access to risk-free debt) is to increase consumption by more than the increase in output. Additionally, the fact that default is more attractive in states with low realization of aggregate productivity

[32]The fact that there is no risk premia priced in debt contributes to the underestimation of the first and second moments of spreads. Chatterjee and Eyigungor (2012) argue that the use of long-term debt in these models can help obtain higher volatilities of spreads.

implies that the interest rate that government faces for issuing debt in low productivity states is higher and this dissuades the government from issuing additional debt to smooth consumption.

The other face of excess volatility of consumption is the counter-cyclicality of trade balance, which is a key feature of emerging markets business cycles. The model yields a negative correlation between trade balance and output. As the interest rate increases in bad states the sovereign borrows less and net exports increase.

Finally, the model is also consistent with a negative co-movement between interest rate spreads and output and a mild positive correlation between public debt and interest rate spreads. The counter-cyclicality of interest rate spreads can be understood with the relative attractiveness of default in low productivity states. The second result is more related to the fact that the total stock of public debt includes external debt and domestic debt which, as was already argued, have different implications for sovereign risk and this attenuates any co-movement between the total stock of public debt and interest rate spreads.

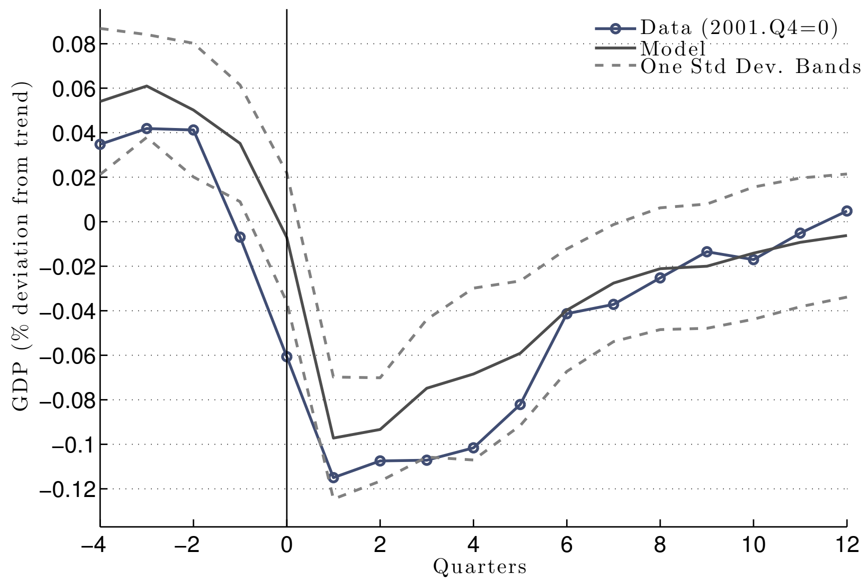
5.3. *Output Dynamics Around Default Episodes*

This section studies the dynamics of output in the model around sovereign default episodes and compares it to the data. The dynamics of output in the model are given by the average path of output around the episodes of default identified in the simulations. Figure 4 plots the model's output dynamics compared to the Argentinean output around the default episode of 2001.Q4. Both series are shown as percentage deviations from trend output.[33] One-standard-deviation bands for the model's average are included. The date of default is set to zero and the window goes from 4 quarters before the default episode to 12 quarters after.

The model's output dynamics are similar to the evolution of output during the 2001 default. The overall behavior of output under the simulated default episode replicates the evolution of output in Argentina both in terms of the magnitude of the fall and the recovery dynamics. In the model, the peak-to-trough fall of output is 15%, the same magnitude as the observed fall in output during the 2001 default. Additionally, the post-default output recovery in the model is consistent with the Argentinean experience. In both cases output starts recovering in the same year of the default and three years later roughly recovers its trend level but is still below its pre default levels. The model also accounts for the fall in output prior to the 2001 default. The fall in output in the model during a default event has two components. One is the exogenous drop in aggregate productivity that triggers the default. The second component is the endogenous

[33]Trend output was obtained by applying an HP filter to both the observed and simulated series of output.

FIGURE 4. Output Dynamics Around Default Episodes



Notes: Model data is obtained from identifying default episodes in simulations and computing the average behavior of output around those episodes.

output cost that comes from the internal costs of default. Both components lead to a lower productivity.[34] The following subsection analyzes the latter part of the fall in output.

5.4. *Disentangling Internal Costs of Sovereign Default*

This section uses the calibrated model to assess the economic impact of the internal costs of sovereign default on the level of output and the degree of government commitment to repay debt, and disentangle the relevance of the balance-sheet effect and the liquidity effect. To do so I perform counterfactual exercises in which I eliminate these mechanisms one at a time and analyze its impact on key macroeconomic variables.

The balance-sheet effect is eliminated in the model by allowing for post-default government bailouts of the banking system. These bailouts consist of a tax to households for an amount equivalent to the aggregate exposure of the banking system to public debt, that is then reimbursed to banks in the form of lump sum transfers.[35] By implementing such a bailout the aggregate banks' net worth is not hit by a sovereign default and therefore the balance-sheet

[34]Consistent with the model's implications, Kehoe (2007) argues that most of the drop of output in the Argentinean crisis was due to a fall in TFP. Additionally, Sandleris and Wright (2014) use firm-level data to show that of the fall in TFP in Argentina, most of it was due to labor misallocation.

[35]A detailed description of how the bailout is implemented in the model is discussed in section 6.

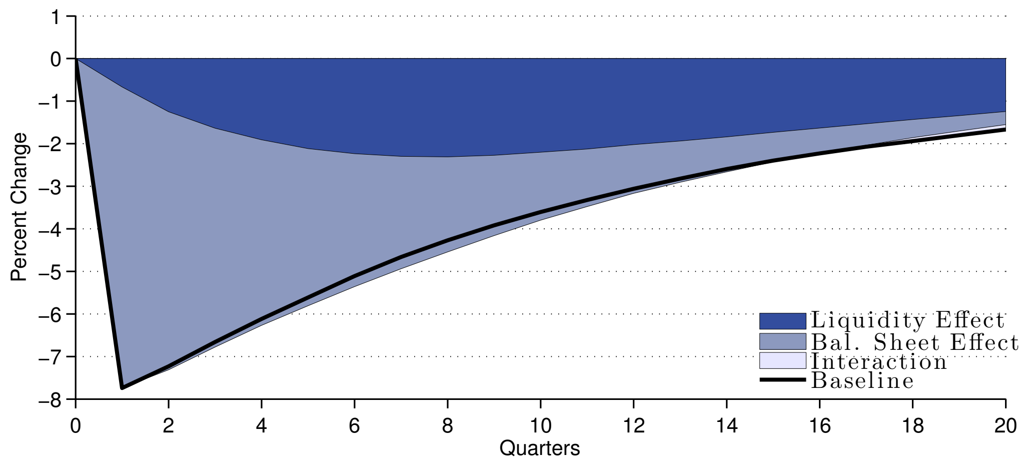
effect is not in place. The liquidity cost is eliminated by setting the return on debt in periods of external financial autarky to be the same as the international interest rate $\zeta = R$. This way the banks can still access the same assets with same expected returns in states in which the government has access to financial markets and in states in which it is in external financial autarky.

The first exercise is from an *ex-post* perspective and studies the effect of a sovereign default on aggregate output and disentangles how much of that effect is due to the balance-sheet effect and how much is due to the liquidity effect. To trace the dynamics of output after a sovereign default I identify those states in the simulation in which default is optimal. For these states I compute the dynamics of output under default and compare them to the output dynamics that would result if the government repaid.[36] This exercise is designed to analyze how much of the fall in output around a default is due to the default decision. Figure 5 shows the average effect of a default on output. A default triggers a drop in output as strong as 7.8% that then recovers gradually and 5 years later is 1.5% below what it would be in the absence of a default. This implies that of the 15% peak-to-trough fall in output in the model around default episodes, half of it is explained by the internal costs (which leaves the other half explained by the fall in exogenous aggregate productivity that triggered the default). In other words, the sovereign default triggers an amplification effect on the contraction of economic activity of approximately 100%. Over the three years following a default, output is on average 5% below what it would be in the absence of default.

