The Currency Composition of Sovereign Debt*

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Abstract. We study the currency composition of sovereign external debt in emerging economies through the lens of a quantitative model in which the government lacks commitment regarding debt and monetary policy. High levels of debt in local currency give rise to incentives to dilute debt repayment through currency depreciation. Governments tilt the currency-composition of debt towards foreign currency to avoid inflationary costs and real exchange rate distortions, at the expense of foregoing the hedging properties of local currency debt. Our model is used to shed light on the recent dynamics of the currency composition of debt and on its cyclical behavior.

Keywords: Sovereign debt, currency composition, monetary policy, time inconsistency.

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1. Introduction

Ever since emerging economies started issuing sovereign debt in global markets, the currency composition of debt has become a central element of the policy debate. During the 1990s, the perception was that governments had difficulties in placing debt denominated in local currency, a phenomenon termed the 'original sin' (Eichengreen et al. (2002)). The predominant concerns at that time were related to the economic vulnerabilities associated to high levels of debt-dollarization.

Over the last decade, this paradigm has changed significantly as governments have recurrently placed debt denominated in local currency, and used the currency denomination of sovereign debt as an active margin of debt management. The surge of local currency borrowing in emerging economies gives rise to a new set of questions that the academic literature has not previously addressed: What allowed governments from emerging economies to escape the 'original sin'? Was it due to changes in policies or changes in macroeconomic fundamentals? How can governments use the mix between local and foreign currency debt in an optimal way?

In this paper we develop a framework to study the what determines the currency composition of sovereign debt and use it to explain the dynamic patterns observed in the data. In the model, a government that lacks commitment chooses debt and monetary policy, weighing the hedging benefits of debt in local currency and its incentive problems. On the one hand, debt in local currency provides insurance for the borrower through the state-contingency created by movements in the real exchange rate and inflation. On the other hand, high levels of debt in local currency induce governments to dilute the value of their debt by generating costly inflation or keeping the real exchange rate excessively depreciated. Governments react to this time inconsistency problem by tilting their debt portfolio towards foreign currency to avoid falling in the temptation of engaging in distortionary nominal exchange rate depreciation.

Our quantitative analysis of the model for a typical emerging economy shows that the presence of these trade-offs can account for the observed degree of 'original sin' as well as its dynamic patterns. We also use our model to assess whether the the gradual dissipation of the 'original sin' was linked to good policies or good fundamentals. We argue that both factors were relevant. In particular, the prolonged economic expansion experienced in emerging economies over the last decade and the change in monetary regime behind inflation stabilization during this period can account for most of the observed change in the currency composition of sovereign debt.

The paper begins by documenting three stylized facts about the currency composition of sovereign debt in emerging economies. Using data on debt stocks for a panel of 18 countries we show that: *(i)* Debt currency composition tends to be tilted towards foreign currency: on average, around three fourths of sovereign external debt is denominated in foreign currency. *(ii)* The original sin has gradually dissipated: in the last decade the average share of debt in local currency increased from 10% to 39%. *(iii)* The currency composition of debt has a strong cyclical component: in economic booms the share of debt denominated in local currency is higher than in recessions. The average correlation between output and the share of debt denominated in local currency is 24%. The first two facts are consistent with previous research (see, for example, Eichengreen and Hausmann (2010) and Du and Schreger (2015)), the third fact is new to the literature. We complement this analysis by studying data on bond issuance for the same countries and show that the behavior of issuance is also in line with the trend and cyclicality of the currency composition of debt. The average share of issuance in local currency increased by 21% in the last decade. Additionally, the correlation between output and the share of debt issuance denominated in local currency is positive in 70% of the countries.

We then layout a general equilibrium model of sovereign debt and monetary policy to study the trade-offs associated with the currency composition of debt and shed light on the above-mentioned facts. In the model the government chooses debt issuance in foreign and local currency and monetary policy without commitment. The exchange rate, and therefore the repayment of debt to foreign investors, is determined endogenously through the interaction of debt choices and the conduct of monetary policy. We study two variants of equilibrium, one in which monetary policy is taken as given and another in which monetary policy is chosen optimally.

We first focus on the equilibrium given a particular monetary policy that replicates the observed co-movement of income and the nominal exchange rate. This approach highlights the role of the hedging properties of debt in local and foreign currency in determining optimal portfolio positions. Given that emerging economies tend to experience currency depreciation in recessions and appreciation during booms, debt in local currency constitutes a useful hedge against income risk. If the government is indebted in local currency, when a negative income shock is realized, the exchange rate likely depreciates which in turn lowers the value of debt repayments in local currency and attenuates the negative effect of the original income shock. The solution to this portfolio problem calls for issuing large levels of debt in local currency and little debt in foreign currency. This position is at odds with observed average levels of debt by currency in emerging economies.

This apparent disconnect is due to the fact that the attractive hedging properties of debt in local currency are partially offset by its associated perverse incentive problems. The government, through its choice of debt issuance and monetary policy can dilute the value of existing debt in local currency. One way to dilute debt in local currency is through inflation. Inflation, although costly, induces nominal exchange rate depreciation which in turn dilutes the value of debt in local currency. Another less obvious way to dilute debt in local currency is through inducing real exchange rate depreciation. By reducing the level of overall debt issuance the government postpones consumption of tradable goods for households which in turn depreciates the real exchange rate and reduces the real value of debt in local currency. Both inflation and excessive real exchange rate depreciation are distortionary from an ex-ante point of view since the effects of debt dilution are anticipated by foreign investors that ask for lower prices for debt in local currency. The government takes this into account when choosing debt in previous periods and tilts the currency composition of debt towards foreign currency denomination to avoid engaging in costly inflation and distorting the real exchange rate in equilibrium. This shift in the currency composition of debt is done at the expense of further consumption insurance given that the model features an endogenous exchange rate that is countercyclical.

