Currency Choice in Contracts*

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Abstract

We study the general equilibrium of an economy in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. The optimal choice of currency trades-off the price risk of each currency with how this risk covaries with the relative consumption needs of the agents signing the contract. When a larger share of private contracts are denominated in local currency, the government can use inflation to redistribute resources more effectively within the economy which, in turn, makes local currency more attractive as a unit of account for private contracts. The use of local currency is more likely when there is low domestic policy risk. Consistent with recent policy initiatives, policies that encourage the denomination of contracts exclusively in local currency can be desirable, since private agents do not internalize the complementarities between private actions and those of the government. We also use our model to explain observed hysteresis of dollarization that occurred in several Latin American countries, and the wide use of the dollar in international trade contracts.

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1 Introduction

One of the central roles of currency is to serve as a unit of account in credit contracts. While in most countries this role is exclusively fulfilled by the local currency, several countries also rely on a foreign currency (for example the dollar) to denominate domestic contracts. The coexistence of multiple currencies in denominating contracts is especially relevant in emerging economies, which are often subject to high levels of government policy risk. In this paper, we address two related questions on the role of currencies as units of account. First, what determines the currency choice of contracts among private agents? Second, how does this collective currency choice affect the government's conduct of monetary policy?

To answer these questions, we study a general equilibrium model in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. These contracts involve the provision of a good in exchange for a future payment in some currency. The optimal choice of currency considers the price risk of each currency and how this risk covaries with the relative consumption needs of the agents signing the contract. The price of the local currency is chosen ex-post by a benevolent government and depends on the use of local currency in private contracts. A key feature of this model is a source of complementarities between the actions of private agents and those of the government. When a larger share of private contracts is denominated in local currency, the government can use inflation to redistribute resources more effectively within the economy which, in turn, makes local currency more attractive as a unit of account for private contracts. Local governments are also subject to exogenous policy risk which reduce the attractiveness of denominating contracts in local currency. We show that the set of equilibria depends crucially on the level of policy risk and multiple equilibria can emerge. We also ask if competitive equilibria are efficient and argue that there might be a role for policy to encourage private agents to denominate contracts exclusively in local currency. This is because private agents do not internalize the complementarities described above. This might help explain recent policy initiatives in many emerging economies aimed at discouraging or prohibiting the use of foreign currency in domestic contracts.

We begin by characterizing the optimal bilateral credit contract. Agents engage in credit contracts to exploit gains from trade of a special good. Credit contracts stipulate the amount of a special good that is provided at the date the contract is signed, in exchange for an amount of local and/or foreign currency to be paid in the future. Currencies serve only as units of account, since the actual payment in the future is made in terms of a numeraire good. Agents have stochastic valuations of the numeraire good which increases the attractiveness of a currency whose price (measured in terms of the numeraire good) covaries with these valuations. We assume that default is costly which implies that a currency with higher price risk lowers the gains from trade. The optimal currency choice features a trade-off between these two forces.

The government's optimal choice of inflation redistributes resources between creditors and debtors in an efficient way. When the debtors have a high valuation (relative to the creditors) the government chooses a higher inflation to lower the burden of debt payments. The degree of redistribution that takes place depends positively on the use of local currency in private contracts. The government's inflation choices are also affected by the degree of policy risk which is independent of the currency choice. The choice of inflation induces a distribution of local currency prices which affect the ex-ante benefits of local currency relative to the foreign one.

We fully characterize the set of equilibria for different levels of policy risk. For low levels of policy risk there is a unique equilibrium in which all contracts are denominated in local currency while for high levels of this risk, all contracts denominated in foreign currency. For intermediate levels of policy risk, there are three equilibria: two which involve exclusive use of either the local or foreign currency and a third interior one in which both the local and foreign currency are used. This characterization helps rationalize why countries with low levels of policy risk like the U.S., Europe, or Japan rely exclusively on their local currency as a unit of account. In contrast, countries with high policy risk such as those in Latin America and Eastern Europe tend to partially or fully rely on foreign currency as a unit of account.

Both recently and historically, many countries have introduced policy initiatives to either encourage or discourage the use of foreign currency as a unit of account. In recent years there have been policy initiatives in a large number of emerging market economies that discourage the use of foreign currency as a unit of account. Two such examples are Brazil and Colombia that currently prohibit the denomination of bank deposits and loans in foreign currency. Other examples of similar initiatives include policies in Croatia, Hungary, Poland which either heavily regulated or forced conversion of foreign currency housing loans to domestic currency.[1] In contrast, two decades ago Ecuador and El Salvador fully dollarized their domestic economies.

Our paper can help explain the prevalence of such policy initiatives. We study the problem of a social planner subject to the same constraints as private agents. We find that the optimal allocation calls for exclusive use of local currency if policy risk is low and exclusive use of foreign currency if policy risk is high. As a result, for regions of the parameter space in which there are multiple equilibria, allocations that involve use of both currencies are dominated by ones in which only one of the currencies is used. In

[1] Another example is Peru which in 2004 prohibited retail price setting in foreign currency.

particular, for relatively low values of policy risk the interior equilibrium is dominated by one in which only the local currency is used while for relatively high values of policy risk the interior equilibrium is dominated by one in which only the foreign currency is used. Moreover, the set of parameters under which the former is true is larger than the latter and this difference increasing in the variance of the process governing the stochastic valuations of the numeraire good.

We then use our model to shed light on the observed hysteresis in the share of foreign currency-denominated contracts. This pattern is most striking in many Latin American economies that still exhibit high levels of financial dollarization in spite of continued success in controlling inflation and inflation risk in the last decade. Figure 2 plots the evolution of deposit dollarization and annual inflation (capped at 100% per annum) for 4 developing countries from 1980 to 2007: Argentina, Bolivia, Peru and Uruguay. All economies went through episodes of rapid increases in the inflation rate, followed by a rapid increase in the fraction of deposits in US dollars. Surprisingly, even though inflation later stabilized, financial dollarization remained high and stable.

To address this empirical pattern, we enhance our baseline model by endowing debtors with claims on local and foreign currency that, as we show, can arise endogenously as a consequence of trading within a credit chain. In this model, currency choice exhibits hysteresis due to the fact that there are benefits of matching the currency of denomination of new debt contracts to the outstanding claims that back the debtors future payments. We illustrate this by showing that even if policy risk is eliminated, in equilibrium, foreign currency (dollars in this case) will still be used a unit of account. The reason is that it is optimal to match the currency of older contracts and only de-dollarize the claims that are backed with future income.

Finally, we extend our model to study currency choice in international trade contracts that involve parties from two different countries. Gopinath (2015) documents that the dollar is widely used as a unit of account in international trade contracts. We extend our model to allow for international trade contracts in which debtors and creditors are from two different economies and contracts can be set in three possible currencies: the currencies of the debtor or creditor country, and a foreign currency (which in this case stands for the dollar). Our model can rationalize the large use of dollars in international contracts relative to domestic contracts. In particular, we find that the range of policy risk for which a full dollar equilibrium exists in the model with international trade contracts is larger than the range that determines the unique full dollar equilibrium in the model with only domestic contracts. The reason is that when contracts involve international parties the ability of each local government to use inflation to redistribute resources efficiently is undermined. While in the model with domestic contracts the government finds it optimal to use inflation to respond to valuation shocks of both creditors and debtors, in the model with international contracts it is not in the interest of the local government to respond to valuation shocks of the agents from the other country.

Related literature. Our paper contributes to the literature that studies the coexistence of currencies in fulfilling the roles of money, and is closely related to the papers that study the use of foreign currency as a unit of account in debt contracts.[2] Ize and Levy-Yeyati (2003) and Rappoport (2009) study models to characterize equilibrium levels of financial dollarization. Other papers study the role of currency denomination of debt in models with financial frictions (see Caballero and Krishnamurthy (2003), Schneider and Tornell (2004) and Bocola and Lorenzoni (2017)).[3] These papers stress the use of both currencies in debt contracts given their differential hedging properties associated with exchange rate fluctuations. In this paper, we build on the framework developed by Doepke and Schneider (2017), who study the general properties of the optimal unit of account, and contribute to this literature in two dimensions. First, we stress the role of coordination in the choice of currency in private contracts and its implications for hysteresis of dollarization and multiplicity of equilibria. Second, in contrast to Doepke and Schneider (2017), in our model prices are determined endogenously and depend on government policy risk. This enables us to study the interaction between government incentives and the distribution of contracts by currency, and assess the efficiency of the competitive equilibrium.

Our paper is also related to a literature the explores the benefits of a common currency/monetary policy to overcome commitment issues by governments. Examples include Chari et al. (2015), Arellano and Heathcote (2010) and Drenik and Perez (2017). While our model also has risky governments who choose policy ex-post, one important difference is that in our model the choice to denominate in a foreign currency might persist even after policy risk has been stabilized.

Finally, our paper contributes to a growing literature on the global role of the dollar (see, for example, Gopinath (2015), Gopinath and Stein (2017) and Chahrour and Valchev (2017)). We contribute to this literature by focusing on the relationship between the use of foreign currency (mostly dollars) and the risk associated to government policy. This cross-country heterogeneity is relevant when explaining the predominance of the dollar and our model is able to rationalize it.

[2]Other strands of the literature have focused on the use of currencies for other purposes. Matsuyama et al. (1993) and Uribe (1997) study the use of a foreign currency as a means of payment. Other papers study the implications of full dollarization (for example, Alesina and Barro (2002), Gale and Vives (2002) and Arellano and Heathcote (2010)) or currency areas (for example, Chari et al. (2015), Aguiar et al. (2015)). A large literature has studied the effects of the currency of denomination of prices. Some examples include Engel (2006) and Gopinath et al. (2010) in the case of international prices and Drenik and Perez (2017) in domestic prices.

[3]Other papers study the optimal choice of currency for corporate debt (see, for example, Aguiar (2005) and Salomao and Varela (2017)) and sovereign debt (see, for example, Ottonello and Perez (2016)).

2 Model

2.1 General Environment

There are two periods $1, 2$. The domestic economy is populated by two types of agents: citizens and a government. Citizens are further divided into sellers and buyers, with a unit measure of each.