The output costs magnitudes are comparable to the magnitudes of default costs considered in other quantitative models of sovereign default. Mendoza and Yue (2012) find that a shift from imported to domestic inputs in the production function due to a sovereign default generates a drop of 5% in Argentinean output which then follows an endogenous recovery. Additionally, Hébert and Schreger (2015) use exogenous variations in default probabilities of the Argentinean government to provide empirical estimation of the elasticity of output drops to default probabilities. They estimate this elasticity to be within 0.04 and 0.1. Interestingly, the implicit elasticity of output drop to default probability that comes out of this exercise is 0.07 which is within their estimated range.

[36] Once identified a state in which default is optimal, the path of output under the decision to repay is simulated 500 times and the average is reported. This way I isolate the effect of the randomness of the time when access to external markets is regained.

FIGURE 5. Disentangling the Output Costs of Default



The model is then solved without the balance-sheet effect and without the liquidity effect and the same exercise is performed over the states in which default is optimal in the baseline model. The output cost that comes out of the model without balance-sheet effect is attributed to the liquidity effect and, similarly, the output cost that comes out of the model without liquidity effect is attributed to the balance-sheet effect. The residual of the output cost in the baseline model that exceeds the sum of the output cost under the model without the balance-sheet effect and the model without the liquidity effect is interpreted as the output cost that is due to the interaction between the balance-sheet and liquidity effect.

As shown in Figure 5 of the total output cost of default (defined as the integral of the output cost over the first 20 quarters following a default) 65% of it is explained by the balance-sheet effect, while the remaining 35% is explained by the liquidity effect. The interaction of both effects plays no role. The relevance of each effect changes as time goes by. The balance-sheet effect is associated with an immediate impact that gradually dissipates as banks earn profits on their investments and the aggregate net worth recovers endogenously. The liquidity effect persists over time as the government remains in financial autarky. It follows that the balance-sheet effect governs the depth of the recession, whereas the liquidity effect determines the persistence of the recession. The liquidity effect, while less important quantitatively, will make the slump last longer.

Two key aggregate moments in the data allow us to identify the quantitative relevance of each channel. The strength of the balance-sheet effect is guided by observed exposure of banks to public debt. The shift in banks liquidity management from public debt to cash holdings is informative of the strength of the liquidity effect. A stronger liquidity effect is associated with a lower return of public debt in autarky and/or a larger period of exclusion.

The second exercise studies how the presence of both effects affects the government's commitment to repay its debt and its ability to credibly issue external debt in international markets. To assess the relevance of each effect I solve and simulate the model under four different specifications: the baseline model, the model with no balance-sheet effect, the model with no liquidity effect and the model with neither balance-sheet nor liquidity effects. Table 4 reports average statistics of key macroeconomic variables for the different specifications.

TABLE 4. Disentangling Default Costs and Government Commitment

Model Specification	Output Cost (next 3 yrs)	Avg. External Debt	Avg. Domestic Debt
Baseline Model	-5.4%	22.1%	9.4%
No Liquidity Effect	-3.6%	13.4%	8.9%
No Balance Sheet Effect	-1.77%	6.96%	9.17%
No Liquidity nor Bal. Sheet Effect	0.00%	0.87%	10.14%

Notes: External and domestic debt are in % of annual GDP. Output cost is the average percentage deviation of output under default with respect to output in absence of default for the following twelve quarters after a default.

The first row shows the average statistics for the baseline model. The average output cost of default (defined as the average percentage deviation of output under default with respect to output under repayment for the 3 years that follow a default) is 5.4%. The average external public debt in the simulations is 22% of GDP and the average domestic public debt is 9.4% of GDP. The later moment was a target of the calibration.

Row 2 reports the average statistics that correspond to the simulations of the model under no liquidity effect. The absence of the liquidity effect attenuates the output cost of default which is 3.6%, 1.8 percentage points lower than that in the baseline specification. This result is consistent with the previous exercise that showed that liquidity costs accounted for approximately 35% of the total output cost and less if we only consider the first three years after a default. This in turn reduces ex-ante commitment for the government and undermines its ability to credibly issue external debt. The average external debt under this specification is 13.3% of GDP, which is 39% lower than the average level under the baseline specification. The stock of domestic debt is 8.9% of GDP, comparable to the average level in the baseline model.

Row 3 reports the statistics for the model without the balance-sheet effect. The implicit output cost of default is 1.8%, which is significantly lower than the 5.4% output cost in the baseline model. The average stock of external public debt is 7.0% of GDP, which is 68% lower than the average level under the baseline model. This suggests that the balance-sheet effect plays an important role enhancing the government's commitment and allowing it to issue higher levels of external debt, significantly more than the liquidity effect. This result is consistent with the fact that the larger fraction of the output cost is explained by the balance-sheet effect. Additionally, as shown in Figure 5 most of the output cost of default in the immediate periods following a default are due to the balance-sheet effect. Given that households are impatient, this implies that the part of the output cost that is more (negatively) valued by the government at the time of making the repayment decision is almost entirely due to the balance-sheet effect. The stock of domestic debt is 9.2% of GDP, in line with the value in the baseline model.

The last row of Table 4 shows the statistics for the model under no balance-sheet nor liquidity effects. In this specification the output cost of default is zero by construction and the only source of commitment for the government comes from the exogenous exclusion period from financial markets after a default. The average external debt that can be sustained with only exclusion costs is only 0.9% of GDP. As previously argued by Arellano (2008) and Aguiar and Gopinath (2006) reputational costs that take the form of exclusion from financial markets cannot account for large amounts of borrowing since the welfare costs of economic fluctuations are small, as originally noted by Lucas (1987). The average level domestic public debt (10.1% of GDP) is in line with the average level of the baseline specification.

In summary, four main conclusions can be drawn from the results presented in this section. First, a sovereign default can trigger a sizable and persistent recession of the order of 5.4% in the three years after the default by affecting the financial system. Second, the presence of the output costs of default are key in inducing government's incentives to repay debt as they account for all of the government's commitment necessary to sustain observed levels of external public debt. Third, the balance-sheet effect is important as it explains 65% of the output cost of default and governs the depth of the output cost. Fourth, the liquidity cost, while less important, accounts for 35% of the output cost of default and accounts for the slow recovery of output after default.

6. Policy Analysis

This section studies the effects two policies that are targeted to address the government's problem of lack of commitment: the implementation of bailouts of the banking system and the implementation of a minimum requirement of public debt holdings in banks.

6.1. *Bailouts of the Banking System*

After a sovereign default the banking system suffers a negative hit to its net worth. Given that the bankers are constrained in the ability to raise funds by the level of their net worth, a social planner would be willing to redistribute resources from the unconstrained agents in the economy (households) towards the constrained agents (banks), to ease the borrowing constraint of banks. A post-default bailout of the banking system is a way of implementing such redistribution. I define a post-default bailout as a lump sum tax to households for an amount equivalent to the aggregate exposure of the banking system to public debt, that is then reimbursed to banks as lump sum transfers. Formally, the bailout is defined as $\tau^b(\mathbf{s})$ where

$$\tau^b(\mathbf{s}) = B^d$$
$$C(\mathbf{s}) = w(\mathbf{s}) + \pi(\mathbf{s}) - \tau(\mathbf{s}) - \tau^b(\mathbf{s})$$
$$N(\mathbf{s}) = \sigma \left(A\mathbb{E}\left[z | z > \underline{z}(\mathbf{s})\right] + \tau^b(\mathbf{s}) \right)$$

where the superscript b indicates bailout. The bailout is assumed to be non-targeted, i.e. transfers are implemented in a uniform way to all banks rather than on an individual basis according to each bank's exposure.[37] Such bailout replicates (at the aggregate level) the allocations of a selective default in which the government defaults on its external debt and repays its domestic debt. Implementing this operation eliminates the balance-sheet effect of a sovereign default. The absence of the balance-sheet effect has a negative impact on the ex-ante government's commitment to repay its debt. By eliminating one source of internal costs of default, the government is more prone to defaulting ex-post and foreign investors anticipate this change in incentives when offering a pricing schedule for public debt. This in turn undermines the government's external debt capacity at the expense of lower consumption front-loading. Therefore,

[37]With non-targeted bailouts the ex-ante incentives of bankers are not changed as the expected return on assets remains the same. If, on the contrary, bailouts are targeted (i.e. on an individual basis according to individual exposures of each bank to public debt), this would create an implicit subsidy to domestic public debt holding that would affect banks' optimal portfolio choices. Since I am interested in isolating the effects of bailouts on the government's incentives I assume the former. See Mengus (2013) for an analysis of post-default bailouts.

the introduction of bailouts entails a benefit of being capable of incurring in a default without suffering the balance-sheet effect on domestic banks and a cost of facing a lower price of debt. In this section I quantify the welfare effects associated to this policy.