A calibrated version of the model, which takes into account the welfare costs of inflation estimated from an independent literature and targets the total level of external public debt and average inflation rate, shows that the equilibrium currency-composition of sovereign debt predicted by the model is close to that observed in the data. The model also correctly predicts the cyclical pattern of the currency composition of sovereign external debt observed in the data. The government optimally chooses to issue a larger share of debt in local currency in booms. This cyclical behavior is not linked to the variation of hedging properties of debt in local currency over the cycle but rather to the cyclical properties of the benefits associated to debt dilution. In recessions the benefit of diluting debt via nominal depreciation is higher since saving resources for consumption is more valuable given that the marginal utility of consumption is higher in those states. This implies an optimal inflation rate that is countercyclical. The government internalizes this when choosing debt by currencies and chooses to reduce its debt issuance in local currency to avoid high levels of costly inflation in recessions.

We use to shed light on the recent observed increase in the share of debt denominated in local currency. This change in the currency composition of external public debt happened simultaneously with a generalized process of economic expansion and disinflation in emerging economies. We argue that these phenomena are related to each other, since a common feature of episodes of expansion and disinflation is an increase in the share of debt denominated in local currency. According to our calibrated model, three fourths of the observed increase in the share of debt in local currency can be explained by positive endowment shocks and monetary shocks (i.e. shocks to the costs of inflation). On the one hand, a sequence of positive output shocks reduce the marginal benefits of diluting debt. On the other hand, the presence of higher inflation costs increase the marginal costs of diluting debt. In reaction to both configurations of shocks the government can afford to increase its share of debt in local currency without increasing inflation ex-post. We also find that, although both the presence of a prolonged expansion and a disinflation process are relevant in explaining the increase in the share of debt in local currency in the model, most of the predicted increase is driven by the output shocks that lead to a prolonged expansion.

In the last part of the paper we comment on the trade-offs associated to the use of alternative debt instruments. It could be thought that one way to get around the possibility of diluting the value of debt in local currency is to issue inflation-linked debt. We argue that this is true only to a partial extent, since the issuance of inflation-linked bonds does not fully overcome its incentive problems. While the government can no longer dilute the value of debt through inflation, it can still do so by postponing tradable consumption and inducing real exchange rate depreciation.

Related Literature. This paper builds upon the literature on sovereign debt and time inconsistency of monetary policy. It is also closely related to a number of recent papers that study the joint determination of sovereign debt issuance and inflation choices.

Following the original framework of sovereign defaultable debt developed in Eaton and Gersovitz (1981), a recent body of literature has studied the quantitative dynamics of sovereign debt. Arellano (2008) and Aguiar and Gopinath (2006) analyze sovereign debt and business cycle properties in emerging economies. Several studies have extended the framework to study different aspects related to debt management such as its maturity composition, its composition by residence of creditors, its liquidity management, roll-over crises and post-default debt renegotiations.[1] Closely related to our work, Arellano and Ramanarayanan (2012) argue that the problem of debt maturity choice involves a trade-off between hedging and disciplining properties. While short-term debt is useful for providing incentives, long-term debt is useful for hedging

[1] For example, Broner et al. (2013), Aguiar et al. (2016) study the optimal maturity composition of sovereign debt. Broner and Ventura (2011) and Perez (2015) analyze how the residence composition of government's creditors can shape the incentives to default. Cole and Kehoe (2000) and Bocola and Dovis (2016) study the

consumption. Similarly, Bianchi et al. (2018) show that the optimal choice of international reserves involves a trade-off between their insurance benefits and incentive problems, the later linked to higher sovereign default risk associated to its financing. Our currency-composition problem bares a resemblance these trade-offs. Here debt in local currency has useful hedging properties and debt in foreign currency does not give rise to perverse incentive problems.

The paper also builds on the literature on the time inconsistency of government polices. Following the seminal contributions of Kydland and Prescott (1977) and Barro and Gordon (1983) a large strand of macroeconomic research has analyzed various contexts in which the ideal policies that a government would want to implement differ from the policies he can credibly promise.[2] In those situations the government would like agents to trust his announcements but agents do not do so because they anticipate the government will have incentives to deviate from that announcement once they have taken their actions. In this paper the source of time inconsistency comes from the fact the government would like foreign investors and himself in the future to trust his desire of not to engage in distortionary exchange rate depreciation in the future but this is not possible as ex-post, once debt has been issued, the incentives to depreciate arise to avoid paying to foreign investors.

Finally, our work is also related to a set of papers that study the effects of issuing nominal debt in local currency. Early work by Fischer (1983) and Calvo (1988) explore the possibility of partially defaulting on nominal debt by engaging in inflation.[3] Bohn (1990) analyzes these time-inconsistency problems in a model in which debt in foreign currency is a hedging device.

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presence of self-fulfilling crises. Yue (2010) and Pitchford and Wright (2012) introduce the dimension of post-default debt renegotiation into defaultable debt models. Aguiar and Amador (2014) survey recent advances in the field.

[2]Examples of applications of time inconsistency problems include capital taxation (Chari and Kehoe (1990)), monetary policy (Rogoff (1985), Calvo (1987), Chari et al. (2015)) and government default and bailouts (Chari and Kehoe (1993), Farhi and Tirole (2012)). A more general treatment of time inconsistency problems can be found in Phelan (2006).

[3]Other papers that focus on the problem of debt dilution through inflation in small open economies include Aguiar et al. (2013) and Da-Rocha et al. (2013), that analyze how the ability to dilute debt via inflation can reduce the likelihood of suffering self-fulfilling debt crises; Araujo et al. (2013) and Nuño and Thomas (2015), that study the welfare implications of issuing debt in local currency; Du and Schreger (2015), that show that inflation can have associated negative effects on the balance-sheet of corporates; and Sunder-Plassmann (2014), that argues that inflation can help the government repay real debt in the long-run. The interaction between inflation and nominal government debt has also been studied in the context of closed economies (for example, Persson et al. (1987), Leeper (1991), Sims (1994) and Diaz-Gimenez et al. (2008)).

Korinek (2009) shows that the presence of endogenous real exchange rates gives rise to hedging properties of debt in local currency. Fanelli (2018) studies how the presence of debt if foreign and local currency shape optimal monetary policy under commitment. Du et al. (2017) argue that various degrees of credibility regarding monetary policy in the presence of risk-averse foreign investors can help reconcile cross-sectional returns of nominal bonds. In contemporaneous and independent work, Engel and Park (2018) study a similar question to ours by analyzing an optimal contract problem with the fear of outright default and debt dilution through inflation. We contribute to this literature by analyzing a quantitative dynamic model of sovereign debt with endogenous currency composition and mapping it to the data.