Buyers have preferences over a special good produced by sellers. Buyers and sellers also value the consumption of a numeraire good which takes place at the end of period 2. Preferences for the representative seller are given by

$$u_s = -x + \mathbb{E}\left[\theta_s c_s\right] - \mathbb{E}l\left(R_l\right),$$

where x is the special good produced by the seller, c_s is the seller's consumption of the numeraire good, and θ_s measures the valuation of the numeraire good. We denote R_l as the price level in the economy, and $l\left(R_l\right)$ captures inflation costs (we define these in detail below). Preferences for the representative buyer are given by

$$u_b = (1+\lambda)\, x + \mathbb{E}\left[\theta_b c_b\right] - \mathbb{E}l\left(R_l\right)$$

where $1+\lambda$ is the valuation of the special good provided by a seller, c_b is the buyer's consumption of the numeraire good, and θ_b is the buyers valuation of numeraire good. The parameter $\lambda > 0$ governs the gains of trading the special good between sellers and buyers. The parameters θ_s and θ_b are independently drawn at period 2 from a distribution with bounded support $\left[\underline{\theta}, \overline{\theta}\right]$ and $\mathbb{E}\left[\theta_i\right] = 1$ for $i = s, b$. The fact that θ_s and θ_b are unknown at period 1 introduces uncertainty in the relative valuations of the numeraire good and gives rise to gains from making relative consumption state-contingent. A high (low) value of θ_b relative to θ_s makes consumption of buyers, relative to sellers, more (less) desirable. The differences in θ_s and θ_b can capture any reason for why it is socially and privately desirable to shift resources between different agents in the population.

The timing of the model is as follows:

1. In the first period sellers produce a special good for buyers in exchange for the promise of payment in period 2.

2. In the second period random variables θ_s and θ_b are realized, the domestic government it chooses its policy which is the aggregate price level, all signed contracts are executed, and consumption of the numeraire good takes place.

Buyers and sellers are endowed with $y > 0$ units of the numeraire good, respectively. Next, we formally define a contract and discuss its properties.

2.2 Bilateral Contracts

A contract between a buyer and a seller consists of a provision of the special good (from the seller to the buyer) in exchange for the promise of future payment (from the buyer to the seller). We impose three important assumptions on the contracting environment. The first is that payments are non-contingent and in particular, cannot depend on the realization of the state (θ_s, θ_b). The second is that payments cannot be made directly in terms of the numeraire good. Instead, payments can only be made in two possible "units of account", which we will call *currencies*. One interpretation of this assumption is that goods are observable but *unverifiable,* as is commonly assumed in the incomplete contracts literature. We will denote the two possible currencies by l (local) and f (foreign). A payment b_l in currency l yields $b_l R_l$ units of the domestic numeraire good in period 2, while a payment b_f in currency f yields $b_f R_f$ units of the domestic numeraire good in period 2. In general, R_l and R_f are random variables from the perspective of private agents that are unknown at the time of the contract being signed. The third assumption is that we assume sufficiently high default costs so that contracts must be *default-free*. In other words, promised payments must be less than or equal to the endowment in each state of the world.

Formally, a bilateral contract signed in sub-period i is a the tuple (x, b_l, b_f), where x indicates the units of special good provided to the buyer and (b_l, b_f) are the units of local and foreign currency promised to be paid to the seller at date 2, respectively. In order to ensure repayment in every state, contracts must satisfy the following condition

$$b_l R_l + b_f R_f \leqslant y \quad \text{for all} \quad R_l, R_f \tag{1}$$

where $\mathbf{R} \subset \mathbb{R}_+^2$ is the compact set of possible price realizations. This inequality states that for all possible price realizations, the income of the buyer should not exceed the promised repayment. Notice also that we implicitly restrict consumption of the numeraire good in period 2 to be non-negative. Agents are exposed to risk from uncertainty about aggregate prices. We adopt the notation convention that R_c is the price of a unit of currency c in terms of the numeraire good of the domestic economy. Therefore, a low (high) R_c indicates a high (low) level of domestic inflation in currency c. Prices in local currency R_l in this economy are endogenous and citizens take them as given. Prices in foreign currency R_f in this economy are exogenous, stochastic with support support $\left[\underline{R}_f, \overline{R}_f\right]$, and independent from the other random variables. We associate the foreign currency with stable currencies like the dollar or the euro, and interpret the risk in R_f as real exchange rate risk.

Without loss of generality, we assume that in each bilateral meeting the buyer makes a take-it-or-leave-it offer to the seller. The seller is willing to participate in the contract as long as

$$-x + \mathbb{E}\left[\theta_s \left(b_l R_l + b_f R_f\right)\right] \geqslant 0 \tag{2}$$

where we normalize the seller's outside option to zero. The optimal contract for the buyer solves

$$\max_{x, b_l, b_f} (1+\lambda)x - \mathbb{E}\left[\theta_b \left(b_l R_l + b_f R_f\right)\right] \tag{3}$$

subject to (1), (2), and non-negativity constraints $b_l, b_f \geqslant 0$.

2.3 Competitive Equilibrium given Government Policy

Local and foreign currency constitute two different units of account that have price risk relative to the domestic numeraire good. The local currency is subject to endogenous inflation risk associated with government policy. From the perspective of citizens, the price level R_l is a random variable with cdf $G(R_l)$ and support $\left[\underline{R}_l, \overline{R}_l\right]$. In the general equilibrium, the shape of $G(R_l)$, and the bounds of the support, $\underline{R}_l$ and $\overline{R}_l$, are endogenous and depend on the choices of the government.

Given the problem defined in the previous section, we can now characterize the optimal bilateral contract between a seller and buyer, taking the distribution of R_l and R_f as given.

Proposition 1. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s \left(b_l R_l + b_f R_f\right)\right]$, *while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right) \frac{R_l}{\overline{R}_l}\right] < \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right) \frac{R_f}{\overline{R}_f}\right]$ *then* $b_l = 0$ *and* $b_f = \frac{y}{\overline{R}_f}$
2. *If* $\mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right) \frac{R_l}{\overline{R}_l}\right] = \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right) \frac{R_f}{\overline{R}_f}\right]$ *then* $b_l = \gamma \frac{y}{\overline{R}_l}$ *and* $b_f = (1-\gamma) \frac{y}{\overline{R}_f}$ *for any* $\gamma \in [0, 1]$.
3. *If* $\mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right) \frac{R_l}{\overline{R}_l}\right] > \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right) \frac{R_f}{\overline{R}_f}\right]$ *then* $b_l = \frac{y}{\overline{R}_l}$ *and* $b_f = 0$.

All proofs are included in the Appendix. First notice that since preferences are linear and $\lambda > 0$, there are positive gains from trading as much of good x as possible. The limit on how much x can be traded is given by the fact that buyers need to be able to pay for that good in the following period. This implies that the no-default constraint ((1)) will always be binding. Additionally, the state for which this constraint will bind is the one in which inflation in both currencies are at their lowest possible realizations (i.e., $R_l = \overline{R}_l$

and $R_f = \overline{R}_f$). If we substitute the participation and no-default constraint into the objective and take derivatives with respect to B_l we obtain

$$\underbrace{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right]}_{\text{Marginal benefit of local currency}(M_l)} \quad - \quad \underbrace{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]}_{\text{Marginal benefit of foreign currency}(M_f)}$$

The expression above represents the difference between the marginal benefit of setting the contract in local currency (first term) and the marginal benefit of setting it in foreign currency (second term). Since the objective is linear, these objects are constant and independent of the choice of b_l. The optimal contract calls for using the currency that has the largest marginal benefit. When the marginal benefit is the same in both currencies, any combination of local and foreign currency is optimal. Using the assumption that θ_s and θ_s have equal means we can rewrite the marginal benefit of currency c as

$$M_c \equiv \lambda\frac{\mathbb{E}\left[R_c\right]}{\bar{R}_c} + \text{cov}\left(\left(\theta_s\left(1+\lambda\right)-\theta_b\right), \frac{R_c}{\bar{R}_c}\right) \tag{4}$$

for $c = l, f$. The marginal benefit of each currency has two components: a price risk term and a covariance term. The ratio $\frac{\mathbb{E}[R_c]}{\bar{R}_c}$ denotes the price risk of denominating contracts in currency c. A higher (lower) value of $\frac{\mathbb{E}[R_c]}{\bar{R}_c}$ represents a lower (higher) risk indexing contracts in currency c. Note that it is the maximal values of R_c that determines price risk due the assumption that contracts must be default-free. The second term is the covariance of relative valuations and currency prices. The marginal benefit of denominating in the foreign currency is exogenous and only given by the price risk term since the covariance term is zero given our assumption of independence between R_f and the valuations θ_b, θ_s. Suppose first that θ_b, θ_s are deterministic. Then the optimal currency choice is determined exclusively by comparing the price risk in both currencies, $\frac{\mathbb{E}[R_l]}{\bar{R}_l} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}$. In this case choosing the currency with the lowest price risk maximizes the gains from trade. In contrast suppose that valuations are stochastic. Now the optimal currency choice also depends on the covariance between prices in local currency and the marginal utility (valuation shocks). For example, from the seller's perspective if R_l is high in states in which it values consumption more (high θ_s) denominating in local currency will be more attractive. The opposite is true for the buyer. As we will see in the next section a benevolent government will choose R_l so that this covariance term is positive. Finally, note that the optimal choice of x can be computed directly from the participation constraint (2).

2.4 Government

We consider a utilitarian government that controls monetary policy and chooses the price level of the domestic economy R_l in the second period to maximize the sum of the utilities of buyers and sellers. As mentioned earlier private agents also suffer inflation losses captured by $l(R_l)$. We assume that $l(R_l) = \frac{\psi}{2}\left(R_l - R^{\dagger}\right)^2$ where $R^{\dagger}$ denotes the price level target for the government. $R^{\dagger}$ is a random variable realized in period 2 and thus is stochastic at the time at which contracts are signed. We assume that $R^{\dagger}$ has bounded support $\left[\underline{R}^{\dagger}, \overline{R}^{\dagger}\right]$. Similar to our definitions of price risk, we will refer to $\frac{\mathbb{E}[R^{\dagger}]}{\overline{R}^{\dagger}}$ as policy risk. This is meant to capture all other sources of uncertainty in monetary policy that are unrelated to the economic developments in the domestic economy. The problem of the government is given by

$$\max_{R_l}\left[\theta_b C_b + \theta_s C_s\right] - 2l\left(R_l - R^{\dagger}\right)$$

where

$$C_b = y - R_l B_l - R_f B_f \tag{5}$$

is the average consumption of the buyer, B_l, B_f are the average levels of contracts denominated in local and foreign currency respectively, and

$$C_s = y + R_l B_l + R_f B_f \tag{6}$$

is the average consumption of the seller. Given the functional form for $l(\cdot)$, the solution to this problem is

$$R_l = R^{\dagger} + \frac{1}{2\psi}\left(\theta_s - \theta_b\right) B_l \tag{7}$$

The optimal choice of inflation redistributes resources between sellers and buyers in an efficient way. When the buyers have a high valuation (relative to the sellers) the government chooses a higher inflation (lower R_l) to lower the burden of debt payments by the buyer and redistribute resources from buyers to sellers. The opposite occurs when the sellers have a high valuation relative to buyers. The level of redistribution depends positively on the use of local currency in private contracts B_l.