Define the welfare benefit (or cost) of allowing for a technology of post-default bailouts of the banking system, denoted $\delta^b(\mathbf{s})$ as the percent change in the lifetime consumption stream required by an individual living in the economy in which post-default bailouts are unfeasible in state $\mathbf{s}$ to be as well off as an individual living in an economy with a technology for implementing post-default bailouts. Formally, $\delta(\mathbf{s})$ is implicitly given by

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u\left(C_t(1+\delta^b(\mathbf{s}))\right|\mathbf{s}\right] = W_h^b\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$$

where $W_h^b\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$ denotes the value function in the economy with the technology to implement bailouts in state $\left\{\left(A_{-1}, g, \underline{z}, B^d, B^x\right), h\right\}$. Since $\delta^b(\mathbf{s})$ is state dependent we compute the unconditional average, denoted δ^b, over all the states of the ergodic set.[38] Formally,

$$\delta^b = \sum_{\mathbf{s}} p(\mathbf{s})\delta^b(\mathbf{s})$$

where $p(\mathbf{s})$ is the unconditional probability of state $\mathbf{s}$.

Results are reported in Table 5. For a household living in an economy without the possibility of bailouts it is welfare increasing to allow for bailouts. The representative household living in this economy requires on average an increase of 0.6% percent in consumption every period to be indifferent between living in this economy and living in an economy with the possibility of post-default bailouts. In this case the benefits associated to the redistribution implied in the bailout are immediate and more valuable for the representative household than the costs associated to lower commitment from the government. In most states of the simulation, the government of the economy with the possibility of bailouts finds it optimal to immediately default on its debt and implement the bailout.

To highlight the pure benefit side of post-default bailouts I compute the welfare effects of announcing a *credible* one-time bailout which does not entail a loss in government commitment. As shown in Table 5, the welfare benefits of implementing this one-time policy are large. The representative household living in an economy without bailouts would be indifferent to live in an economy that implements a one-time bailout if permanent consumption increases by 2.6%.

A reverse exercise is performed to assess what are the welfare effects of prohibiting the implementation of bailouts in an economy in which bailouts are already feasible. I compute

[38] To compute the unconditional average I use the simulations of the baseline model without bailouts.

Table 5. Welfare Analysis of Bailouts

Bailout Technology	Δ Permanent Consumption
Allowing Bailouts in Economy without Bailouts	
Permanently	0.58%
One-time	2.60%
Prohibiting Bailouts in Economy with Bailouts	
Permanently	2.56%

Notes: Values are expressed in equivalent change in permanent consumption. The first two rows report the welfare effects of allowing bailouts in the ergodic set of an economy without bailouts. The last row reports the welfare effects of prohibiting bailouts in the ergodic set of an economy where bailouts are feasible.

the percent change in lifetime consumption required by an individual living in the economy in which post-default bailouts are feasible to be as well off as an individual living in an economy in which bailouts are unfeasible. Results are shown in the last column of Table 5. In this case, the welfare effects change direction: an individual living in an economy with bailouts would require an increase of 2.6% in consumption every period to be indifferent between living in this economy and living in an economy where bailouts are unfeasible. These significant welfare gains are related to the possibility of issuing higher levels of external debt (given a higher government commitment) and front-load more consumption.[39]

Overall, the results presented in Table 5 highlight a time inconsistency problem associated with the implementation of bailouts. In an economy in which investors anticipate that the government can implement bailouts, it is attractive for the same government to tie its own hands and commit to permanently prohibit the implementation of bailouts. This aligns government's incentives to repay its debt and allows for higher levels of external debt in equilibrium and higher consumption front-loading. On the other hand, once the government is already with high levels of external debt that are consistent with foreign investors internalizing that bailouts are unfeasible, the implementation of bailouts becomes attractive. In this case the benefits associated to defaulting on a large stock of external debt and avoiding the balance-sheet effect are more valuable than the costs associated to a lower external debt capacity in the future.

[39]By moving into an economy without bailouts would allow to issue roughly 15% of annual GDP worth of external debt. Given the calibrated discount factor of $\beta = 0.9$, this implies a significant welfare improvement just by means of consumption front-loading.

6.2. *Policies Oriented at Enhancing Banks' Exposure to Public Debt*

This section analyzes the welfare and economic effects of policies targeted at increasing the banks' exposure to public debt. These type of policies can have a positive effect on welfare given the presence of a positive externality generated by banks' holdings of public debt. When individual banks solve their portfolio problem, they do not take into account the fact that by investing in public debt they enhance the government's commitment to repay its debt by increasing the cost of default. This in turn allows the government to credibly issue higher levels of external debt in equilibrium and households to benefit from higher consumption front-loading.[40]

I consider the implementation of a minimum requirement of public debt holdings in every bank. The policy is characterized by the parameter χ that specifies the minimum requirement of public debt as a share of each bank's net worth. The policy introduces the following additional constraint in the bank's problem 23

$$q^b(\tilde{\mathbf{s}})b^d \geq \chi n$$

This constraint will not be binding for low-productivity banks that are indifferent between buying public debt and lending to other banks. However, it will be binding for high-productivity banks since they are forced to allocate part of their asset portfolio in public debt that would otherwise be invested it in their productive technology. A minimum requirement of public debt therefore crowds out investment in productive technology from high-productivity banks. This in turn reduces the demand for aggregate labor, which lowers wages and attracts low-productivity banks to invest in their technology. As a result, the aggregate level of output falls as labor is allocated into technologies with lower productivities on average. The formal derivations of the solution to the banks' problem, as well as the equations that characterize the competitive equilibrium be found in Appendix A.

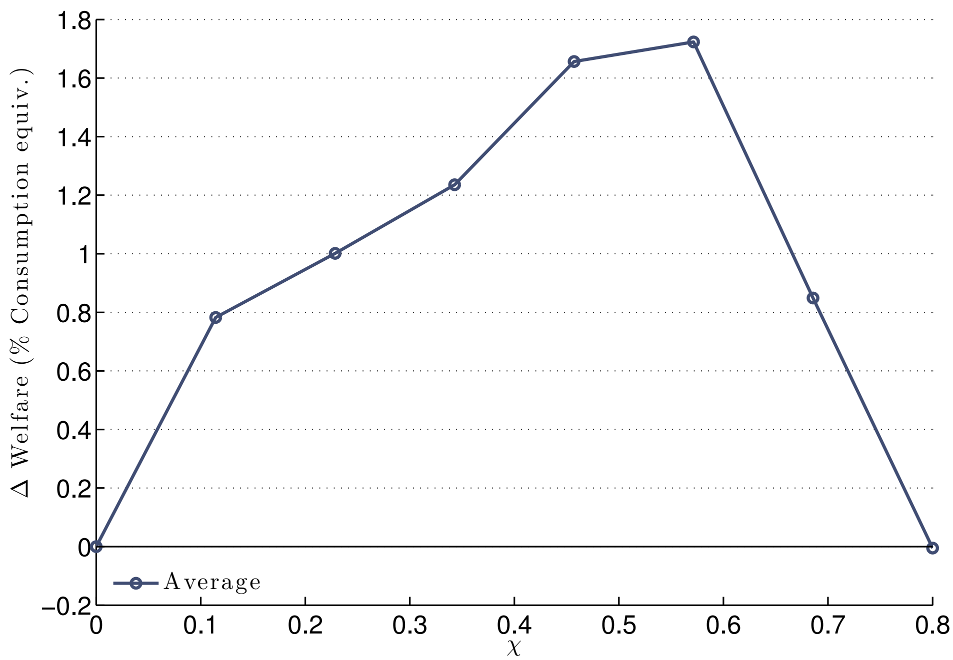
The implementation of a minimum requirement therefore entails a trade-off between lower output due to a lower average productivity and higher external debt issuance due to the enhancement of government commitment. We define the welfare benefit (or cost) of implementing a policy of minimum requirement χ, denoted δ^χ as the unconditional average across all states of the percent change in the lifetime consumption stream required by an individual living in

[40]This idea is also explored in contemporaneous work in Chari et al. (2014) that explore conditions under which imposing public debt on financial intermediaries can be optimal.

the economy with no minimum requirement of public debt in a given state to be as well off as an individual living in an economy with a minimum public debt requirement χ.[41]

The welfare effects of the implementation of a minimum requirement policy χ are shown in Figure 6. The considered values of χ ranged from 0 to 80% of a bank's net worth. Welfare is maximized with the implementation of a minimum requirement of 55%, which is equivalent to 6.5% of total assets for the high-productivity banks. The welfare gains associated to the implementation of this policy are equivalent to an increase of 1.7% in permanent consumption.