Layout. The remaining of the paper is organized as follows. Section 2 presents the data on currency composition of sovereign debt for a set of emerging economies. Section 3 describes the model and defines the relevant notion of equilibria. Section 4 characterizes the main trade-offs involved in the choice of the currency composition of debt. Section 5 analyzes the quantitative properties of the model and compares them to their data counterparts. Section 6 comments on the use of indexed debt. Finally, section 7 concludes.

2. Debt Currency-Composition in Emerging Economies: Stylized Facts

This section documents the main patterns of the currency composition of sovereign debt observed in emerging economies. We are interested in studying how large are debt positions in local currency relative to foreign currency, how these relative positions have changed over the past decade and how they fluctuate over the business cycle. To this end, we compute two complementary analyses using aggregate data on sovereign external debt decomposed by currency denomination and data on issuances at the bond level, for a panel of emerging economies. All countries included in the sample are middle income economies integrated to global capital markets, as measured for example, by the fact that at some point in time they were part of the set of countries included in J.P. Morgan's EMBI tracking sovereign debt in emerging economies.

2.1. *Data on Debt Stocks*

The source of these data is the new dataset constructed by Arslanalp and Tsuda (2014), which includes foreign holdings of both internationally- and domestically-issued government debt for the period 2004-2014 on a quarterly basis, and World Development Indicators (WDI), which covers annual data on total external public debt for a longer period of time. Table 1 reports four key moments for a sample of emerging economies. One is related to the total level of external public debt and the remaining three are related to the share of debt denominated in local currency. The first column shows the average level of external public debt for all countries. Emerging economies display average levels of external public debt of of 22% of GDP.

The last three columns illustrate the behavior of debt by currency denomination. The currency composition of external public debt is tilted towards debt in foreign currency. As shown in the second column, on average, one quarter of the public debt held abroad is denominated in local currency. There is heterogeneity in the average composition across countries.[4] While in 5 countries the share of debt in local currency is less than 10%, in 4 countries the share exceeds 50%. Moreover, the share of debt in local currency increased significantly in the last decade (third column). The average share of debt in local currency was 10% in 2004, compared to 39% in 2014. This increase was generalized, with 14 of the 18 countries shifting towards more local currency denominated debt (see Figures A.1 and A.2 for the dynamics of debt and the share of debt in local currency for all countries).

Finally, the share of public debt in local currency is on average procyclical, with booms characterized by larger positions of debt in local currency than busts. The average correlation between the cyclical components of output and the share of external public debt denominated in local currency is 24% (see last column).[5] Again, only a few countries deviate from this pattern and show a countercyclical share in local currency. Table A1 shows that this comovement is driven both by a procyclical stock of debt in local currency and a countercyclical stock of debt in foreign currency.

One disadvantage of the data on stocks is that the data availability on the currency composition starts in 2004 and hence the cyclical properties can be strongly influenced by the episode of the global financial crisis. To address this issue we analyze data on debt issuance for a longer period of time. In addition we look at the episode of the Tequila crisis in Mexico which has historical data on debt stocks available.[6] From the beginning to the end of 1994 the share of sovereign debt denominated in foreign currency increased from less than 5% to more than 50% (Figure A.3).

The cyclicality in the share of debt in local currency can be driven by differential issuance decisions by currency, differential changes in the valuation of debt by currency and/or differential trading between foreign and domestic investors. We argue that changes in the valuation

[4]In this paper we don't explore this cross-sectional heterogeneity that has been analyze in related work (see Hausmann and Panizza (2003), Eichengreen and Hausmann (2010) and Du et al. (2017)).

[5]To compute the cyclical components of both variables we de-trend them using an HP filter. Main results are invariant to using a linear trend to filter the data (see Table A1).

[6]For a detailed study of debt dynamics during this episode see Calvo and Mendoza (1996).

Table 1. Facts on Sovereign External Debt by Currency

Country	Total Debt Average (% of GDP)	Share of Debt in LC Average (% of Debt)	Share of Debt in LC Δ 2014-04 (% of Debt)	Share of Debt in LC Correlation with Output
Argentina	28%	6%	-2%	24%
Brazil	12%	50%	57%	72%
Bulgaria	43%	0%	1%	53%
China	7%	8%	45%	-23%
Egypt	34%	10%	0%	31%
Hungary	38%	34%	-15%	50%
India	14%	8%	23%	15%
Indonesia	30%	15%	27%	27%
Lithuania	19%	3%	-3%	-70%
Malaysia	23%	56%	74%	44%
Mexico	17%	41%	62%	59%
Peru	31%	13%	27%	64%
Philippines	35%	10%	25%	-7%
Poland	22%	37%	12%	5%
Russia	5%	13%	46%	28%
South Africa	9%	55%	46%	16%
Thailand	12%	57%	79%	-11%
Turkey	20%	28%	24%	62%
Average	22%	25%	29%	24%
Median	21%	14%	26%	27%
Std. Dev.	11%	20%	28%	36%

Notes: The average external public debt is computed over the period 1990-2014. The share of debt in LC refers to the share of external public debt denominated in local currency. The correlation between output and the share of debt in LC refers to the correlation between the cyclical component of real GDP and the cyclical component the share of external public debt denominated in local currency. Both variables are detrended using the HP filter. The share of debt in local currency and the correlation with GDP is computed for the period 2004-2014, when data by currency becomes available. Δ 2014-04 refers to the difference between the share of debt in local currency in 2014 and in 2004. For countries in which the 2004 data was not available, the difference is taken from the earliest data available.

of debt are not the main drivers. Since the data on debt stocks is measured at face value, the only valuation change can come from movements in the exchange rate. Table A1 shows the correlation of de-trended output and the share of debt in local currency computed at constant exchange rates over time. The individual correlations are similar to those computed at current exchanges rate suggesting that these are not driven by valuation effects. In the following subsection we argue that the data on active issuance decisions show cyclical patterns that are similar to those observed in the stocks data.