The government's choices of inflation also affect the marginal benefit of local currency (M_l) (defined in equation (4)) in the first period. On the one hand, the redistribution that the government attains using monetary policy induces a positive covariance between relative valuations and prices in local currency, thereby increasing the marginal benefit of local currency. The higher the use of local currency B_l, the higher the endogenous positive covariance for local currency. On the other hand, by reacting to valuation shocks the government also affects the price risk of local currency. Recall that we defined price risk of local currency as the ratio $\frac{\mathbb{E}[R_l]}{\bar{R}_l}$. Given the optimal choice of R_l, we have that $\mathbb{E}[R_l] = \mathbb{E}\left[R^\dagger\right]$ and the maximal value of R_l is given by

$$\bar{R}_l = \bar{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l. \tag{8}$$

The higher the use of local currency B_l, the higher $\overline{R}_l$ and the lower $\frac{\mathbb{E}[R_l]}{\bar{R}_l}$ (or the higher the price risk of local currency). Throughout our baseline analysis we make the following parametric assumption.

Assumption 1. *Assume that*

$$\frac{var(\theta)}{\left(\overline{\theta} - \underline{\theta}\right)} > 1.$$

Denote $M_l(B_l)$ as the marginal benefit of denominating in local currency (defined in equation (4)), once we substitute in the optimal choice of R_l by the government. This assumption guarantees that $M_l(B_l)$ is increasing in B_l. In particular, it guarantees that the positive effect of higher B_l on the covariance more than offsets the effect of higher B_l on higher price risk of local currency. Therefore, under this assumption, the benefit of denominating contracts in local currency is increasing in B_l, thus generating complementarities in denomination choices. .

Given this we can now define a competitive equilibrium for this economy.

Definition 1. A competitive equilibrium is an allocation for private citizens (x, B_l, B_f) and an inflation choice for the government R_l such that, given R_l the allocations solve contracting problem defined in (3), and given B_l, R_l satisfies (7).

2.5 Equilibrium Characterization

We now provide a characterization of the set of competitive equilibria. The main objective of this exercise is to understand how the set of equilibria changes as we vary the level of policy risk $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$. As we will show, for low levels of risk, there is a unique equilibrium in which all contracts are denominated in local currency. For intermediate levels of this risk there are three equilibria: two in which all contracts are completely denominated in either local or foreign currency and an interior equilibrium. Finally, for high enough levels of policy risk there is a unique equilibrium in which all contracts are denominated in the foreign currency.

To vary policy risk, we fix $\overline{R}^\dagger$ and vary $\mathbb{E}\left[R^\dagger\right]$. In particular, a higher value of $\mathbb{E}\left[R^\dagger\right]$ denotes a lower level of policy risk. The set of equilibria is characterized in the following proposition.

Proposition 2. *There exist thresholds* $\mu_1 = \frac{\mathbb{E}[R_f]}{\bar{R}_f}$ *and* $\mu_2 < \mu_1$ *such that:*

1. *If* $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} > \mu_1$ *there exists a unique equilibrium in which* $B_l = \frac{y}{\bar{R}^*}$ *where* $\bar{R}^*$ *is the solution to*

$$\bar{R}_l^* = \bar{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right)\frac{y}{\bar{R}_l^*}.$$

2. *If* $\mu_2 < \frac{\mathbb{E}[R^\dagger]}{R^\dagger} \leqslant \mu_1$ *there exist three equilibria with* $B_l = \frac{y}{\bar{R}^*}$*,*$B_l = 0$ *and* $B_l \in \left(0, \frac{y}{\bar{R}^*}\right)$ *respectively.*

3. *If* $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} \leqslant \mu_2$ *there exists a unique equilibrium in which* $B_l = 0$*.*

The thresholds μ_1 and μ_2 depend on parameters and are defined in the appendix. The figure below presents a graphical depiction of the set of equilibria. The blue line is the competitive equilibrium for a given government policy and thus, for a given M_l. When $M_l > M_f$ private agents denominate in local currency and when $M_l < M_f$ they denominate in foreign currency. The red lines depict the marginal benefit of local currency as a function of B_l for different values of policy risk. All lines are increasing since our assumption implies $M_l(B_l)$ is increasing. To understand the role of policy risk in the determination of equilibria it is useful to analyze how policy risk affects the marginal benefit of local currency. Note that when there are no contracts in local currency, the marginal benefit of local currency is given by policy risk, i.e., $M_l(0) = \lambda\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$. As we increase price risk (decrease the ratio) the marginal benefit of local currency decreases for all possible values of B_l. When the policy risk is lower than price risk of foreign currency (case 1) the only equilibrium is full use of local currency, shown at the intersection of the red and blue solid lines. This is because even when no contracts are set in local currency it is still worthwhile to denominate contracts in local currency due to lower price risk. As more contracts are signed the attractiveness of local currency increases as the government endogenously uses inflation to redistribute resources more effectively.

When the policy risk is intermediate (case 2) we have multiple equilibria. Multiplicity arises due to the complementarities between the private and government actions. As more contracts are set in local currency the government uses inflation to provide more insurance through better redistribution. One of the equilibria involves full use of foreign currency. If all private contracts are set in foreign currency, there are no incentives for the government to use inflation in order to redistribute. Therefore the marginal benefit of local currency is only given by policy risk which in this region is higher than the price risk of foreign currency. Another equilibrium involves full use of local currency. If all private contracts are denominated in local currency the government is incentivized to use inflation to redistribute efficiently, and this makes local currency more attractive than

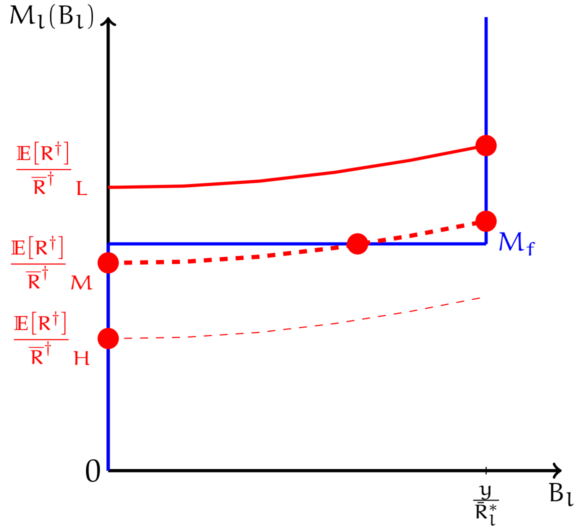

Figure 1: Characterization of Competitive Equilibrium

foreign currency. Finally, there is a third interior equilibrium at which the level of B_l is such that the marginal benefit of local and foreign currency are equal. In the Figure the three equilibria correspond to the three intersections of the blue and the middle red dashed line.

When the policy risk is high enough (case 3) the unique equilibrium involves full use of foreign currency. This equilibrium exists since the marginal benefit of local currency is policy risk when all contracts are set in foreign currency, and policy risk is larger than price risk of foreign currency. The equilibrium is unique since even if all contracts are set in local currency, the government's use of inflation to redistribute does not compensate for the high levels of policy risk. In the Figure, this case corresponds to the intersection of the bottom dashed red line with the blue line.

3 Constrained Efficiency

We now consider the problem of a social planner who chooses the allocation and inflation rate and is subject to the same constraints as private agents. The utilitarian social planner solves

$$\max \mathbb{E}\left(-x + C_s + (1+\lambda)\,x + C_b - 2\psi l\left(R_l - R^{\dagger}\right)\right)$$

subject to the definitions of C_b and C_s in (5) and (6) respectively, the participation constraint for the seller, (2), the no-default constraint (1), and the best reposes of the government (7), and (8). Note that we assumed that the participation constraint for the buyer is slack and we will check that it is satisfied ex-post.

Analogous the competitive equilibrium, we now characterize the solution to the planner's problem given different values of policy risk.

Proposition 3. *There exists a threshold* μ^{sp} *with* $\mu_2 < \mu^{sp} < \mu_1$

1. *If* $\frac{\mathbb{E}[R^{\dagger}]}{R^{\dagger}} \geqslant \mu^{sp}$ *the solution to the Social Planner's problem is* $B_l^{sp} = \frac{y}{\bar{R}^*}$
2. *If* $\frac{\mathbb{E}[R^{\dagger}]}{R^{\dagger}} \leqslant \mu^{sp}$ *the solution to the Social Planner's problem is* $B_l^{sp} = 0$

This result illustrates that an interior equilibrium can never be efficient. In particular, for policy risk in (μ_{sp}, μ_1), the full local currency equilibrium dominates the interior and full foreign currency equilibrium while for policy risk in (μ_2, μ_{sp}) the full foreign currency dominates the other two equilibria. In contrast, if policy risk is either very low or very high, the unique competitive equilibrium (full local in the former, full foreign in the latter) is constrained efficient.

The proof follows from the observation that Assumption 1 implies that the Social Planner's problem is strictly convex. As a result computing the solution of this problem involves comparing end points. The relative value of the end-points depends on whether policy risk is high or low. Intuitively, low policy risk increases the value of the full local currency equilibrium relative to the full foreign currency one while high policy risk does the opposite.

The combination of the equilibrium characterization and the above result help rationalize some of the policies described in the introduction. Consider a country with very low policy risk. The model predicts that contracts signed within the country will be denominated in local currency and it is efficient to do so. For slightly higher levels of policy risk, equilibria in which contracts are denominated in foreign currency exist but these are inefficient. Optimal policy would prescribe limits on how much contracts should be denominated in the foreign currency. This might help explain the prevalence of policies in a variety of Latin American countries, including Brazil, Colombia and Peru, that call for forced de-dollarization of contracts.[4]In contrast, for high enough levels of policy risk policy would encourage and incentivize the use of foreign currency. Examples of these types of policies are the forced dollarization adopted by Ecuador in the year 2000.