FIGURE 6. Welfare Effects of a Minimum Requirement of Public Debt in Banks



7. Conclusion

This paper develops a dynamic model of endogenous default with heterogeneous banks to explore two mechanisms through which a sovereign default can affect the domestic economy through its banking system. In the model economy the role of public debt is dual. First, it is a security that allows to perform inter-temporal trade when the holders of this security are foreign

[41]Formally, $\delta^\chi = \sum_{\mathbf{s}} p(\mathbf{s})\delta^\chi(\mathbf{s})$ where $\delta^\chi(\mathbf{s})$ solves

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u\left(C_t(1+\delta^\chi(\mathbf{s}))\right)\middle|\, \mathbf{s}\right] = W_h^\chi\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$$

where $W_h^\chi\left(A_{-1}, g, \underline{z}, B^d, B^x\right)$ denotes the value function in the economy with policy χ.

investors. Second, it provides liquidity to the domestic financial system given the presence of financial frictions in the domestic economy.

A negative liquidity effect arises following a sovereign default as the supply of public debt is relatively scare and its return low. This makes banks substitute the use of government securities to transfer wealth across periods for investments in less productive projects. A negative balance-sheet effect of default arises due to a tightening in the banks' borrowing constraint that reduces their ability to raise funds and prevents the flow of resources to productive investments.

When quantifying the model to match the Argentinean economy I find that these two mechanisms can generate a deep and persistent fall in output. Additionally, the presence of an endogenous cost of default is important in aligning the government's incentives to repay. The joint presence of the balance-sheet effect and the liquidity effect can help explain the observed levels of external debt issuance. When disentangling the strength of each effect I find that the balance-sheet effect is more important as it generates a larger output cost of default ex-post and a stronger ex-ante commitment for the government.

Finally, the model is used to explore the welfare and economic effects of post-default bailouts of the banking system and a minimum requirement of public debt in banks. Although highly desirable ex-post, post-default bailouts can be welfare reducing ex-ante as they weaken the government's commitment. On the other hand, implementing a minimum requirement of public debt on banks of 55% of their net worth maximizes welfare.

References

Aguiar, M. and M. Amador (2014): *Sovereign Debt*, North-Holland, 647–87.

Aguiar, M. and G. Gopinath (2006): "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 69, 64–83.

——— (2007): "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 115, 69–102.

Aiyagari, S. R. and E. R. McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 98, 690–712.

Arellano, C. and A. Ramanarayanan (2012): "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120, 187–232.

Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2015): "Sovereign Default: The Role of Expectations," Manuscript, Federal Reserve Bank of Minneapolis.

Basu, S. (2009): "Sovereign Debt and Domestic Economic Fragility," Manuscript, Massachusetts Institute of Technology.

Benjamin, D. and M. L. Wright (2009): "Recovery Before Redemption: A Theory of Delays in Sovereign Debt Renegotiations," Manuscript, University of California at Los Angeles.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1, 1341–1393.

Bianchi, J. and E. G. Mendoza (2013): "Optimal Time-Consistent Macroprudential Policy," NBER Working Paper No. 19704.

Bocola, L. (2014): "The Pass-Through of Sovereign Risk," Manuscript, University of Pennsilvania.

Broner, F. and J. Ventura (2011): "Globalization and Risk Sharing," *The Review of Economic Studies*, 78, 49–82.

Brunnermeier, M. K. and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *The American Economic Review*, 104, 379–421.

Brutti, F. (2011): "Sovereign Defaults and Liquidity Crises," *Journal of International Economics*, 84, 65–72.

BULOW, J. AND K. ROGOFF (1989): "Sovereign Debt: Is to Forgive to Forget?" *The American Economic Review*, 79, 43–50.

CALVO, G. A. (1988): "Servicing the Public Debt: The Role of Expectations," *The American Economic Review*, 647–661.

CHARI, V. V., A. DOVIS, AND P. J. KEHOE (2014): "On the Optimality of Financial Repression," Manuscript, University of Minnesota.

CHATTERJEE, S. AND B. EYIGUNGOR (2012): "Maturity, Indebtedness, and Default Risk," *The American Economic Review*, 102, 2674–2699.

COLE, H. AND T. KEHOE (2000): "Self-Fulfilling Debt Crises," *The Review of Economic Studies*, 67, 91–116.

CUADRA, G. AND H. SAPRIZA (2008): "Sovereign Default, Interest Rates and Political Uncertainty in Emerging Markets," *Journal of International Economics*, 76, 78–88.

D'ERASMO, P. (2011): "Government Reputation and Debt Repayment in Emerging Economies," Manuscript, University of Maryland, College Park.

DIAS, D. A. AND C. RICHMOND (2008): "Duration of Capital Market Exclusion: Stylized Facts and Determining Factors," Manuscript, UCLA.

DRENIK, A. AND D. J. PEREZ (2014): "Price Setting Under Uncertainty About Inflation," Manuscript, Stanford University.

DU, W. AND J. SCHREGER (2015): "Sovereign Risk, Currency Risk, and Corporate Balance Sheets," Manuscript, Harvard University.

EATON, J. AND M. GERSOVITZ (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 48, 289–309.

EDWARDS, S. (1984): "LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80," *The American Economic Review*, 74, 726–734.

GELOS, R. G., R. SAHAY, AND G. SANDLERIS (2011): "Sovereign Borrowing by Developing Countries: What Determines Market Access?" *Journal of International Economics*, 83, 243–254.

GENNAIOLI, N., A. MARTÍN, AND S. ROSSI (2014): "Sovereign Default, Domestic Banks, and Financial Institutions," *The Journal of Finance*, 69, 819–866.

GERTLER, M. AND P. KARADI (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58, 17–34.

GERTLER, M. AND N. KIYOTAKI (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis," *Handbook of Monetary Economics*, 3, 547–599.

Gopinath, G. and B. Neiman (2014): "Trade Adjustment and Productivity in Large Crises," *The American Economic Review*, 104, 793–831.

Guembel, A. and O. Sussman (2009): "Sovereign Debt without Default Penalties," *The Review of Economic Studies*, 76, 1297–1320.

Hatchondo, J., L. Martinez, and C. S. Padilla (2012): "Debt Dilution and Sovereign Default Risk," Working Paper 10-08, Federal Reserve Bank of Richmond.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration Bonds and Sovereign Defaults," *Journal of International Economics*, 79, 117–125.

Hébert, B. and J. Schreger (2015): "The Costs of Sovereign Default: Evidence from Argentina," Manuscript, Stanford University.

Holmström, B. and J. Tirole (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106, 1–40.

IMF (2002): "The Design and Effectiveness of Collective Action Clauses," IMF Working Paper.

Kehoe, T. J. (2007): "What can we learn from the 1998-2002 depression in Argentina?" *Great depressions of the twentieth century.*

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

——— (2005): "2002 Lawrence R. Klein Lecture Liquidity and Asset Prices*," *International Economic Review*, 46, 317–349.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

Lorenzoni, G. and I. Werning (2013): "Slow Moving Debt Crises," NBER Working Paper No. 19228.

Lucas, R. E. (1987): *Models of Business Cycles*, New York: Basil Blackwell.

Melitz, M. J. and S. J. Redding (2014): "New Trade Models, New Welfare Implications," Manuscript, Harvard University.

Mendoza, E. G. and V. Z. Yue (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 127, 889–946.

Mengus, E. (2013): "Honoring Sovereign Debt or Bailing Out Domestic Residents: A Theory of Internal Costs of Default," Manuscript, Toulouse School of Economics.

Na, S., S. Schmitt-Grohé, M. Uribe, and V. Yue (2015): "A Model of the Twin Ds: Optimal Default and Devaluation," Manuscript, Columbia University.

Reinhart, C. M. and K. S. Rogoff (2011): "From Financial Crash to Debt Crisis," *The American Economic Review*, 101, 1676–1706.