2.2. *Data on Debt Issuance*

We repeat the same analysis using data on bond issuances. This data measures directly the changes in debt that are due to active issuance decisions of governments, isolating from changes in the currency composition of debt due to cross-border flows of already issued debt. Additionally, this data has the advantage that it is available for a longer period of time. We collected data on sovereign bonds issued by all countries in our sample for the period 1990-2014. The source of the data is Bloomberg. Data on each bond includes the institutional name of the debtor, the issue date, maturity, face value, and currency denomination. In Appendix B.1 we provide details on the construction of our dataset and assess its consistency with the data on debt stocks at the country level. One drawback of this data is that we do not know the identity of the creditor. This implies that some of the bonds can be acquired by domestic residents. We address this issue by dropping bonds that are either issued by central banks, have very small face values or very short maturities. These securities are more likely to be purchased by domestic investors.[7] In Appendix B.1 we show that results remain unchanged when we do not drop these bonds.

Table 2 reports similar moments to those in the previous table but for our issuance data. The first column shows the average annual level of debt issuance as a percentage of GDP. Total issuance is computed as the sum of the face value of debt issued in a given year for a given country. The average level of issuance in our dataset is 7.8% of GDP, suggesting that our dataset has broad coverage. As in the case of debt stocks, this average figure hides significant levels of heterogeneity across countries.

The second and third columns report the average share of debt issuance denominated in local currency for the periods 1990-2003 and 2004-2014, respectively. Consistent with the data on debt stocks, the generalized increase in the share of debt in local currency is also present in

[7]For example, IMF and World Bank (2016) argue that 'local banks continue to dominate holdings of local market debt in many emerging markets, particularly at the shorter end of the maturity spectrum'.

Table 2. Facts on Sovereign Debt Issuance by Currency

Country	Annual Issuance	Share of Issuance in LC		
	Average (% of GDP)	Avg. 90-03 (% of Issuance)	Avg. 04-14 (% of Issuance)	Correlation with Output
Argentina	4.3%	5%	29%	-22%
Brazil	12.8%	68%	99%	32%
Bulgaria	8.3%	42%	80%	21%
China	3.9%	91%	100%	-6%
Egypt	12.9%	76%	95%	2%
Hungary	16.3%	95%	88%	9%
Indonesia	2.1%	25%	62%	-18%
Lithuania	5.9%	76%	72%	42%
Malaysia	6.1%	94%	100%	35%
Mexico	4.6%	59%	93%	47%
Peru	7.0%	21%	58%	-31%
Philippines	7.0%	67%	88%	3%
Poland	10.6%	87%	88%	32%
Russia	3.8%	62%	96%	43%
South Africa	10.5%	82%	98%	-0%
Thailand	3.5%	66%	96%	-64%
Turkey	12.5%	52%	93%	42%
Average	7.8%	63%	84%	10%
Median	7.0%	67%	93%	9%
Std. Dev.	4.1%	27%	19%	31%

Notes: The average annual issuance is computed over the period 1990-2014. It is computed as the sum of the face value of all bonds issued in a given year as a percentage of GDP and then averaged across years. The share of issuance in LC refers to the share of bond issuance denominated in local currency. The correlation between output and the share of debt in LC refers to the correlation between the cyclical component of real GDP and the cyclical component the share of bond issuance denominated in local currency. Both variables are detrended using the HP filter. See Appendix B.1 for details on the issuance data.

the issuance data. The average share of issuance in local currency in the last decade is 84%, significantly higher than the 63% average for the 1990-2003 period. Additionally, the increase was generalized, happening in all countries except two. It is worth noting that the average share of debt issuance in local currency is higher than the share of debt in local currency in the previous table. This is to be expected because of several reasons. First, issuance (imperfectly) measures the change in debt and given the dedollarization trend the marginal unit of debt should be more tilted towards local currency than the average unit of debt (reflected by the total debt level). Second, local currency debt typically has shorter maturities implying a larger flow of issuance for a given stock. Finally, in our issuance data we are imperfectly filtering out debt held by domestic investors and this debt tends to be more tilted towards local currency than the external sovereign debt.

The last column reports the correlation of the cyclical component of the share of issuance in local currency and the cyclical component of GDP for the 1990-2014 period. Consistent with the data on debt stocks, for 70% of the countries in our sample this correlation is positive. Governments tilt their issuance towards debt denominated in local currency in booms. The average correlation across countries is 10%, which is lower than the correlation between the share of debt stock in local currency and output. In this case issuance data is much more volatile than stock data since issuances are made only occasionally, and this attenuates the correlations. This cyclical pattern is more evident when we analyze the evolution of the country-average share of debt in local currency and the country-average cyclical component of GDP (see Figure A.4). Not only does the average share of debt in local currency fall during the global financial crisis, but it also falls during the Tequila and Russian crises. The correlation between these two variables is 31%.

In summary, in this section we use data on debt stocks and issuance to document three stylized facts regarding the currency-composition of external sovereign debt that will be the main focus of the paper. These facts are: *(i)* an average currency composition of sovereign external debt that is tilted towards foreign currency, *(ii)* the recent increase in the share of debt denominated in local currency, and *(iii)* its procyclical behavior.[8] The first two facts are consistent with previous work of Du and Schreger (2015), that document the currency composition of external liabilities in 14 emerging economies. The third fact is new to the literature.

3. A Model of Debt Currency Composition

In this section we construct a dynamic model to study the currency composition of sovereign debt. The environment is a small open economy in which a representative risk-averse household

[8] These patterns are not necessarily present in the case of private external debt. Du and Schreger (2015) show that this debt is mostly denominated in foreign currency and did not exhibit an increase in the share of debt in local currency. This facts has not yet been studied theoretically and goes beyond the scope of this paper.

consumes tradable and non-tradable goods and faces a stochastic endowment. The government makes the economy's savings decisions through the choice of debt issuance, and conducts monetary policy through the choice of inflation. The government is benevolent but lacks commitment in debt issuance and the choice of inflation. There exists a foreign and a local monetary unit of account, in which prices and contracts are denominated. Two securities are available for saving and borrowing purposes: debt denominated in foreign and local currency. In the next subsection we present the theoretical framework and discuss the role of key assumptions.