It is worth comparing the relative sizes of the intervals (μ_{sp}, μ_1) and (μ_2, μ_{sp}). Given the definitions of these thresholds, it is easy to show that $\mu_{sp} - \mu_1 > \mu_2 - \mu_{sp}$. This implies that within the range of policy risk for which the economy is susceptible to multiple equilibria, the full use of local currency is the most efficient outcome for a wider part of that range of policy risk. This asymmetry is due to the presence of the complementarities between actions of privates and the government, through which a larger use of

[4]In the case of Brazil and Colombia there are restrictions to denominating bank deposits or loans in foreign currency. In the case of Peru, the restrictions are less severe. While bank deposits and loans can be denominated in foreign currency, the government prohibits retail price setting in foreign currency.

local currency in private contracts incentivizes the government to use inflation to redistribute resources efficiently and thus, makes local currency more attractive as a unit of account. Moreover, the relative size of these intervals is increasing in the variance of θ. This suggests that if there is a large need for discretion in monetary policy, for a large range of policy risk, optimal policy would prescribe a move away from foreign currency. As discussed earlier, such policies have been enacted in large number of countries.

4 Hysteresis

As discussed in the introduction, a distinctive feature among many Latin American countries is the hysteresis of dollarization even after inflation risk stabilized. The model presented above suggests that the set of equilibria can change dramatically for small changes in policy risk around the threshold which might seem to be at odds with this observation. However, the above analysis ignores the fact that citizens might be part of credit chains and thus might also have endowments of obligations in both currencies. Here, we present a simple extension in which the buyer is endowed with claims $\left(\hat{b}_f, \hat{b}_l\right)$ payable to the buyer in the second period. In the appendix we present a model of a credit chain in which these endowments arise endogenously a consequence of trading within the chain. The optimal contract solves

$$\max_{b_l, b_f} (1+\lambda)x - \mathbb{E}\theta_b\left(R_l\left(b_l - \hat{b}_l\right) + R_f\left(b_f - \hat{b}_f\right)\right)$$

subject to (2) and the no-default constraint

$$R_l\left(b_l - \hat{b}_l\right) + R_f\left(b_f - \hat{b}_f\right) \leqslant y, \forall (R_l, R_f)$$

Assumption 2. *Assume that*

$$\frac{1}{2\psi}\left(var(\theta) - \left(\overline{\theta} - \underline{\theta}\right)\frac{\mathbb{E}[R_f]}{\underline{R}_f}\right)\frac{y}{\bar{R}_l^*} < \overline{R}^\dagger\left(\frac{\mathbb{E}[R_f]}{\underline{R}_f} - 1\right)$$

This assumption requires an upper bound on the variance of θ. Note that this bound contains a free parameter $\underline{R}_f$, and which can be made arbitrarily small in order to satisfy this and Assumption 1.

Proposition 4. *Under Assumption 2,* $b_f \geqslant \hat{b}_f$ *and* $b_l \geqslant \hat{b}_f$.

The proposition says that even if policy risk is small, the optimal contract will still use a combination of foreign and local currency to denominate contracts. In particular, the optimal contract will feature currency matching of stocks but the flows will denominated in the currency with the largest marginal benefit. To illustrate this result, suppose that $\text{var}(\theta) = 0$. Then, we know from earlier that the optimal currency choice only involves comparing price risk. Notice that with existing obligations, the price level that makes the no-default constraint bind will now depend on when $b_i \leqslant \hat{b}_i$ or not. In the former, the relevant price is $\underline{R}_i$ while in the latter it is $\bar{R}_i$. The difference in price risk is

$$\frac{\mathbb{E}[R_l]}{\tilde{R}_l} - \frac{\mathbb{E}[R_f]}{\tilde{R}_f}$$

where $\tilde{R}_i \in \{\underline{R}_i, \bar{R}_i\}$. Suppose that $b_f < \hat{b}_f$. Then the difference in price risk is

$$\frac{\mathbb{E}[R_l]}{\bar{R}_l} - \frac{\mathbb{E}[R_f]}{\underline{R}_f} < 0$$

which implies that $b_f < \hat{b}_f$ can never be part of an equilibrium contract. A similar argument holds for the local currency. This suggests that currency is mismatch is costly and tightens the no default constraint. As a result, the optimal contract currency matches stocks and prices the flows in single currency. The argument in the appendix shows that if the above argument goes through as long as the variance of θ is not too large. If the variance was very large then it might be optimal to cannibalize the stocks of foreign currency.

This analysis can shed light into the observed de-dollarization process of several Latin American countries. These countries experienced high levels of policy risk (measured for example, by average levels of inflation and inflation volatility) and high levels of dollarization of private contracts during the decade of 1990s. In early 2000s policy risk significantly subsided, yet the levels of dollarization of credit and deposits only decreased mildly. Our model would predict this behavior since it is optimal to currency-match previously accumulated contracts when these are used to back newly issued debt.

5 Contracts in International Trade

One of the facts mentioned in the introduction is that there is an extensive use of the dollar as a unit of account in international trade contracts. Here, we extend the model to include a third country in order to show how our theory can rationalize this fact. More specifically, the objective is to show that contracting with parties located in other countries, as opposed to signing contracts with domestic counter-parties, makes the use of a third, external currency, more likely.

We analyze a model in which buyers living in country b sign contracts with sellers living in country s. A contract between a buyer and a seller consists of a provision of the special good in exchange for the promise of future payment. Relative to the baseline version of the model, here we allow payments to be made in three possible "units of account": currencies from country b and s, and the foreign currency f. Despite the fact that agents take prices as given, the realizations of prices in countries b and s (denoted by R_b and R_s) are endogenously determined by governments' choices and the price of foreign currency still remains exogenous.

Given the introduction of a third unit of account, the optimal private contract now solves

$$\max_{x, b_b, b_s, b_f} (1+\lambda)\, x - \mathbb{E}\theta_b \left(R_b b_b + R_s b_s + R_f b_f\right)$$

subject to the participation constraint

$$-x + \mathbb{E}\theta_s \left(R_b b_b + R_s b_s + R_f b_f\right) \geqslant 0,$$

the no-default constraint

$$\bar{R}_b B_b + \bar{R}_s B_s + \bar{R}_f B_f \leqslant y, \tag{9}$$

and the non-negativity constraints $b_b, b_s, b_f \geqslant 0$. The solution to this problem can be summarized in the following proposition:

Proposition 5. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s \left(b_b R_b + b_s R_s + b_f R_f\right)\right]$, *while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_i}{\bar{R}_i}\right)\right] > \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_j}{\bar{R}_j}\right)\right] \geqslant \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_k}{\bar{R}_k}\right)\right]$ *then* $b_i = \frac{y}{\bar{R}_i}, b_j = 0, b_k = 0$

2. *If* $\mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_i}{\bar{R}_i}\right)\right] = \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_j}{\bar{R}_j}\right)\right] > \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_k}{\bar{R}_k}\right)\right]$ *then* $b_i = \gamma \frac{y}{\bar{R}_j}$, $b_j = (1-\gamma)\frac{y}{\bar{R}_j}$ *and* $b_k = 0$ *for any* $\gamma \in [0, 1]$.

3. *If* $\mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_i}{\bar{R}_i}\right)\right] = \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_j}{\bar{R}_j}\right)\right] = \mathbb{E}\left[\left(\theta_s (1+\lambda) - \theta_b\right)\left(\frac{R_k}{\bar{R}_k}\right)\right]$ *then* $0 \leqslant b_i \leqslant \frac{y}{\bar{R}_j}$, $0 \leqslant b_j \leqslant \frac{y}{\bar{R}_j}$ *and* $0 \leqslant b_k \leqslant \frac{y}{\bar{R}_k}$ *such that* $\bar{R}_i B_i + \bar{R}_j B_j + \bar{R}_k B_k = y$

The intuition behind this results is the same as in Proposition (1). Taking prices as given, agents write contracts using the currency that has the largest marginal benefit, allowing for combinations of two or three currencies whenever the buyer is indifferent.

Next, we revisit the government's problem. There are two utilitarian governments that control monetary policy and choose the price level of the b and s economies. The only departure we introduce relative to the baseline model, is that each government cares only about the welfare of its citizens. In all other aspects, such as the levels of policy risk in each country $\frac{\mathbb{E}\left[R_b^\dagger\right]}{\bar{R}_b^\dagger}$ and $\frac{\mathbb{E}\left[R_s^\dagger\right]}{\bar{R}_s^\dagger}$, we assume that governments are symmetric. Thus, the problem of the government in country $i \in \{b, s\}$ is given by

Lets consider the government's problem in b

$$\max_{R_b} [\theta_i C_i] - 2 \cdot \psi l \left(R_i - R_i^\dagger \right),$$

where

$$C_i = y - R_b B_b - R_s B_s - R_f B_f \tag{10}$$

is the average consumption of agent i, B_b, B_s, B_f are the average levels of contracts denominated in country's b and s currency, and foreign currency, respectively. The solution to this problem of government of country b is

$$R_b = R_b^\dagger - \frac{1}{2\psi} \theta_b B_b, \tag{11}$$

with the largest feasible price level the government can implement being given by

$$\bar{R}_b = \bar{R}_b^\dagger - \frac{1}{2\psi} \underline{\theta} B_b.$$

The solution to this problem of government of country s is

$$R_s = R_s^\dagger + \frac{1}{2\psi} \theta_s B_s, \tag{12}$$

with the largest feasible price level the government can implement being given by

$$\bar{R}_s = \bar{R_s}^\dagger + \frac{1}{2\psi} \bar{\theta} B_s.$$

The reason why these expressions differ is that the government of country b cares only about the welfare of buyers, whereas the government of country s cares only about the welfare of sellers. Therefore, ex-post the government in b wants to increase the welfare of buyers by decreasing the real burden of debt denominated in currency b, which can be achieved by deviating from the target $R_b^\dagger$ and implementing a *lower* price of a unit of currency b in terms of the numeraire good. Similarly, ex-post the government in s increases the welfare of sellers by deviating from the target $R_s^\dagger$ and implementing a *higher* price of a unit of currency s in terms of the numeraire good. Given these results, we can now define a competitive equilibrium for this economy.