Sandleris, G. and M. L. Wright (2014): "The Costs of Financial Crises: Resource Misallocation, Productivity, and Welfare in the 2001 Argentine Crisis," *The Scandinavian Journal of Economics*, 116, 87–127.

Sosa Padilla, C. (2012): "Sovereign Defaults and Banking Crises," Manuscript, McMaster University.

Sturzenegger, F. and J. Zettelmeyer (2008): "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998–2005," *Journal of International Money and Finance*, 27, 780–805.

Tomz, M. and M. L. J. Wright (2007): "Do Countries Default in "Bad Times" ?" *Journal of the European Economic Association*, 5, 352–360.

Uribe, M. and V. Z. Yue (2006): "Country Spreads and Emerging Countries: Who Drives Whom?" *Journal of International Economics*, 69, 6–36.

Woodford, M. (1990): "Public Debt as Private Liquidity," *The American Economic Review*, 80, 382–388.

Yue, V. Z. (2010): "Sovereign Default and Debt Renegotiation," *Journal of International Economics*, 80, 176–187.

Appendix A. Omitted Proofs and Results (For Online Publication)

Recursive Representation of Banks' Problem

The bank's problem admits the following recursive representation which depends on future government policies $(\mathcal{B}'(\mathbf{s}), \mathcal{I}(\mathbf{s}))$ and on the law of motion of the aggregate state $\Gamma(\mathbf{s'}, \mathbf{s}, B', \iota)$. Define $\tilde{\mathbf{s}} = (\mathbf{s}, B', \iota)$ the augmented aggregate state and (abusing notation) $\tilde{\mathbf{s}}' = (\mathbf{s'}, \mathcal{B}'(\mathbf{s}), \mathcal{I}(\mathbf{s})))$ next period's augmented aggregate state given future government policies. Denote $v(n, z; \tilde{\mathbf{s}})$ the value of an individual bank with net worth n, idiosyncratic productivity (for next period) z, in augmented aggregate state $\tilde{\mathbf{s}}$, that solves the bank's problem in recursive form. After knowing his idiosyncratic productivity, a banker faces the following recursive problem

$$v(n, z; \tilde{\mathbf{s}}) = \max_{l' \geq 0, b^{d'} \geq 0, d'} \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \left((1-\sigma)n' + \sigma v(n', z'; \tilde{\mathbf{s}}')\right) | \tilde{\mathbf{s}}\right] \tag{23}$$

subject to:

$$n = w(\tilde{\mathbf{s}})l' + q^b(\tilde{\mathbf{s}})b^{d'} + q^d(\tilde{\mathbf{s}})d' \tag{24}$$

$$n' = A'zl' + \iota(\tilde{\mathbf{s}}')b^{d'} + d' \tag{25}$$

$$q^d(\tilde{\mathbf{s}})d' \geq -\kappa n \tag{26}$$

Proof of Proposition 1

We first conjecture that the value function is linear in net worth, i.e. $v(n, z; \tilde{\mathbf{s}}) = \nu(z; \tilde{\mathbf{s}})n$, then solve the portfolio problem of the banks and finally verify our conjecture. Using our conjecture and equation (24) to substitute away d' we can re-write the recursive problem of the banks as

$$\nu(z; \tilde{\mathbf{s}})n = \max_{l' \geq 0, b^{d'} \geq 0} \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \left((1-\sigma) + \sigma \nu(z'; \tilde{\mathbf{s}}')\right) n' | \tilde{\mathbf{s}}\right] \tag{27}$$

subject to:

$$\begin{aligned} n' = &\left(R^l(z; \tilde{\mathbf{s}}, \tilde{\mathbf{s}}') - R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right) w(\tilde{\mathbf{s}})l' + \left(R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') - R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right) q^b(\tilde{\mathbf{s}})b^{d'} \\ &+ R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')q^d(\tilde{\mathbf{s}})d' \end{aligned} \tag{28}$$

$$(1+\kappa)n \geq w(\tilde{\mathbf{s}})l' + q^b(\tilde{\mathbf{s}})b^{d'} \tag{29}$$

where $R^l(z; \tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \equiv \frac{A'z}{w(\tilde{\mathbf{s}})}$, $R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \equiv \frac{\mathcal{I}(\mathbf{s}')}{q^b(\tilde{\mathbf{s}})}$ and $R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}') \equiv \frac{\mathcal{I}(\mathbf{s}')}{q^d(\tilde{\mathbf{s}})}$. This problem is linear in $l', b^{d'}$ and its solution involves corners.

If $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}}, \tilde{\mathbf{s}}')\right]$, the expected risk-adjusted return on deposits and public debt are the same and the solution to the portfolio problem depends on z.

– If $z > \underline{z}'(\tilde{\mathbf{s}})$: $\quad w(\tilde{\mathbf{s}})l' = (1+\kappa)n \quad q^d(\tilde{\mathbf{s}})d' = -\kappa n \quad q^b(\tilde{\mathbf{s}})b^{d'} = 0$
– If $z \leq \underline{z}'(\tilde{\mathbf{s}})$: $\quad w(\tilde{\mathbf{s}})l' = 0 \quad q^d(\tilde{\mathbf{s}})d' = x \in [0, n] \quad q^b(\tilde{\mathbf{s}})b^{d'} = n - x$

Now we verify our conjecture of linearity. Substituting the solution to the problem in (27) the level of net worth scales away and we obtain a law of motion for the marginal value of one unit of net worth.

– For $z \leq \underline{z}'(\tilde{\mathbf{s}})$:

$$\nu(z;\tilde{\mathbf{s}}) = \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\left(1-\sigma+\sigma\nu(z',\tilde{\mathbf{s}}')\right)R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$$

– For $z > \underline{z}'(\tilde{\mathbf{s}})$:

$$\nu(z;\tilde{\mathbf{s}}) = \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\left(1-\sigma+\sigma\nu(z',\tilde{\mathbf{s}}')\right)R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\left[1+(\kappa+1)\left(\frac{R^l(z;\tilde{\mathbf{s}},\tilde{\mathbf{s}}')}{R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')}-1\right)\right]\right]$$

Proof of Proposition 2

The aggregate demand for labor is determined by the amount of resources that high productivity banks can raise in the interbank deposit market which is given by

$$\begin{aligned} w(\tilde{\mathbf{s}})L(\tilde{\mathbf{s}}) &= \int\int_{z>\underline{z}'(\tilde{\mathbf{s}})}(1+\kappa)n dG(z)d\mathcal{G}(n,z_{-1};\tilde{\mathbf{s}}) \\ &= N(\tilde{\mathbf{s}})(1+\kappa)\left[1-G(\underline{z}'(\tilde{\mathbf{s}}))\right] \end{aligned}$$

where the second equality uses the independence between the net worth with which banks arrive to the period and the level of idiosyncratic productivity. Given that the aggregate supply of labor is normalized to one and using the market clearing condition we obtain equation (19).

Now we determine the equilibrium in the interbank market. First note that market clearing in the interbank market implies that $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right] \geq \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$. This is shown by contradiction. Suppose $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right] < \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$, then any bank, regardless of its productivity, would borrow up to its constraint raising interbank deposits (some of them would use it to invest in their technology, others to buy public debt). This implies that the interbank market for deposits would not clear at that price.

Now suppose that $q^b(\tilde{\mathbf{s}}) < \overline{q}(\tilde{\mathbf{s}})$. Under this condition we show that $q^d(\tilde{\mathbf{s}}) = q^b(\tilde{\mathbf{s}})$ or equivalently $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$. We show this by contradiction. Suppose $\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^d(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right] > \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$, then no bank would buy public debt and there is a cutoff $\overline{z}$ such that banks with $z < \overline{z}$ lend to other banks and banks with $z < \overline{z}$ borrow and invest everything in their production. Market clearing in the deposits market implies

$$\begin{aligned} 0 &= \int\int_{z\leq\underline{z}'(\tilde{\mathbf{s}})} n dG(z)d\mathcal{G}(n,z_{-1};\tilde{\mathbf{s}}) - \int\int_{z>\underline{z}'(\tilde{\mathbf{s}})}\kappa n dG(z)d\mathcal{G}(n,z_{-1};\tilde{\mathbf{s}}) \\ &= N(\tilde{\mathbf{s}})\left[G(\underline{z}'(\tilde{\mathbf{s}}))(1+\kappa)-\kappa\right]. \end{aligned}$$

which implies $\overline{z} = G^{-1}\left(\frac{\kappa}{1+\kappa}\right)$. Using the indifference condition for $\overline{z}$ and the expression for wages, the risk-adjusted return of interbank deposits would be

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')A'\right]\frac{G^{-1}\left(\frac{\kappa}{\kappa+1}\right)}{N(\tilde{\mathbf{s}})}$$

which is lower than the risk-adjusted return from investing in public debt which is a contradiction.