3.1. *The Model Economy*

Households. Preferences are defined over an infinite stream of consumption and inflation:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\{u(c_t)-l(\pi_t)\}\right], \tag{1}$$

where $\beta\in(0,1)$ is the subjective discount factor, c_t denotes consumption in period t, $u:\mathbb{R}_+\rightarrow\mathbb{R}$ is a differentiable, increasing and concave function, $\pi_t\equiv\frac{P_t}{P_{t-1}}$ is the gross inflation rate at period t, P_t denotes the aggregate level of prices in period t (to be described in more detail later), and $l:\mathbb{R}_+\rightarrow\mathbb{R}$ is a differentiable, convex function with $l'(\pi^*)=0$. The disutility from inflation, discussed in more detail in section 3.3, captures the distortionary costs associated to inflation.

The consumption good is assumed to be a composite of tradable and non-tradable goods

$$c_t=C(c_{T,t},c_{N,t}), \tag{2}$$

where $c_{T,t}$ is consumption of tradable goods in period t, $c_{N,t}$ is consumption of non-tradable goods in period t, and $C:\mathbb{R}_+^2\rightarrow\mathbb{R}_+$ is a differentiable function, increasing in both arguments, concave, and homogeneous of degree one.

Households receive a stream of endowments of tradable goods $y_{T,t}$ and non-tradable goods $y_{N,t}$. They are assumed to be hand-to-mouth and the government makes savings decisions for them through an active management of lump-sum transfers to them. The budget constraint of households each period is then given by:

$$p_{T,t}c_{T,t}+p_{N,t}c_{N,t}=p_{T,t}y_{T,t}+p_{N,t}y_{N,t}+T_t, \tag{3}$$

where $p_{T,t}$ and $p_{N,t}$ are prices of tradables and non-tradables and T_t are lump-sum transfers from the government, all measured in local currency.

The *household's problem* is to choose state-contingent plans $\{c_t,c_{T,t},c_{N,t}\}_{t=0}^{\infty}$ that maximize utility (1), subject to the aggregation technology (2), the sequence of budget constraints (3), the given sequence of prices $\{p_{T,t}, p_{N,t}\}_{t=0}^{\infty}$, and the given sequence of endowments and government transfers $\{y_{T,t}, y_{N,t}, T_t\}_{t=0}^{\infty}$. Combining the first-order conditions of the household's problem, the optimal choice between tradable and non-tradable goods can be described by:

$$\frac{C_{c_T,t}}{C_{c_N,t}} = \frac{p_{T,t}}{p_{N,t}}, \tag{4}$$

where we denote $f_{x,t}$ the derivative of f with respect to the variable x_t to simplify notation.

Government. The government chooses inflation and sovereign external debt, making lump-sum transfers from the proceedings of net borrowing to households. Two securities are available for saving and borrowing purposes: debt in foreign and local currency. Debt consists of long-term bonds with a deterministic decay rate. We set the same duration for both types of debt to restrict our analysis to the effect of the currency denomination of debt. By issuing one bond in foreign (local) currency in period t the government promises to repay one unit of foreign (local) currency in period $t+1$, δ in period $t+2$, δ^2 in period $t+3$, and so on, and in exchange receives q_t^* (q_t), units of foreign (local) currency in period t. The case of $\delta = 0$ corresponds to short-term debt, as δ increases the maturity of the bond increases. The convenience of this type of contracts is its recursive structure. Let b_t^* and b_t denote the amount coupons in foreign and local currency to be paid in period t, respectively. The law of motion of these coupons is given by

$$b_{t+1}^* = \delta b_t^* + i_t^*,$$
$$b_{t+1} = \delta b_t + i_t,$$

where i_t^* and i_t denote the period t issuance of new bonds in foreign and local currency, respectively. The budget constraint of the government expressed in local currency is then given by

$$T_t = e_t q_t^* \left(b_{t+1}^* - \delta b_t^*\right) + q_t \left(b_{t+1} - \delta b_t\right) - e_t b_t^* - b_t, \tag{5}$$

where e_t is the exchange rate (i.e., the price of foreign currency in terms of local currency).

Foreign Lenders. We assume securities are priced by risk-neutral foreign investors that have access to a risk-less security denominated in foreign currency that pays the gross international interest rate R. The price of both securities are then given by

$$q_t^* = \frac{1}{R-\delta}, \tag{6}$$

$$q_t = \frac{1}{R}\mathbb{E}_t\left[\frac{e_t}{e_{t+1}}\left(1+\delta q_{t+1}\right)\right]. \tag{7}$$

Equilibrium. The market for non-tradable goods clears:

$$c_{N,t} = y_{N,t}. \tag{8}$$

Assuming the law of one price holds for tradable goods and normalizing the international price of tradables to one, we get that $p_{T,t} = e_t$. Given the assumed preferences and aggregation technology the aggregate price level of the economy is given by $P_t = e_t\left(C_{c_T}\left(\frac{c_{T,t}}{y_{N,t}},1\right)\right)^{-1}$.[9] It will be convenient to use this expression to denote the inverse of the equilibrium nominal exchange rate as:

$$e_t^{-1} = r\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right), \tag{9}$$

where $r\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right) = \frac{1}{P_t}\left(C_{c_T}\left(\frac{c_{T,t}}{y_{N,t}},1\right)\right)^{-1}$ is a differentiable function, decreasing in its first argument and increasing in its second argument.

Aggregating households and government budget constraints and imposing condition (8), we obtain the resource constraint of the economy, expressed in foreign currency as:

$$c_{T,t} = y_{T,t} + q_t^*\left(b_{t+1}^* - \delta b_t^*\right) - b_t^* + r\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right)\left[q_t\left(b_{t+1}-\delta b_t\right) - b_t\right]. \tag{10}$$

We can now define the competitive equilibrium of this economy as follows:

Definition 1 (Competitive equilibrium). *Given initial debt positions, b_0^* and b_0, initial price level, P_0, a state-contingent sequence of endowments, $\{y_{T,t}, y_{N,t}\}_{t=0}^{\infty}$, and government policies $\{\pi_t, b_{t+1}^*, b_{t+1}\}_{t=0}^{\infty}$* a competitive equilibrium *is a state-contingent sequence of allocations $\{c_{T,t}, c_{N,t}, T_t\}_{t=0}^{\infty}$ and prices $\{q_t^*, q_t, e_t, P_t\}_{t=0}^{\infty}$, such that:*

(1) Allocations solve the household's problem given equilibrium prices,

(2) Transfers satisfy the government budget constraint,

(3) Debt prices satisfy (6) and (7),

(4) The market for non-tradable goods clears.