Definition 2. A competitive equilibrium is an allocation for private citizens (x, B_b, B_s, B_f) and an inflation choice for governments R_b and R_s such that, given R_b and R_s, the allocations solve contracting problem defined in (9), and given B_b and B_s, R_b and R_s satisfy (11) and (12).

We make the following assumption throughout this subsection.

Assumption 3. *Assume that* $\overline{\theta}$ *is such that*

$$\underline{R}_b^\dagger - \frac{2}{\psi}\overline{\theta}\frac{y}{\bar{R_b}^\dagger} > 0,$$

where $\overline{R}_b^* = R_b^\dagger + \frac{1}{2\psi}\left(-\theta_b\right)\frac{y}{\bar{R}_b^*}$*, and* $var(\theta) > \bar{\nu}$.

The first part of the assumption simply guarantees that prices are well defined (i.e., prices cannot be negative and $\bar{R}_b^*$ is real). The second part is a sufficient condition to prove our next result and requires that agents' valuations of the numeraire good are volatile enough (the lower bound of $\bar{\nu}$ is defined in the Appendix). It imposes a lower bound on the volatility of agents' marginal valuation of the numeraire good, which might be more stringent than Assumption 1.

Proposition 6. *There exist a threshold* μ_2^I *such that, if* $\frac{\mathbb{E}\left[R_b^\dagger\right]}{\bar{R}_b^\dagger} = \frac{\mathbb{E}\left[R_s^\dagger\right]}{\bar{R}_s^\dagger} \leqslant \mu_2^I$ *there exists a unique equilibrium in which* $B_b = B_s = 0$*. Furthermore,* $\mu_2^I > \mu_2$.

The thresholds μ_2^I depends on parameters and is defined in the appendix. Similar to what we found in the baseline model, there exists a threshold μ_2^I such that if policy risk in country b and s is high enough, the unique equilibrium displays the use of the foreign currency as the sole unit of account. However, the most important result of this proposition is that $\mu_2^I > \mu_2$, that is, the threshold obtained in the three country model is larger than the one found in the baseline model. This implies that for levels of policy risk such that $\mu_2^I > \frac{\mathbb{E}\left[R_b^\dagger\right]}{\bar{R}_b^\dagger} = \frac{\mathbb{E}\left[R_s^\dagger\right]}{\bar{R}_s^\dagger} > \mu_2$, the introduction of a foreign counter-party to buyers in country b renders the equilibrium with $B_f = \frac{y}{\bar{R}_f}$ unique. More broadly, equilibria with contracts signed in foreign currency only are more likely to occur in a model with international trade. The intuition behind this result is that, as each country's government is concerned about the welfare of its citizens only, when contracts involve international parties the ability of each local government to use inflation to redistribute resources efficiently is undermined. While in the model with domestic contracts the government finds it optimal to use inflation to respond to valuation shocks of both buyers and sellers, in the model with international contracts it is not in the interest of the local government to respond to valuation shocks of agents from the other country.

6 Conclusion

This paper develops a framework to study the optimal choice of currency in the denomination of private credit contracts in general equilibrium. A key feature of the studied economy is a source of complementarities between the actions of private agents and those of the government. When more private contracts are denominated in local currency, the government has more incentives to use inflation to redistribute resources efficiently within the economy which, in turn, makes local currency even more attractive as a unit of account for private contracts. We argue that the degree of policy risk determines the type of equilibria that emerge. For low policy risk, the unique equilibrium is with full use of local currency, whereas for high levels of policy risk, the unique equilibrium is with full use of foreign currency. For intermediate levels of policy risk both equilibria co-exist with an additional interior equilibrium. Our constrained efficiency analysis argues that, for the majority of the parameter space in which the economy is vulnerable to multiple equilibria, the efficient outcome involves exclusive use of local currency. This result can help rationalize various policy initiatives aimed at de-dollarizing domestic economies. Finally, we use our model to shed light on the observed hysteresis in the dollarization of contracts, and also to analyze the case of international debt contracts, in which dollars are even more prevalent.

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A Omitted Proofs

Proof of Proposition 2

The following definitions will be useful for this proof. Define,

$$\mathcal{H}(B) \equiv (1+\lambda)\, M_2(B) - M_1(B)$$

where

$$\begin{aligned} M_2(B) &\equiv \mathbb{E}\left[\theta_s\left(R_l(B) - \frac{R_f}{\bar{R}_f}\bar{R}_l(B)\right)\right] \\ &= \bar{R}^{\dagger}\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) B_l \end{aligned}$$

and

$$\begin{aligned} M_1(B) &\equiv \mathbb{E}\left[\theta_b\left(R_l(B) - \frac{R_f}{\bar{R}_f}\bar{R}_l(B)\right)\right] \\ &= \bar{R}^{\dagger}\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) - \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) B_l \end{aligned}$$

where we have used the best response of the government

$$R_l(B) = R^{\dagger} + \frac{1}{2\psi}\left(\theta_b - \theta_s\right) B_l$$

$$\tilde{y} \equiv \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} y$$

It will also be useful to compute

$$M_1'(B) = -\frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

and

$$M_2'(B) = \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

Notice that the function $\mathcal{H}(B)$ is useful for characterizing the set of equilibria in this model. There are three types of equilibria that can exist. First, an equilibrium with $B_l = 0$ exists iff $\mathcal{H}(B) \leqslant 0$. Next, an equilibrium in which $B_f = 0$ can exist iff $\mathcal{H}\left(\frac{y}{\bar{R}^*}\right) \geqslant 0$, where $\frac{y}{\bar{R}^*}$ corresponds to the maximal feasible value of B_l and $\bar{R}_l^*$ solves

$$\bar{R}_l^* = \bar{R}^{\dagger} + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right)\frac{y}{\bar{R}_l^*}$$

or

$$\bar{R}_l^* = \frac{\bar{R}^\dagger + \sqrt{\left(\bar{R}^\dagger\right)^2 + 2\frac{y}{\psi}\left(\overline{\theta} - \underline{\theta}\right)}}{2}$$

Finally, an interior equilibrium exists iff there exists some $B_l \in \left(0, \frac{y}{\bar{R}^*}\right)$ such that $\mathcal{H}(B_l) = 0$.

Define $\mu_1 \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f}$. We will show that if $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f} > 0$, then there is a unique equilibrium in which $B_l = \frac{y}{\bar{R}^*}$.We need to show that $\mathcal{H}\left(\frac{y}{\bar{R}_l^*}\right) \geqslant 0$. Notice that under Assumption 1, $M_2(B) > 0$. Therefore, for any B.

$$\mathcal{H}(B) \geqslant M_2(B) - M_1(B) = \frac{\text{var}(\theta)}{\psi} > 0$$

and in particular, $\mathcal{H}\left(\frac{y}{\bar{R}_l^*}\right) > 0$. Moreover, given our characterization of equilibria above, this also implies that we have a unique equilibrium.

Next, define

$$\mu_2 \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{1}{2\psi}\frac{y}{\bar{R}^\dagger \bar{R}^*}\left(\frac{(2+\lambda)}{\lambda}\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

Notice that $\mu_2 < \mu_1$. We show that for $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \in (\mu_2, \mu_1]$, there exist three equilibria. First, we show an equilibrium exists in which $B_l = 0$. We know from above that for this equilibrium to exist it must be that $\mathcal{H}(0) \leqslant 0$. Using the expressions, we derived earlier

$$\mathcal{H}(0) = \lambda \bar{R}^\dagger \left[\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right] \leqslant 0$$

which is implied by our assumption. Next, we want to show that there exists an interior equilibrium, i.e. there exists a B such that $\mathcal{H}(B) = 0$ or

$$(1+\lambda) = \frac{M_1(B)}{M_2(B)}$$

Using our definitions, we have

$$\frac{M_1(B)}{M_2(B)} = \frac{\bar{R}^\dagger\left(\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) - \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)B_l}{\bar{R}^\dagger\left(\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)B_l}$$

We know this equal to 1 at $B = 0$. Lets consider the slope which has the sign

$$\begin{aligned}
& M_2(B)\, M_1'(B) - M_1(B)\, M_2'(B) \\
=& \left[\bar{R}^\dagger \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) B_l\right] \left(-\frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)\right) \\
& - \left[\bar{R}^\dagger \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) - \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) B_l\right] \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) \\
=& -\bar{R}^\dagger \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) \frac{1}{\psi} \text{var}(\theta) \\
>& 0
\end{aligned}$$

Therefore, since the ratio equals one at $B_l = 0$ and in strictly increasing, there is a unique solution at some B^*. B^* solves

$$B^* = \frac{2\psi\lambda\bar{R}^\dagger \left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}\right)}{\left((2+\lambda)\,\text{var}(\theta) - \lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)}$$

For this to be strictly interior a necessary and sufficient condition is

$$B^* < \frac{y}{\bar{R}^*}$$

or

$$\frac{2\psi\lambda\bar{R}^\dagger \left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}\right)}{\left((2+\lambda)\,var(\theta) - \lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)} < \frac{y}{\bar{R}^*}$$

or

$$\bar{R}^\dagger \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\frac{y}{\bar{R}^*}\left(\frac{(2+\lambda)}{\lambda} var(\theta) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) > 0$$

or

$$\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} > \mu_2$$

Finally, given monotonically of the ratio $\frac{M_1}{M_2}$, it follows that if there is a interior there is a full peso equilibrium, since it must be that

$$\frac{M_1\left(\frac{y}{\bar{R}^*}\right)}{M_2\left(\frac{y}{\bar{R}^*}\right)} > \frac{M_1(B^*)}{M_2(B^*)} = 1 + \lambda$$

which implies that $\mathcal{H}\left(\frac{y}{\bar{R}^*}\right) > 0$.

Finally, assume that $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \leqslant \mu_2$. Given the above analyses, it is straightforward to see that in this case there is a unique equilibrium in which $B_l = 0$. In particular, in this interval, it must be that $\mathcal{H}(B) \leqslant 0$. Q.E.D.