Now we prove that the law of motion for the threshold productivity and aggregate level of domestic debt solve (17)-(18).Given that the risk-adjusted return of public debt and deposits is the same, the productivity level $\overline{z}(\tilde{\mathbf{s}})$ that would make a bank indifferent between investing in their own technology and lending to other bank (or buying public debt) must deliver the same risk-adjusted return as the other two options

$$\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')A'\right]\frac{\underline{z}'(\tilde{\mathbf{s}})}{w(\tilde{\mathbf{s}})} = \mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]$$

According to proposition 1 the banks with $z < \underline{z}(\tilde{\mathbf{s}})$ are indifferent between buying public debt or lending to other banks. Therefore, the volume of interbank lending is demand-determined and the aggregate demand for public debt is determined residually

$$\begin{aligned} q^b(\tilde{\mathbf{s}})B^d(\tilde{\mathbf{s}}) &= \int\int_{z\leq \underline{z}'(\tilde{\mathbf{s}})} n dG(z) d\mathcal{G}(n, z_{-1}; \tilde{\mathbf{s}}) - \int\int_{z>\underline{z}'(\tilde{\mathbf{s}})} \kappa n dG(z) d\mathcal{G}(n, z_{-1}; \tilde{\mathbf{s}}) \\ &= N(\tilde{\mathbf{s}})\left[G(\underline{z}'(\tilde{\mathbf{s}}))(1+\kappa) - \kappa\right]. \end{aligned}$$

This is part of an equilibrium if within the banks that are indifferent between buying public debt and lending to other banks there is enough resources to satisfy the demand for interbank lending at that price, or equivalently, if the residual demand for public debt is non-negative. This is true if the following inequality holds

$$G(\underline{z}'(\tilde{\mathbf{s}})) \geq \frac{\kappa}{1+\kappa}$$

which is true given our original assumption.

Competitive Equilibrium with a Minimum Requirement of Public Debt in Banks

The bank's problem with the minimum requirement of public debt is

$$v(n, z; \tilde{\mathbf{s}}) = \max_{l'\geq 0, b^{d'}\geq 0, d'} \mathbb{E}\left[\Lambda(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\left((1-\sigma)n' + \sigma v(n', z'; \tilde{\mathbf{s}}')\right)|\tilde{\mathbf{s}}\right] \tag{30}$$

subject to:

$$n = w(\tilde{\mathbf{s}})l' + q^b(\tilde{\mathbf{s}})b^{d'} + q^d(\tilde{\mathbf{s}})d'$$
$$n' = A'zl' + \iota b^{d'} + d'$$
$$q^d(\tilde{\mathbf{s}})d' \geq -\kappa n$$
$$q^b(\tilde{\mathbf{s}})b^{d'} \geq \chi n.$$

Following the same argument as in proposition 1 the solution of this problem in the relevant case of $q^d(\tilde{\mathbf{s}}) = q^b(\tilde{\mathbf{s}})$ is given by

$$w(\tilde{\mathbf{s}})l' = (1+\kappa-\chi)n \qquad q^d(\tilde{\mathbf{s}})d' = -\kappa n \qquad q^b(\tilde{\mathbf{s}})b^{d'} = \chi n, \quad \text{for } z > \underline{z}'(\tilde{\mathbf{s}})$$
$$w(\tilde{\mathbf{s}})l' = 0 \qquad q^d(\tilde{\mathbf{s}})d' = x \in [0, (1-\chi)n] \qquad q^b(\tilde{\mathbf{s}})b^{d'} = n - x, \quad \text{for } z \leq \underline{z}'(\tilde{\mathbf{s}})$$

Using the labor market clearing condition, the indifference condition for the threshold bank and the aggregate demand for domestic public debt we obtain expressions for the wage, threshold productivity and domestic public debt.

$$w(\tilde{\mathbf{s}}) = N(\tilde{\mathbf{s}})(1+\kappa-\chi)\left[1 - G(\underline{z}'(\tilde{\mathbf{s}}))\right]$$
$$\underline{z}'(\tilde{\mathbf{s}}) = \frac{\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')R^b(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')\right]}{\mathbb{E}\left[\tilde{\Lambda}(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')A'\right]} w(\tilde{\mathbf{s}})$$
$$q^b(\tilde{\mathbf{s}})B^d(\tilde{\mathbf{s}}) = N(\tilde{\mathbf{s}})\left[G(\underline{z}'(\mathbf{s}))(1+\kappa-\chi) - \kappa + \chi\right].$$

Appendix B. Numerical Solution (For Online Publication)

De-trending of the Bank's and Government Problems

First I derive the de-trended recursive banks' problem and then the government problem. The state variables for the banks problem are given by $(n, z; A_{-1}, g, \underline{z}, B^d, B^x)$.[42] The banker's problem is given by

$$v(n_0, z_0; A_{-1}, g_0, \underline{z}_0, B_0^d, B_0^x) = \max_{\{n_t, l_t, b_t^d, d_t\}_{s=1}^{\infty}} \mathbb{E}_0 \sum_{t=1}^{\infty} (1-\sigma)\sigma^{t-1}\Lambda_{0,t} n_t \tag{31}$$

subject to

$$n_t = \prod_{s=0}^{t} R_s^d n_0 + \sum_{s=1}^{t} \prod_{u=s}^{t-1} R_u^d \left[\left(R_s^l - R_s^d\right) w_{s-1} l_s + \left(R_s^b - R_s^d\right) q_{s-1}^b b_s^d \right] \tag{32}$$

$$q_t^b b_{t+1} \geq \kappa n_t \tag{33}$$

$$b_{t+1}^d \geq 0 \tag{34}$$

Equation (32) is obtained by iterating over the definition of net worth. Now we argue that the constraint set of this maximization problem is homogeneous of degree one in $(n; A_{-1}, B^d, B^x)$. Consider a new initial state given by $(\alpha n, z; \alpha A_{-1}, g, \underline{z}, \alpha B^d, \alpha B^x)$ with $\alpha > 0$. Conjecture that new wages are given by αw_t and that $q_t^d, q_t^b, \Lambda_{0,t}$ are not affected by the change in state. Then given the balance-sheet constraints, it follows that if $\{n_t, l_t, b_t^d, d_t\}_{s=1}^{\infty}$ is feasible under the initial state, then $\{\alpha n_t, l_t, \alpha b_t^d, \alpha d_t\}_{s=1}^{\infty}$ is feasible under the new initial state $(\alpha n, z; \alpha A_{-1}, g, \underline{z}, \alpha B^d, \alpha B^x)$ with $\alpha > 0$. Given that the objective function is homogeneous of degree one on n_t it follows that $v(\alpha n, z; \alpha A_{-1}, g, \underline{z}, \alpha B^d, \alpha B^x) = \alpha v(n_0, z_0; A_{-1}, g_0, \underline{z}_0, B_0^d, B_0^x)$.

Now consider the recursive problem of the bank. Consider $\alpha_t = (A_{t-1}\mu_g)^{-1}$ and denote $\hat{x} = (A_{t-1}\mu_g)^{-1} x$ the de-trended version of variable x and $\hat{\mathbf{s}} = (g, \underline{z}, \hat{B}^d, \hat{B}^x)$. The normalization results in an aggregate productivity level with unconditional average of one. Conjecture the that the price of debt is homogeneous of degree zero, i.e. $q^b(\hat{\mathbf{s}}) = q^b(\mathbf{s})$. Then, using the definition of aggregate consumption and the stochastic discount factor it can be shown that

$$\Lambda(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \Lambda(\mathbf{s}, \mathbf{s}') \exp(g)^{-\gamma} \tag{35}$$

[42] To simplify notation I consider private allocations to depend only on the aggregate state. This already assumes that private allocations correspond to a Markov equilibrium in which government policies are optimal and depend on the aggregate state $\mathbf{s}$.