[9] Formally, P_t is the ideal price index, which solves the following cost minimization problem

$$P_t \equiv \min_{c_{T,t},c_{N,t}} e_t c_{T,t} + p_{N,t}c_{N,t} \quad s.t. \quad C(c_{T,t}, c_{N,t}) = 1.$$

3.2. *Optimal Government Policies*

In this section we formulate the problem of optimal policy for a benevolent government that lacks commitment regarding debt issuance and monetary policy. We focus on the notion of a Markov Perfect Equilibrium in which policies depend on payoff-relevant states. We assume that y_N is constant over time and y_T follows a Markov process with transition probability $g_y(y_T, y_T')$.

Optimal debt and monetary policy. Define $\mathbf{s} = \{b^*, b, y_T, P_{-1}\}$ as the aggregate state. Since the planner lacks commitment, it takes as given the optimal monetary policy of future planners. Let $\mathcal{R}(\mathbf{s})$ be the perceived inverse of the nominal exchange rate policy function of future planners. The planner's problem written in recursive form is given by:

$$V(\mathbf{s}) = \max_{b^{*\prime}, b', \pi, c_T} u\left(C(c_T, y_N)\right) - l\left(\pi\right) + \beta \mathbb{E}\left[V(\mathbf{s}')\right] \tag{P1}$$

subject to

$$c_T = y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) b + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \tilde{q}\left(b' - \delta b\right),$$

$$\tilde{q} = \frac{1}{R}\mathbb{E}\left[\mathcal{R}(\mathbf{s}') + \delta \tilde{q}(\mathbf{s}')\right],$$

$$P = \pi P_{-1},$$

where $\tilde{q} = qe^{-1}$ is the price of debt in local currency expressed in foreign currency. The first restriction corresponds to the resource constraint of the economy where the prices of both debts have been already replaced. The second restriction is the recursive definition of the price of local currency debt. The third restriction defines inflation as the increase in the aggregate level of prices. The government is also subject to a debt limit that prevents Ponzi schemes.

Provided that inflation is non-zero in equilibrium, the price level and the nominal level of debt denominated in local currency will exhibit a stochastic trend. We deal with this property by detrending the variables denominated in local currency by the lagged level of aggregate prices P_{t-1}. We denote by $\hat{x}_t = \frac{x_t}{P_{t-1}}$ the detrended version of a variable x_t. In Appendix C.1 we show that the problem (P1) has an equivalent detrended specification in which the state space is reduced by one dimension by combining b and P_{-1} into one single variable $\hat{b}$.

Given the recursive formulation of the planner's problem we define equilibrium as follows:

Definition **2** (Markov Perfect Equilibrium). *A Markov Perfect Equilibrium consists of a set of policy functions* $\{c_T(\boldsymbol{s}), b^{*\prime}(\boldsymbol{s}), b'(\boldsymbol{s}), \pi(\boldsymbol{s})\}$*, value function* $V(\boldsymbol{s})$*, perceived policies* $\mathcal{R}(\boldsymbol{s})$ *and prices* $\{q^*(\boldsymbol{s}), q(\boldsymbol{s})\}$ *such that*

- *Policies* $\{c_T(\boldsymbol{s}), b^{*\prime}(\boldsymbol{s}), b'(\boldsymbol{s}), \pi(\boldsymbol{s})\}$ *solve the planner's problem* (P1).

– *Prices* $\{q^*(\boldsymbol{s}), q(\boldsymbol{s})\}$ *solve* (6), (7).
– *Perceived policies coincide with actual policies:* $\mathcal{R}(\boldsymbol{s}) = r\left(P(\boldsymbol{s}), \frac{c_T(\boldsymbol{s})}{y_N}\right)$.

Optimal debt given monetary policy. To characterize the optimal policy trade-offs, it is useful to consider the problem of a government that can only choose debt policies for a given monetary policy. In particular, in this alternative problem it is assumed that inflation follows a response function $\Pi(\mathbf{s}, b^{*'}, b^{'}, \epsilon^\pi)$, where the exogenous term ϵ^π is stochastic and follows a Markov process potentially correlated with y_T, with transition probability $\tilde{g}(y_T, \epsilon^\pi, y_T', \epsilon^{\pi\prime})$. Each period, after $\{y_T, \epsilon^\pi\}$ is realized, the government chooses $\{b^{*'}, b^{'}, c_T\}$. Define $\tilde{\mathbf{s}} = \{\mathbf{s}, \epsilon^\pi\}$ as the aggregate state for this problem. The planner's problem for a given monetary policy written in recursive form is given by:

$$\tilde{V}(\tilde{\mathbf{s}}) = \max_{b^{*\prime}, b^\prime, c_T} u\left(C(c_T, y_N)\right) - l\left(\Pi(\tilde{\mathbf{s}}, b^{*'}, b^{'})\right) + \beta \mathbb{E}\left[\tilde{V}(\tilde{\mathbf{s}}')\right] \tag{P2}$$

subject to

$$c_T = y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) b + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \tilde{q}\left(b' - \delta b\right),$$

$$\tilde{q} = \frac{1}{R}\mathbb{E}\left[\mathcal{R}(\mathbf{s}') + \delta \tilde{q}(\mathbf{s}')\right],$$

$$P = \Pi(\tilde{\mathbf{s}}, b^{*'}, b^{'}) P_{-1},$$

and subject to the debt limit. Appendix C.1 also shows the de-trended specification of problem (P2) that reduces the state space dimensionality. We define equilibrium for a given monetary policy as follows:

Definition **3** (Markov Perfect Equilibrium Given Monetary Policy)**.** *A Markov Perfect Equilibrium given monetary policy* $\Pi(\mathbf{s}, b^{*'}, b^{'}, \epsilon^\pi)$ *consists of a set of policy functions* $\{c_T(\tilde{\mathbf{s}}), b^{*\prime}(\tilde{\mathbf{s}}), b'(\tilde{\mathbf{s}})\}$, *value function* $V(\tilde{\boldsymbol{s}})$, *perceived policies* $\mathcal{R}(\tilde{\boldsymbol{s}})$ *and prices* $\{q^*(\tilde{\boldsymbol{s}}), q(\tilde{\boldsymbol{s}})\}$ *such that*

– *Policies* $\{c_T(\tilde{\boldsymbol{s}}), b^{*\prime}(\tilde{\boldsymbol{s}}), b'(\tilde{\boldsymbol{s}})\}$ *solve the planner's problem* (P2).
– *Prices* $\{q^*(\tilde{\boldsymbol{s}}), q(\tilde{\boldsymbol{s}})\}$ *solve* (6), (7).
– *Perceived policies coincide with actual policies:* $\mathcal{R}(\boldsymbol{s}) = r\left(P(\boldsymbol{s}), \frac{c_T(\boldsymbol{s})}{y_N}\right)$.

3.3. *Discussion of Assumptions*

The model features an endogenous nominal exchange rate. Fluctuations in the nominal exchange rate, which are associated with endogenous movements in the real exchange rate and in the domestic inflation rate, are key for the hedging properties associated to debt in local currency in the model. The fluctuations in the real exchange rate come from movements in the relative price of tradable and non-tradable goods which are relevant in the data in emerging economies (see for example, Burstein et al. (2005)), hence the relevance of our tradable/non-tradable structure in our model.[10] We abstract from fluctuations in foreign inflation, which in the data are an order of magnitude smaller than the shocks we analyze. Theoretically, fluctuating foreign inflation can deliver additional hedging properties for debt in foreign currency (see Bohn (1990)).

A consequence of the simplifying assumption of hand-to-mouth households is that in the model the stock of external public debt coincides with the stock of total external debt. We follow most of the sovereign debt literature and make this assumption based on technical reasons.[11] However, we conjecture that the the omission of private external debt might not be quantitatively large since, as we document in Table A2, in the data its size is relatively small and it does not feature strong cyclical properties.

In our baseline model we assume that the maturity of debt in local and foreign currency is the same in order to make the two securities differ only in their currency of denomination. We also assume that the government cannot engage in outright default. This assumption is made to focus on the trade-offs associated to the currency composition of debt. In Appendix D we layout and solve an extended version of the model in which we allow for outright default. The model with outright default adds a trade-off associated to the decision of repayment/default that is mostly governed by the total stock of outstanding debt, whereas the currency split of debt is still mostly determined by the hedging/incentives trade-off analyzed in our baseline model.

Finally, we model the cost of inflation as a separable loss function to ease the exposition of the main trade-offs. These costs can reflect losses due to economic inefficiencies associated to inflation, such as losses of real balances or price dispersion. In Appendix D we show that models of cash in advance or money in the utility function can feature a disutility term that is increasing in inflation, similar to the one we model. We also show that our main quantitative results are in line with a calibrated version of a model with preferences for real money balances. In the same Appendix, we also discuss the implications of expressing the cost function in terms of exchange rate depreciation, which could be relevant for countries that face currency mismatch in private balance-sheets.

[10]If the model would feature a single good the hedging properties of debt in local currency would only stem from movements in inflation, which are not very cyclical in the data.

[11]An economy with both private and public debt portfolios would feature four endogenous state variables and two Euler equations as implementability constraints, quickly running into the curse of dimensionality.

4. The Trade-Offs of the Debt Currency Composition

This section describes the main trade-offs associated to the choice of the currency composition of sovereign debt. We argue that the optimal currency composition of debt weighs the hedging benefits of debt in local currency against its incentive problems. On the one hand, debt in local currency is useful for hedging if the economy features countercyclical exchange rates (i.e. local currencies that depreciate in recessions). On the other hand, debt in local currency is harmful by giving rise to incentives to dilute debt repayment through distortionary nominal currency depreciation.

4.1. *The Hedging Benefits of Debt in Local Currency*

This section highlights the hedging benefits of debt in local currency. To do so we focus on the Markov equilibrium given monetary policy (Definition 3), for a particular case of $\delta = 0$, $l(\pi) = 0$ and in which $\Pi(\tilde{\mathbf{s}}, b^{*'}, b')$ is such that

$$r\left(P, \frac{c_T}{y_N}\right) = \frac{\epsilon^\pi}{P_{-1}}. \tag{11}$$

This approach facilitates the characterization of the hedging properties in debt in local currency in terms of the co-movement of the inverse of the detrended nominal exchange rate $\hat{e}^{-1} = \epsilon^\pi$ and the tradable endowment. Since the payoffs of local currency debt are exogenous and the government doesn't face losses from inflation, the government problem (P2) in this particular case is equivalent to a portfolio problem of an agent that faces income risk and can choose saving and borrowing in foreign and local currency. This particular type of portfolio problem with incomplete markets and income risk does not have closed-form solutions.[12] The optimal consumption process satisfies the following optimality conditions (see proof of Proposition 1 for their derivation)

$$u'(c_t)C_{c_T,t} = \beta R \mathbb{E}\left[u'(c_{t+1})C_{c_T,t+1}\right], \tag{12}$$

$$\mathbb{COV}\left[u'(c_t)C_{c_T,t}, \hat{e}_t^{-1}\right] = 0. \tag{13}$$

Along the optimal consumption path, the consumption process features the Euler equation for tradable goods (equation (12)). Equation (13) comes from the possibility of issuing debt in local currency which is a security that is priced by risk-neutral investors that care about the expected return in foreign currency. According to this equation, along the optimal consumption path the marginal utility of tradable consumption is isolated from any risk that stems from fluctuations

[12] Duffie et al. (1997) characterize the solution to this class of problem in continuous time. Viceira (2001) provides an analytical characterization of an approximate solution to a similar problem.

in the exchange rate. This consumption path is attained with certain positions in foreign and local currency that depend on the correlation between tradable income and the inverse of the nominal exchange rate. This is made clear in the following proposition that characterizes the optimal debt position in local currency for a particular joint stochastic process for tradable endowment and the inverse of the nominal exchange rate.