Proof of Proposition 3

Given that both the participation constraint and the no-default constraint will bind, we can write the planner's problem is

$$\max_{B_l}\left(\mathbb{E}\left([(1+\lambda)\,\theta_s - \theta_b]\left(\left(R_l - \frac{R_f}{\bar{R}_f}\bar{R}_l\right)B_l + \frac{R_f}{\bar{R}_f}y\right)\right) + 2y\right) - 2\psi l\left(R_l - R^\dagger\right)$$

subject to (7), and (8). Given our previous definitions, it will be useful to define the planning problem as follows:

$$SP(B) \equiv \max_{B}\left[(1+\lambda)\left(M_2(B)\,B + \tilde{y}\right) - \left(M_1(B)\,B + \tilde{y}\right) + 2y - 2\psi\mathbb{E}l\left(R_l(B) - R^\dagger\right)\right]$$

where $\tilde{y} = \frac{\mathbb{E}[R_f]}{\bar{R}_f}y$ subject to

$$R_l(B) = R^\dagger + \frac{1}{2\psi}\left(\theta_s - \theta_b\right)B$$

The first order condition is

$$\begin{aligned}SP'(B) &= \left[(1+\lambda)\left[M_2(B) + M_2'(B)\,B\right] - M_1(B) - M_1'(B)\,B - 2\psi\mathbb{E}l'\left(R_l(B) - R^\dagger\right)R_l'(B)\right]\\ &= \left[(1+\lambda)\,M_2(B) - M_1(B) + \Delta(B)\,B\right]\end{aligned}$$

where

$$\Delta(B) \equiv (1+\lambda)\,M_2'(B) - M_1'(B) - \mathbb{E}\left(\theta_s - \theta_b\right)R_l'(B)$$

Next, lets check the second order condition of the planner's problem. First, we have

$$\begin{aligned}\Delta'(B) = {} & h''\left(M_2(B)\,B + x\right)\left[M_2'(B)\,B + M_2(B)\right]M_2'(B) + h'\left(M_2(B)\,B + x\right)M_2''(B)\\ & - \mathbb{E}M_1''(B) - \mathbb{E}\left(\theta_s - \theta_b\right)R_l''(B) = 0\end{aligned}$$

which implies that

$$
\begin{aligned}
SP''(B) =& (1+\lambda) M_2'(B) - M_1'(B) + \Delta(B) \\
=& 2(1+\lambda) M_2'(B) - 2M_1'(B) - \mathbb{E}(\theta_s - \theta_b) R_l'(B) \\
=& 2\left((1+\lambda)\frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right) + \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)\right) - \frac{1}{\psi}\text{var}(\theta) \\
>& 2\left(\frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right) + \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)\right) - \frac{1}{\psi}\text{var}(\theta) \\
\geqslant& \frac{1}{\psi}\text{var}(\theta)
\end{aligned}
$$

where the inequality in line 3 follows from Assumption 1. Therefore, the Planner's problem is strictly convex which implies that computing the solution involves comparing end points $B_l = 0$ and $B_l = \frac{y}{\bar{R}^*}$. Note that the maximal feasible level of B_l depends only on parameters and thus is identical across both the competitive equilibrium and the Planning problem.

Define

$$
\mu^{sp} = \frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{1}{2\psi}\frac{y}{\bar{R}^\dagger \bar{R}^*}\left(\frac{(1+\lambda)}{\lambda}\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)
$$

We have

$$
SP(0) = \lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}y + 2y
$$

and

$$
\begin{aligned}
SP\left(\frac{y}{\bar{R}^*}\right) =& \left[(1+\lambda)\left(M_2\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} + \tilde{y}\right) - \left(M_1\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} + \tilde{y}\right) + 2y - \psi\mathbb{E}\left(\frac{1}{2\psi}(\theta_s - \theta_b)\frac{y}{\bar{R}^*}\right)^2\right] \\
=& \left[\lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}y + 2y + (1+\lambda) M_2\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - M_1\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - \psi\mathbb{E}\left(\frac{1}{2\psi}(\theta_s - \theta_b)\frac{y}{\bar{R}^*}\right)^2\right]
\end{aligned}
$$

Thus to compare the above two terms, we need to compute the sign of

$$
\begin{aligned}
& (1+\lambda) M_2\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - M_1\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - \psi\mathbb{E}\left(\frac{1}{2\psi}(\theta_s - \theta_b)\frac{y}{\bar{R}^*}\right)^2 \\
=& \left[\left(\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\frac{y}{\bar{R}^\dagger \bar{R}^*}\left(\frac{(1+\lambda)}{\lambda}\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)\right]\lambda\bar{R}^\dagger\frac{y}{\bar{R}^*}
\end{aligned}
$$

which immediately implies the result given threshold μ^{sp}. Lets now check that the participation constraint of the buyer is satisfied. The buyer's payoff is

$$\begin{aligned}
&(1+\lambda)\,x + y - \mathbb{E}\theta_s\,[R_l B_l + R_f B_f]\\
=&y + \mathbb{E}\,[(1+\lambda)\,\theta_b - \theta_s]\,[R_l B_l + R_f B_f]\\
\geqslant&y + \lambda\mathbb{E}\,[R_f]\,\frac{y}{\bar{R}_f}\\
>&0
\end{aligned}$$

which implies that the participation constraint is satisfied. Finally, it is easy to see that $\mu^{sp} < \mu_1$ and a simple computation implies that

$$\mu^{sp} - \mu_2 = \frac{1}{2\lambda\psi}\frac{y}{\bar{R}^*}\mathrm{var}\,(\theta) > 0$$

which proves that $\mu_2 < \mu^{sp} < \mu_1$. Q.E.D.

Proof of Proposition 4

As before, we can substitute the participation and no-default constraint to write the contracting problem as

$$\max_{b_l}\,(1+\lambda)\left(\mathbb{E}\theta_s\left(\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)(b_l - \hat{b}_l) + \frac{R_f}{\tilde{R}_f}y\right)\right) - \mathbb{E}\theta_b\left(\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)(b_l - \hat{b}_l) + \frac{R_f}{\tilde{R}_f}y\right)$$

where $\tilde{R} = \{\overline{R}, \underline{R}\}$ depending on whether $b \geqslant \hat{b}$. The first order condition is

$$(1+\lambda)\,\mathbb{E}\left[\theta_s\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)\right] - \mathbb{E}\left[\theta_b\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)\right] \geqslant 0$$

First, suppose that $b_l < \hat{b}_l$. Then the foc is

$$\underline{R}_l\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\left(\frac{R^\dagger + \frac{1}{2\psi}(\theta_s - \theta_b)\,B_l}{\underline{R}^\dagger} - \frac{R_f}{\bar{R}_f}\right)\right] - \mathbb{E}\left[\theta_b\left(\frac{R^\dagger + \frac{1}{2\psi}(\theta_s - \theta_b)\,B_l}{\underline{R}^\dagger} - \frac{R_f}{\bar{R}_f}\right)\right]\right]$$

$$\begin{aligned}
&(1+\lambda)\left[\left(\frac{\mathbb{E}\left[R^\dagger\right] + \frac{1}{2\psi}\mathrm{var}\,(\theta)\,B_l}{\underline{R}^\dagger} - \frac{\mathbb{E}\,[R_f]}{\bar{R}_f}\right)\right] - \left(\frac{\mathbb{E}\left[R^\dagger\right] - \frac{1}{2\psi}\mathrm{var}\,(\theta)\,B_l}{\underline{R}^\dagger} - \frac{\mathbb{E}\,[R_f]}{\bar{R}_f}\right)\\
\geqslant&(1+\lambda)\left(\frac{\mathbb{E}\left[R^\dagger\right]}{\underline{R}^\dagger} - \frac{\mathbb{E}\,[R_f]}{\bar{R}_f}\right) - \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\underline{R}^\dagger} - \frac{\mathbb{E}\,[R_f]}{\bar{R}_f}\right)\\
>&0
\end{aligned}$$

so that $b_l < \hat{b}_l$ can never be part of an equilibrium. Next, suppose that $b_f < \hat{b}_f$. Then the foc is

$$\bar{R}_l \left[(1+\lambda)\,\mathbb{E}\left[\theta_s \left(\frac{R^\dagger + \frac{1}{2\psi}(\theta_s - \theta_b) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l} - \frac{R_f}{\underline{R}_f}\right)\right] - \mathbb{E}\left[\theta_b \left(\frac{R^\dagger + \frac{1}{2\psi}(\theta_s - \theta_b) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l} - \frac{R_f}{\underline{R}_f}\right)\right]\right]$$

$$= \bar{R}_l \left[(1+\lambda) \left(\frac{\mathbb{E}\left[R^\dagger\right] + \frac{1}{2\psi}\mathrm{var}(\theta) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l} - \frac{\mathbb{E}[R_f]}{\underline{R}_f}\right) - \left(\frac{\mathbb{E}\left[R^\dagger\right] - \frac{1}{2\psi}\mathrm{var}(\theta) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l} - \frac{\mathbb{E}[R_f]}{\underline{R}_f}\right)\right]$$

$$= \bar{R}_l \left[(1+\lambda) \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l} - \frac{\mathbb{E}[R_f]}{\underline{R}_f}\right) - \left(\frac{\mathbb{E}\left[R^\dagger\right]}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l} - \frac{\mathbb{E}[R_f]}{\underline{R}_f}\right)\right]$$

$$+ \bar{R}_l \left[(1+\lambda) \left(\frac{\frac{1}{2\psi}\mathrm{var}(\theta) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l}\right) - \left(\frac{\frac{1}{2\psi}\mathrm{var}(\theta) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l}\right)\right]$$

$$= \bar{R}_l \lambda \left[\left(\frac{\frac{1}{2\psi}\mathrm{var}(\theta) B_l}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l}\right) - \left(\frac{\mathbb{E}[R_f]}{\underline{R}_f} - \frac{\mathbb{E}\left[R^\dagger\right]}{\overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l}\right)\right]$$

For the model to display hysteresis we need the expression above to be less than zero. The sign of the expression above is equal to the sign of

$$\frac{1}{2\psi}\left(\mathrm{var}(\theta) - \left(\overline{\theta} - \underline{\theta}\right)\frac{\mathbb{E}[R_f]}{\underline{R}_f}\right) B_l - \overline{R}^\dagger \left(\frac{\mathbb{E}[R_f]}{\underline{R}_f} - \frac{\mathbb{E}\left[R^\dagger\right]}{\overline{R}^\dagger}\right) <$$
$$\frac{1}{2\psi}\left(\mathrm{var}(\theta) - \left(\overline{\theta} - \underline{\theta}\right)\frac{\mathbb{E}[R_f]}{\underline{R}_f}\right) B_l - \overline{R}^\dagger \left(\frac{\mathbb{E}[R_f]}{\underline{R}_f} - 1\right) < 0$$

where the last inequality follows from Assumption 2. Q.E.D.