Using the homogeneity of the bank's value function we can obtain the de-trended bank's recursive problem

$$
\begin{aligned}
v(\hat{n}, z; \hat{\mathbf{s}}) &= (A_{-1}\mu_g)^{-1} v(n, z; \mathbf{s}) \\
&= (A_{-1}\mu_g)^{-1} \max_{l' \geq 0, b^{d\prime} \geq 0, d' \geq -\kappa n/q^d} \mathbb{E}\big[\Lambda(\mathbf{s}, \mathbf{s}')\left((1-\sigma)n' + \sigma v(n', z'; \mathbf{s}')\right)|\mathbf{s}\big] \\
&= \max_{l' \geq 0, \hat{b}^{d\prime} \geq 0, \hat{d}' \geq -\kappa \hat{n}/q^d} \mathbb{E}\big[\Lambda(\mathbf{s}, \mathbf{s}')\exp(g)\left((1-\sigma)\hat{n}' + \sigma v(\hat{n}', z'; \hat{\mathbf{s}}')\right)|\mathbf{s}\big] \\
&= \max_{l' \geq 0, \hat{b}^{d\prime} \geq 0, \hat{d}' \geq -\kappa \hat{n}/q^d} \mathbb{E}\big[\Lambda(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\exp(g)^{1-\gamma}\left((1-\sigma)\hat{n}' + \sigma v(\hat{n}', z'; \hat{\mathbf{s}}')\right)|\mathbf{s}\big] \qquad (36)
\end{aligned}
$$

where in the third equality I use the definition of $\hat{n}'$ and the homogeneity of the value function, and in the third equality I use equation (35).

Now I derive the de-trended recursive problem for the government. Denote $\Phi(\mathbf{s}_t)$ the budget set of associated to the government problem. Using a similar argument it can be shown that if $(\iota_t, C_t, \underline{z}_{t+1}, B^d_{t+1}, B^x_{t+1}) \in \Phi(\mathbf{s}_t)$ then $(\iota_t, \hat{C}_t, \underline{z}_{t+1}, \hat{B}^d_{t+1}, \hat{B}^x_{t+1}) \in \Phi(\hat{\mathbf{s}}_t)$. Then using homogeneity of degree $1-\gamma$ of the utility function we can write the recursive problem of the government as

$$
W(g, \underline{z}, \hat{B}^d, \hat{B}^x) = \max_{\iota \in \{0,1\}} \iota W^m(g, \underline{z}, \hat{B}^d, \hat{B}^x) + (1-\iota)W^a(g, \underline{z}) \qquad (37)
$$

where the value of repayment and keeping access to external financial markets is

$$
W^m(g, \underline{z}, \hat{B}^d, \hat{B}^x) = \max_{B^{x\prime}} u(\hat{C}(m, \hat{s})) + \beta \exp(g)^{1-\gamma}\mathbb{E}\left[W(g', \underline{z}', \hat{B}^{d\prime}, \hat{B}^{x\prime})|\hat{s}\right] \qquad (38)
$$

subject to

$$
\begin{aligned}
\hat{C}(s, m) &= \frac{\exp(g)}{\mu_g}\frac{\lambda \underline{z}}{\lambda - 1} - \hat{B}^x + q^b(\hat{s}, \hat{B}^{x\prime})\hat{B}^{x\prime} \\
\underline{z}' &= \underline{z}'\left(\hat{s}, m; \hat{B}^{x\prime}, \iota\right) \\
\hat{B}^{d\prime} &= B^{d\prime}\left(\hat{s}, m; B^{x\prime}, \iota\right)
\end{aligned}
$$

and where value of defaulting and loosing access to external financial markets is

$$
W^a(g, \underline{z}) = u(\hat{C}(a, \hat{s})) + \beta \exp(g)^{1-\gamma}\mathbb{E}\left[\phi W(g', \underline{z}', \hat{B}^{d\prime}, 0) + (1-\phi)W^a(g', \underline{z}', \hat{B}^{d\prime})|\hat{s}\right] \qquad (39)
$$

where

$$
\begin{aligned}
\hat{C}(s, a) &= \frac{\exp(g)}{\mu_g}\frac{\lambda \underline{z}}{\lambda - 1} \\
\underline{z}' &= \underline{z}'\left(\hat{s}, a; \hat{B}^{x\prime}, \iota\right) \\
\hat{B}^{d\prime} &= B^{d\prime}\left(\hat{s}, a; B^{x\prime}, \iota\right)
\end{aligned}
$$

Note that the endogenous law of motion of the cutoff productivity and the stock of domestic debt, $\underline{z}\left(\hat{s}, m; \hat{B}^{x\prime}, \iota\right)$ and $\hat{B}^{d}\left(\hat{s}, m; \hat{B}^{x\prime}, \iota\right)$ correspond to the solution of the competitive equilibrium.

Numerical Algorithm

The model is solved using a global solution that uses projection methods. The competitive equilibrium given any government policy is solved using Euler equation iteration and the government problem is solved using value function iteration methods.

Denote $\hat{x} = \frac{x}{A_{-1}\mu_g}$ the de-trended version of variable x. Let $\hat{\mathbf{s}} = \left\{\left(g, \underline{z}, \hat{B}^d, \hat{B}^x\right), h\right\}$ denote the de-trended aggregate state. First I solve for the set of competitive equilibrium given any *current* government policies $\left\{\hat{B}^{x\prime}, \iota\right\}$, *expected* government policies $\left\{\hat{\mathcal{B}}^{x\prime}(\hat{\mathbf{s}}), \mathcal{I}(\hat{\mathbf{s}})\right\}$ and associated functions of expected consumption and price of public debt $\left\{\hat{\mathcal{C}}(\hat{\mathbf{s}}), q^b(\hat{\mathbf{s}}, \hat{B}^x)\right\}$. This implies solving for equilibrium functions $\left\{\underline{z}'(\hat{\mathbf{s}}; \hat{B}^{x\prime}, \iota), \hat{B}^{d\prime}(\hat{\mathbf{s}}; \hat{B}^x, \iota), \hat{N}(\hat{\mathbf{s}}; \hat{B}^x, \iota), \nu(\hat{\mathbf{s}}; \hat{B}^x, \iota)\right\}$, using the following set of equations

$$\underline{z}'(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \left[(\kappa+1)\hat{N}\frac{\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\right]}{\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\exp(g')\right]}\right]^{\frac{1}{1+\lambda}} \tag{40}$$

$$q^b(\hat{\mathbf{s}}, \hat{B}^x)\hat{B}^{d\prime}(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \hat{N}\left((1-\underline{z}^{-\lambda})(1+\kappa)-\kappa\right) \tag{41}$$

$$\hat{N}(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \sigma\left(\frac{\exp(g)}{\mu_g}\frac{\lambda\underline{z}}{\lambda-1} + \iota\hat{B}^d\right) \tag{42}$$

$$\nu(\hat{\mathbf{s}}; \hat{B}^x, \iota) = \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\left[1 + \frac{(\kappa+1)}{\lambda-1}\underline{z}'(\hat{\mathbf{s}})^{-\lambda}\right]\right] \tag{43}$$

where

$$\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \beta\exp(g)^{1-\gamma}\left(\frac{\hat{\mathcal{C}}(\hat{\mathbf{s}}')}{\hat{C}(\hat{\mathbf{s}})}\right)^{-\gamma}\left(1-\sigma+\sigma\nu(\hat{\mathbf{s}}')\right) \tag{44}$$

Note that I have used the functional forms used in the calibration to substitute for $u(\cdot), G(\cdot)$ and I have also used the case of $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')R^d(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\right] = \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')R^b(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\right]$. Additionally, equation (43) comes from solving the expectation over z' in equation (16).

The algorithm to solve for the competitive equilibrium given expected and current government policies follows these steps:

(1) Generate a discrete grid for variable x state space $G_x = x_1, x_2, ...x_{N_x}$, for $x = g, \underline{z}, \hat{B}^d, \hat{B}^x$. The total aggregate state space is given by $\mathcal{S} = G_g \times G_{\underline{z}} \times G_{\hat{B}^d} \times G_{\hat{B}^x} \times \{m, a\}$.

(2) Feed in some expected government policies $\left\{\hat{\mathcal{B}}^x(\hat{\mathbf{s}}), \mathcal{I}(\hat{\mathbf{s}})\right\}$.

(3) Conjecture a functional forms $\mathcal{E}_1(\mathbf{s}, B^{x\prime}, \iota)$ and $\mathcal{E}_2(\mathbf{s}, B^{x\prime}, \iota)$ for all $(\mathbf{s}, B^{x\prime}, \iota) \in \mathcal{S} \times G_{\hat{B}^x} \times \{0,1\}$, that will be guesses for $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\right]$ and $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\exp(g')\right]$, respectively.

(4) Solve for $\left\{\underline{z}'(\hat{\mathbf{s}}), \hat{B}^{d\prime}(\hat{\mathbf{s}}), \hat{N}(\hat{\mathbf{s}}), \nu(\hat{\mathbf{s}})\right\}$ using (40)-(43).[43] Check whether $\hat{B}^{d\prime}(\hat{\mathbf{s}}) \geq 0$ in every grid point (this ensures that we are under the equilibrium in which $q^d(\mathbf{s}) = q^b(\mathbf{s})$).

(5) Compute $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\right]$ and $\mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\exp(g')\right]$ using quadrature methods for computing expectations. For evaluation of the functions outside grid points I use piecewise linear interpolation.

(6) If $\sup_{\mathbf{s}, B^{x\prime}, \iota}\left\|\mathcal{E}_1(\mathbf{s}, B^{x\prime}, \iota) - \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\frac{\mathcal{I}(\hat{\mathbf{s}}')}{q^b(\hat{\mathbf{s}}, \hat{B}^x)}\right]\right\| < \epsilon$ and $\sup_{\mathbf{s}, B^{x\prime}, \iota}\left\|\mathcal{E}_2(\mathbf{s}, B^{x\prime}, \iota) - \mathbb{E}\left[\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}')\exp(g')\right]\right\| < \epsilon$ then the conjecture is an competitive equilibrium. If not, update (using some dampening) and start again from step two until convergence.

Given the set of competitive equilibria the second part of the algorithm solves for the government problem, given its time inconsistency problem. Following Bianchi and Mendoza (2013), a solution to the Markov Perfect Equilibrium can be found by solving a fixed point between the expected government policies and the optimal one-period deviation policies that solve government problem (37)-(39) in its de-trended recursive representation.

The algorithm to solve for the Markov Perfect equilibrium follows these steps:

(1) Conjecture expected policies $\left\{\hat{\mathcal{B}}^x(\hat{\mathbf{s}}), \mathcal{I}(\hat{\mathbf{s}})\right\}$ and a price schedule for public debt $q^b(\hat{\mathbf{s}}, \hat{B}^{x\prime})$ for any $\hat{\mathbf{s}}$ in the previously defined state space $\mathcal{S}$.

(2) Solve for the set of competitive equilibria given any possible current government policy and the conjectured expected government policy. This is done using the first part of the algorithm.

(3) Solve for the recursive government problem (37) - (39). The problem is solved using value function iteration. The choice of external debt in the maximization problem is done over a finer grid to improve accuracy.

(4) Compute $q^b(\mathbf{s}, \hat{B}^{x\prime}) = \mathbb{E}\left[\iota(\hat{\mathbf{s}}')|\hat{\mathbf{s}}\right]/R$ using quadrature methods.

(5) If $\sup_{\mathbf{s}}\|\mathcal{X}(\mathbf{s}) - X(\mathbf{s})\| < \epsilon$ for $X = B^x, q^b$ (where $\mathcal{X}$ refers to the expected version of X) then stop. Otherwise update conjectures of expected policies and price of debt (using some dampening parameter) and start from the first step.

[43] To solve for equations (40) and (43) the following numerical approximation is made: $\tilde{\Lambda}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \sim \beta\exp(g)^{1-\gamma}\left(1 - \sigma + \sigma\nu(\hat{\mathbf{s}}')\right)$. The approximation is shown to be accurate along the simulations as the approximation error in $\underline{z}'(\hat{\mathbf{s}})$ is negligible.

Appendix C. Sensitivity Analysis (For Online Publication)

This section analyzes the sensitivity of the main results to certain key parameters in the model. In particular, I consider the effects of different specifications for the degree of tightness of the limited commitment constraint of banks (captured by parameter κ), the discount factor of households (parameter β) and the dispersion of idiosyncratic bank productivities (captured by the shape of the Pareto distribution of idiosyncratic productivities λ). Results are reported in Table 6. Column 1 shows the main summary statistics for the baseline model.

Table 6. Sensitivity Analysis

Statistic	(1) Baseline Model	(2) Tighter Banks LC constraint $\kappa = 4.5$	(3) Higher Discount Factor $\beta = 0.96$	(4) Lower Prod. Dispersion $\lambda = 4$
Δ Output (wrt Baseline)	0.0%	-12.3%	-0.5%	-17.6%
External Public Debt	22.1%	22.4%	21.7%	9.1%
Domestic Public Debt	9.4%	9.0%	9.1%	4.5%
Outptut Cost of Default	-5.36%	-5.26%	-5.50%	-2.62%
No Liquidity Effect				
External Public Debt	13.4%	12.6%	11.3%	5.8%
Outptut Cost of Default	-3.55%	-3.64%	-3.53%	-1.58%
No Balance Sheet Effect				
External Public Debt	7.0%	6.7%	8.6%	6.2%
Outptut Cost of Default	-1.77%	-1.88%	-1.87%	-1.68%
Neither Effect				
External Public Debt	0.9%	0.5%	0.7%	1.7%
Outptut Cost of Default	0.00%	0.00%	0.00%	0.00%

Notes: All statistics are averages from each model's simulations. Δ output is the variation in average output with respect to the baseline model. External and domestic debt are in % of annual GDP. Output cost is the average percentage deviation of output under default with respect to output in absence of default for the following twelve quarters after a default.

Column 2 shows the summary statistics for an alternative specification in which all the parameters of the model are the same as in the baseline case, except for the parameter associated to the banks' limited commitment constraint which is set to $\kappa = 4.5$. This value is in line with that considered in Gertler and Kiyotaki (2010) and Bocola (2014) which study developed economies. A tighter limited commitment constraint for banks has associated a lower level of output. The average level of output in the simulations of this economy is 12% lower than the average level of output in the simulations of the baseline model. Given a tighter limited commitment constraint banks with high productivities can borrow less from banks with low productivities and can demand less labor. This reduces equilibrium wages and attracts banks with lower productivities to invest in their technology. This in turn reduces the level of output since labor is allocated to technologies that are, on average, of lower productivity. The tighter constraint also increases the liquidity value of public debt given that there is less lending in the interbank market and the availability of public debt helps alleviate the inefficiencies introduced by the limited commitment constraint. Having said this, the variation in the tightness of the limited commitment constraint does not significantly change the economic relevance of the balance-sheet and liquidity effect. The level of external public debt in the model with both effects and the the specifications with only one of them do not change significantly with respect to those in the baseline parametrization.

Column 3 reports the results for an alternative specification of the model with a higher discount factor of $\beta = 0.96$ which is closer to the discount factor considered in standard models of business cycles. A higher discount factor reduces the value of issuing external public debt to front-load consumption. However, the average levels of external debt do not change much with respect to the baseline scenario since the government is constrained in how much debt in can issue in both cases by its lack of commitment. The average level of domestic public debt and the output cost of default are similar to those in the baseline model. The relevance of the liquidity effect in generating commitment is higher than in the baseline parametrization. In the model without the balance-sheet effect the average level of external public debt issuance 8.6% compared to 7.7% in the baseline parametrization. The reason is that given the higher discount factor, households value more the output cost of default due to the liquidity effect (that is persistent over time) and this in turns generates more commitment for the government.

Finally, column 4 considers an alternative model parametrization with a lower dispersion in the distribution of idiosyncratic productivity. In particular, I consider the shape of the Pareto distribution of productivities of $\lambda = 4$ which is close to the value used in trade studies for advanced economies (e.g. Melitz and Redding (2014)). A lower dispersion in the idiosyncratic productivity of banks implies that negative shocks to the financial system translate into shocks of smaller magnitude to output. The reason is that idiosyncratic productivities are more concentrated and hence changes in the composition of banks that are using their production technology will not have large effects on the average productivity and hence on output. This implies a sovereign default has a smaller effect on output and thus on the government's commitment. Under this specification the average output cost of default in the 3 years that follow a default is 2.6%, compared to 5.4% in the baseline parametrization. Consequently, the average level of external public debt is 9.1% of GDP, less than half of the levels in the baseline parametrization.