Proof of Proposition 6

The proof of this proposition relies on $var(\theta)$ being high enough. In order to get started, we assume that $var(\theta) > max\left\{\lambda, \left(\overline{\theta} - \underline{\theta}\right)\right\}$. Given that both the participation constraint and the no-default constraint will bind, we can write the buyer's problem as

$$\max_{b_b, b_s, b_f} \mathbb{E}\left((1+\lambda)\theta_s - \theta_b\right)\left(R_b b_b + R_s b_s + R_f b_f\right) + \gamma\left(y - \bar{R}_b b - \bar{R}_s b_s - \bar{R}_f b_f\right),$$

where γ is the Lagrange multiplier on the no-default constraint. The first order conditions of this problem are given by

$$\frac{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)R_i\right]}{\bar{R}_i}-\gamma,$$

for currencies $i \in \{b, s, f\}$. The proof of the proposition proceeds in two steps. First, we show the existence of an equilibrium with $B_b = 0$, $B_s = 0$ and $B_f = \frac{y}{\bar{R}_f}$. Second, we show its uniqueness.

In order for $B_b = 0$, $B_s = 0$ and $B_f = \frac{y}{\bar{R}_f}$ to be an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency b and s:

$$\frac{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)R_b\right]}{\bar{R}_b}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)R_s\right]}{\bar{R}_s}.$$

After substituting in the governments' best responses and evaluating these expressions at $B_b = 0$, $B_s = 0$ and $B_f = \frac{y}{\bar{R}_f}$, these optimality conditions simplify to $\mu_1 = \frac{\mathbb{E}(R_f)}{\bar{R}_f} > \frac{\mathbb{E}\left(R_b^{\dagger}\right)}{\bar{R_b}^{\dagger}}$ and $\mu_1 = \frac{\mathbb{E}(R_f)}{\bar{R}_f} > \frac{\mathbb{E}\left(R_s^{\dagger}\right)}{\bar{R_s}^{\dagger}}$. These are identical to the condition obtained in the baseline model. For this to be an unique equilibrium, it must also be true that the previous inequalities hold for all $B_b \in \left[0, \frac{y}{\bar{R}_b^*}\right]$ and for all $B_s \in \left[0, \frac{y}{\bar{R}_s^*}\right]$. These constraints are given by

$$\bar{R_b}^{\dagger}\left(\frac{\mathbb{E}\left(R_b^{\dagger}\right)}{\bar{R_b}^{\dagger}}-\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f}\right)+\frac{1}{2\psi}B_b\left(\frac{var\left(\theta\right)-\lambda}{\lambda}+\underline{\theta}\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f}\right)<0 \tag{13}$$

and

$$\bar{R}_s^{\dagger}\left(\frac{\mathbb{E}\left(R_s^{\dagger}\right)}{\bar{R}_s^{\dagger}}-\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f}\right)+\frac{1}{2\psi}B_s\left(\frac{var\left(\theta\right)\left(1+\lambda\right)+\lambda}{\lambda}-\bar{\theta}\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f}\right)<0. \tag{14}$$

In order to find out the values of B_b and B_s at which these constraint are more likely to bind, we show that the terms multiplying B_b and B_s are positive. The fact that $\frac{var(\theta)-\lambda}{\lambda}$ +

$\underline{\theta}\frac{\mathbb{E}(R_f)}{\bar{R}_f} > 0$ simply follows from Assumption 3. The fact that $\frac{var(\theta)(1+\lambda)+\lambda}{\lambda} - \overline{\theta}\frac{\mathbb{E}(R_f)}{\bar{R}_f} > 0$ follows from

$$\frac{var(\theta)(1+\lambda)+\lambda}{\lambda} - \frac{\mathbb{E}(R_f)}{\bar{R}_f}\overline{\theta} > \frac{(\overline{\theta}-\underline{\theta})}{\lambda} + (\overline{\theta}-\underline{\theta}) + 1 - \frac{\mathbb{E}(R_f)}{\bar{R}_f}\overline{\theta} > \frac{(\overline{\theta}-\underline{\theta})}{\lambda} > 0,$$

where the first inequality follows from Assumption 3, and the second inequality follows from the fact that $\mathbb{E}[\theta_i] = 1$ (which implies $\overline{\theta} \geqslant 1$ and $\underline{\theta} \leqslant 1$) and $\mathbb{E}(R_f) < \bar{R}_f$. Since both terms are positive, we only need to verify that equations (13) and (14) hold when evaluated at $B_b = \frac{y}{\bar{R}_b^*}$ and $B_s = \frac{y}{\bar{R}_s^*}$. Finally, define the threshold μ_2^I as

$$\mu_2^I \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{1}{2\psi\bar{R}^\dagger}\max\left\{\frac{y}{\bar{R}_b^*}\left(\frac{var(\theta)-\lambda}{\lambda} + \underline{\theta}\frac{\mathbb{E}(R_f)}{\bar{R}_f}\right), \frac{y}{\bar{R}_s^*}\left(\frac{var(\theta)(1+\lambda)+\lambda}{\lambda} - \overline{\theta}\frac{\mathbb{E}(R_f)}{\bar{R}_f}\right)\right\}.$$

Recall that the condition that guarantees uniqueness of foreign currency equilibrium in the baseline model is

$$\mu_2 \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{1}{2\psi\bar{R}^\dagger}\frac{y}{\bar{R}^*}\left(\frac{(2+\lambda)}{\lambda}var(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta}-\underline{\theta})\right) < 0.$$

Thus, the final step consists of showing that $\mu_2^I > \mu_2$ under the assumption of symmetric countries (i.e., $\mathbb{E}\left(R_b^\dagger\right) = \mathbb{E}\left(R_s^\dagger\right) = \mathbb{E}\left(R^\dagger\right)$ and $\bar{R_b}^\dagger = \bar{R_s}^\dagger = \bar{R}^\dagger$). This comparison narrows down to showing that

$$\frac{y}{\bar{R}_b^*}\left(\frac{var(\theta)-\lambda}{\lambda} + \underline{\theta}\frac{\mathbb{E}(R_f)}{\bar{R}_f}\right) < \frac{y}{\bar{R}^*}\left(\frac{(2+\lambda)}{\lambda}var(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta}-\underline{\theta})\right) \tag{15}$$

and

$$\frac{y}{\bar{R}_s^*}\left(\frac{var(\theta)(1+\lambda)+\lambda}{\lambda} - \overline{\theta}\frac{\mathbb{E}(R_f)}{\bar{R}_f}\right) < \frac{y}{\bar{R}^*}\left(\frac{(2+\lambda)}{\lambda}var(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta}-\underline{\theta})\right). \tag{16}$$

After substituting in the expressions for $\bar{R}_b^*$ and $\bar{R}^*$, equation (15) becomes

$$0 < 1 - \underline{\theta}\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f} + \frac{var\left(\theta\right)}{\lambda}\left(\frac{\bar{R_b}^{\dagger} + \sqrt{\bar{R_b}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y}}{\bar{R}^{\dagger} + \sqrt{\bar{R}^{\dagger 2} + 2\frac{1}{\psi}\left(\bar{\theta} - \underline{\theta}\right)y}}\left(2+\lambda\right) - 1\right)$$
$$- \frac{\bar{R_b}^{\dagger} + \sqrt{\bar{R_b}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y}}{\bar{R}^{\dagger} + \sqrt{\bar{R}^{\dagger 2} + 2\frac{1}{\psi}\left(\bar{\theta} - \underline{\theta}\right)y}}\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right).$$

Note that the term multiplying $var\left(\theta\right)$ is positive:

$$\frac{\bar{R_b}^{\dagger} + \sqrt{\bar{R_b}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y}}{\bar{R}^{\dagger} + \sqrt{\bar{R}^{\dagger 2} + 2\frac{1}{\psi}\left(\bar{\theta} - \underline{\theta}\right)y}}\left(2+\lambda\right) - 1 > \frac{\bar{R_b}^{\dagger} + \sqrt{\bar{R_b}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y}}{\bar{R}^{\dagger} + \sqrt{\bar{R}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y} + \sqrt{2\frac{1}{\psi}\bar{\theta}y}}\left(2+\lambda\right) - 1$$
$$= \frac{\left(1+\lambda\right)\bar{R_b}^{\dagger} + \left(1+\lambda\right)\sqrt{\bar{R_b}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y} - \sqrt{2\frac{1}{\psi}\bar{\theta}y}}{\bar{R}^{\dagger} + \sqrt{\bar{R}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y} + \sqrt{2\frac{1}{\psi}\bar{\theta}}} > 0,$$

where the last inequality follows from the assumption that $\underline{R}_b^{\dagger} - \frac{2}{\psi}\bar{\theta}\frac{y}{\bar{R_b}^{\dagger}} > 0$, which implies that $\bar{R_b}^{\dagger} > \sqrt{2\frac{1}{\psi}\bar{\theta}y}$. Then, there exists a threshold $\tilde{\nu}$ such that when $var\left(\theta\right) > \tilde{\nu}$ equation (15) is satisfied. Now we are in a position to define the lower bound for the proposition to be true: $\bar{\nu} \equiv max\left\{\tilde{\nu}, \lambda, \left(\bar{\theta} - \underline{\theta}\right)\right\}$. In order to show that equation (16) holds, it is sufficient to show that

$$\frac{y}{\bar{R}_s^*}\left(\frac{var\left(\theta\right)\left(1+\lambda\right)+\lambda}{\lambda} - \bar{\theta}\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f}\right) < \frac{y}{\bar{R}_s^*}\left(\frac{\left(2+\lambda\right)}{\lambda}var\left(\theta\right) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right),$$

where we have used the fact that $R^* < R_s^*$. This condition can be simplified to $0 < \frac{var(\theta)-\lambda}{\lambda} + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\underline{\theta}$, which trivially holds given our assumption that $var(\theta) > max\left\{\tilde{\nu}, \lambda, \left(\bar{\theta} - \underline{\theta}\right)\right\}$. Q.E.D.

B A Model of a Credit Chain

We now present a simple credit chain model that endogenizes the stocks of foreign and local currency in Section 4.

Suppose that citizens are further divided into one of I sub-types $\mathcal{I} \in \{1, 2, ..., I\}$ with continuum of each. A citizen of type i has preferences over a special good produced by type $i+1$ and produces a special good valued by type $i-1$. All types also value the consumption of the numeraire good which takes place at the end of period 2. Preferences for the representative citizen type i are given by

$$u_i = (1+\lambda)\ {}_ix_{i+1} - {}_{i-1}x_i + \mathbb{E}\left[\theta_i c_i\right]$$

where ${}_ix_{i+1}$ is the special good produced by a citizen of type $i+1$ for a citizen of type i and ${}_{i-1}x_i$ is the special good produced by a citizen of type i for a citizen of type $i-1$. We assume that ${}_0x_1 = {}_Ix_{I+1} = 0$ so that type 1 does not produce a special good for any other type and type I does not consume a special good. As in the baseline we assume that $\theta_i \in \left[\overline{\theta}, \underline{\theta}\right]$ is independent across sub-types and that $\mathbb{E}\left[\theta_i\right] = 1$.

The timing of the model is as follows:

1. The first period $t = 1$ is divided into $I-1$ sub-periods in which trade takes place sequentially:
   (a) In sub-period 1, citizens of type 2 produces a special good for citizens of type 1 in exchange for the promise of payment in period 2.
   (b) Similarly, in sub-period i, citizens of type $i+1$ produce a special good for citizens of type i in exchange for the promise of payment in period 2
2. The second period $t = 2$ is divided into three sub-periods:
   (a) In sub-period 1, the type of the domestic government is realized and it chooses its policy which is the aggregate price level
   (b) In sub-period 2, endowments for all citizens are realized
   (c) In sub-period 3, all signed contracts are executed in the order in which they were signed and finally, consumption of the composite good takes place.

Assume that all citizens are endowed with y units of the numeraire good. The definition of a bilateral contract between i and $i+1$ is identical to Section 4. Note that in this contract $i+1$ is the seller and and i is the buyer. Given the structure of the credit chain, $\left(\hat{b}_f, \hat{b}_l\right)$ is the promised payment to type i from types $i-1$.

We can then use Propositions 2 and 4 to characterize the bilateral contract.

Proposition 7. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l R_l + b_f R_f\right)\right]$, *while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{R_l}{\bar{R}_l}\right] < \mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \hat{b}_l$ *and* $b_f = \hat{b}_f + \frac{y}{\bar{R}_f}$
2. *If* $\mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{R_l}{\bar{R}_l}\right] = \mathbb{E}\left[\left(\theta_s(1+\lambda) - \theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \hat{b}_l + \gamma\frac{y}{\bar{R}_l}$ *and* $b_f = \hat{b}_f + (1-\gamma)\frac{y}{\bar{R}_f}$ *for any* $\gamma \in [0,1]$.

3. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] > \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \hat{b}_l + \frac{y}{\bar{R}_l}$ *and* $b_f = \hat{b}_f$.

The result follows immediately from Propositions 2 and 4. In particular, the optimal contract will feature currency matching of stocks and will denominate the flows in the currency with the largest marginal benefit. As a corollary of this Proposition we can compute the aggregate stock of local currency which will be needed for the government's problem. Let (B_{fi}, B_{li}) denote the aggregate stock of local and foreign currency obligations in the contract signed between $i+1$ and i.

Corollary 1. *In equilibrium,*

1. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] < \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $B_{li} = 0$ *and* $B_{fi} = i\frac{y}{\bar{R}_f}$
2. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] = \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $B_{li} = \sum_{j=1}^{i}\gamma_j\frac{y}{\bar{R}_l}$ *and* $B_{fi} = \sum_{j=1}^{i}\left(1-\gamma_j\right)\frac{y}{\bar{R}_l}$
3. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] > \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $B_{li} = i\frac{y}{\bar{R}_l}$ *and* $B_{fi} = 0$.

Next, the government's problem is

$$\max_{R_l}\sum_i\left[\theta_i C_i\right] - I\cdot l\left(R_l - R^\dagger\right)$$

where

$$C_i = y - R_l\left(B_{li} - B_{li-1}\right) - R_f\left(B_{fi} - B_{fi-1}\right)$$

The best response of the government is given by

$$R_l = R^\dagger - \frac{1}{I\psi}\sum_{i=1}^{I}\theta_i\left(B_{li} - B_{li-1}\right)$$

and so

$$\bar{R}_l = \overline{R}^\dagger - \max_\theta\left(\frac{1}{I\psi}\sum_{i=1}^{I}\theta_i\left(B_{li} - B_{li-1}\right)\right)$$

Given this lets understand how the optimal contract changes in the credit chain. Recall that the marginal benefit of denominating the contract signed between i and $i+1$ in currency l is

$$\lambda\frac{\mathbb{E}\left[R_l\right]}{\bar{R}_l} + \mathrm{cov}\left(\left(\theta_{i+1}\left(1+\lambda\right)-\theta_i\right), \frac{R_l}{\bar{R}_l}\right)$$

Using the best response of the government the previous equation for the contract signed between 1 and 2 can be written as

$$\begin{aligned}&\frac{1}{\bar{R}_l}\left(\lambda\mathbb{E}\left[R^\dagger\right]+\operatorname{cov}\left(\left(\theta_2\left(1+\lambda\right)-\theta_1\right),R_l\right)\right)\\=&\frac{1}{\bar{R}_l}\left(\lambda\mathbb{E}\left[R^\dagger\right]+\frac{1}{I\psi}\left(\left[\left(2+\lambda\right)B_{l1}-\left(1+\lambda\right)B_{l2}\right]\right)\operatorname{var}\left(\theta\right)\right)\end{aligned}$$

Recall the expression for marginal benefit in the baseline model

$$\frac{1}{\bar{R}_l}\left(\lambda\mathbb{E}\left[R^\dagger\right]+\frac{1}{2\psi}\left(\left[\left(\operatorname{var}\left(\theta\right)\left(2+\lambda\right)\right)\right]\right)B_{l1}\right)$$

since $B_{l2} = 0$. On comparing the two we see that the term in parenthesis is smaller in the environment with the credit chain. In particular the covariance term is smaller owing to the fact that each government policy will respond less to individual shocks within the chain. However the the term $\bar{R}_l$ also changes and the direction of this change is ambiguous. So it is hard to say anything generally about the set of equilibria.

For illustrative purposes suppose only sub-type one is endowed with y units of the numeraire good while all other sub-types are endowed with zero units of the good. In this case

$$\bar{R}_l = \overline{R}^\dagger + \frac{1}{I\psi}\left(\overline{\theta}-\underline{\theta}\right)\left(B_{l1}\right)$$

while in the baseline

$$\bar{R}_l = \overline{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)\left(B_{l1}\right)$$

which implies that in the credit chain, the price risk of local currency is lower. Thus, we have two competing effects. In particular, as $I \to \infty$, the marginal benefit converges to

$$\lim_{I\to\infty}\frac{\left(\lambda\mathbb{E}\left[R^\dagger\right]+\frac{1}{I\psi}\left(\left[B_{l1}\right]\right)\operatorname{var}\left(\theta\right)\right)}{\overline{R}^\dagger+\frac{1}{I\psi}\left(\overline{\theta}-\underline{\theta}\right)\left(B_{l1}\right)}=\frac{\lambda\mathbb{E}\left[R^\dagger\right]}{\overline{R}^\dagger}$$

so the the optimal currency choice only involves comparing the price risk.

C Figures

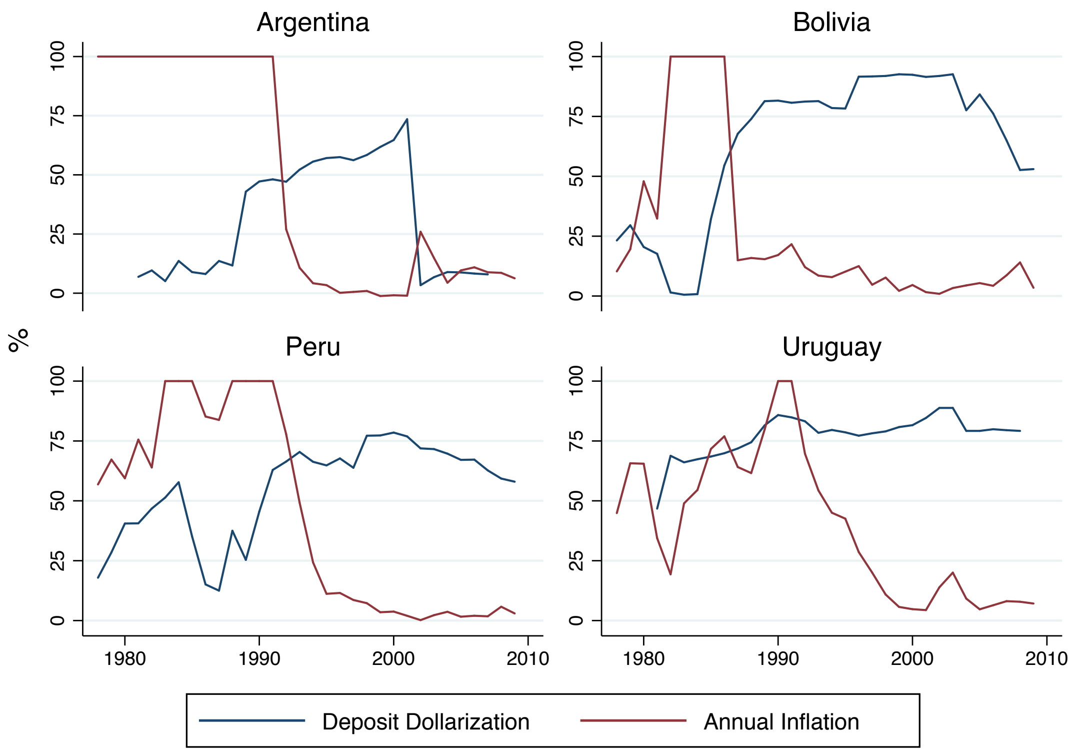

Figure 2: Persistence of Financial Dollarization
*Sources:*Levy-Yeyati (2006) and IFS