PROPOSITION **1.** *Consider the equilibrium given monetary policy (Definition 3), in which $\hat{e}_t^{-1}$ is exogenous and stochastic. Assume $y_{T,t}$ and $\hat{e}_t^{-1}$ follow a joint stochastic process which is i.i.d. over time, that $y_{N,t} = y_N$ for all t, no losses of inflation ($l(\pi_t) = 0$), and short-term debt ($\delta = 0$). Define the standard deviation of random variable X as σ_X and correlation coefficient between X and Y as $\rho(X, Y)$. Then:*

(1) If $\rho(y_{T,t}, \hat{e}_t^{-1}) = 1$ then $b_t = P_{t-1}\frac{\sigma_{y_T}}{\sigma_{\hat{e}^{-1}}} > 0$ is part of the solution set of (P2) and $\{c_t\}$ follows a deterministic path.

(2) If $\rho(y_{T,t}, \hat{e}_t^{-1}) = 0$ then $b_t = 0$ is part of the solution set of the (P2).

(3) If $\rho(y_{T,t}, \hat{e}_t^{-1}) = -1$ then $b_t = -P_{t-1}\frac{\sigma_{y_T}}{\sigma_{\hat{e}^{-1}}} < 0$ is part of the solution set of (P2) and $\{c_t\}$ follows a deterministic path.

Proof. See Appendix E. □

This proposition illustrates the hedging properties of debt in local currency. When tradable income is perfectly positively correlated with the inverse of the nominal exchange rate, the government can attain perfect tradable consumption smoothing for households. It does so by issuing positive debt in local currency. This way, when there is a negative tradable income shock the currency depreciates and the government needs to repay less in terms of tradable consumption, muting this way the negative effect of the income shock. When income is perfectly negatively correlated with the inverse of the exchange rate the government can also attain tradable consumption smoothing for the households by saving in local currency. This way, when a negative tradable income shock realizes the currency appreciates and the return on its savings, measured in tradable consumption, is higher. These results on full insurance, which are also stressed in Korinek (2009), follow from the fact that the economy has an asset whose payoff comoves perfectly with the only shock that hits the economy. When tradable income is uncorrelated with the exchange rate debt in local currency provides no hedging against income risk. In this case taking any position in local currency exposes households to an additional source of risk without perceiving any excess expected return for this since debt pricing is done by risk-neutral investors.

A well-known stylized fact in emerging economies, documented for our sample of economies in Table A3, is that exchange rates are negatively correlated with aggregate tradable income (that is, $\rho(y_{T,t}, e_t^{-1}) > 0$). Given the results shown in Proposition 1, this introduces a reason for why debt in local currency can be a useful hedge against income risk. In next section we use the patterns observed in the data between income and exchange rate to conduct a quantitative assessment of this channel based on the debt problem for given monetary policy.

4.2. *The Incentive Problems of Debt in Local Currency*

This section highlights the incentive problems associated to debt in local currency. These problems arise due to the fact that the value of repayment of debt in local currency, $r\left(P_t, \frac{c_{Tt}}{y_{Nt}}\right)$, can be diluted ex-post. There are two channels of dilution. One is *dilution through inflation*, which operates through the effect that inflation choices have on the nominal exchange rate, and the other one is *dilution through real exchange rate*, which operates through the effect that debt choices have on tradable consumption and on the nominal exchange rate. We analyze each channel separately by focusing on the Markov equilibrium with optimal time-consistent monetary policy in a deterministic setting without shocks to endowments and short-term debt ($\delta = 0$).

To analyze the dilution through inflation channel we focus on the case in which $r_c(P_t, c_{Tt}) = 0$. The trade-offs associated to the choice of monetary and debt policies emerge from the analysis of the first order conditions of the detrended government problem. We provide a derivation of these conditions in Appendix E. Consider a case in which the outstanding level of detrended debt in local currency is positive ($\hat{b}_t > 0$). The trade-off involved in the choice of inflation can be seen from the first order condition associated to inflation:

$$-l'(\pi_t) = u'(c_t)C_T(c_{T,t}, y_{N,t})r_P(\pi_t, c_{Tt})\hat{b}_t.$$

On the one hand, higher inflation entails direct losses for $\pi_t \geq \pi^*$. On the other hand, higher inflation dilutes the value of repayment of local-currency debt measured in tradable consumption units and allows for higher consumption by saving resources. A marginal increase in inflation reduces debt repayment and increases tradable consumption by $r_P(\pi_t, c_{Tt})\hat{b}_t$. This marginal benefit is higher, the higher the outstanding stock of debt in local currency. This implies that, in most cases, the optimal level of inflation is increasing in the level of debt in local currency.

Additionally, to the extent that tradable consumption is procyclical in equilibrium, the optimal inflation rate will be countercyclical. This is due to the fact the benefits of diluting debt through inflation are countercyclical. In periods of low tradable endowment, the marginal utility of consumption is high and so is the marginal benefit of saving resources for consumption by reducing debt repayments. This increases the attractiveness of diluting debt in local currency through inflation.

Inflation is distortionary from an ex-ante perspective. The reason is that in equilibrium foreign investors anticipate the optimal inflation choices and offer lower debt prices for higher levels of debt in local currency to compensate for the future debt dilution. The lower debt prices offset the ex-post benefits of debt dilution. Therefore, from an ex-ante perspective inflation only entails the direct welfare costs. The government takes this into account when making debt choices. In particular, the government chooses a lower level of debt in local currency than what he would choose if he could commit not to inflate ex-post, precisely to avoid incurring in costly inflation in equilibrium. This can be seen in the modified Euler equation that characterizes the choices of debt in local currency in the detrended government problem:

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1}\underbrace{\frac{1}{1+\frac{\hat{b}_{t+1}r_{P,t+1}\pi_{\hat{b},t+1}}{r(\pi_t,c_{Tt})}}}_{\text{Discipline Effect}}.$$

The Euler equation is augmented by a disciplining effect that acts as a higher interest rate as long as the choice of debt in local currency for next period is positive. The choice of debt in foreign currency is governed by the following Euler equation

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1}\underbrace{\frac{1}{1+\hat{b}_{t+1}r_{P,t+1}\pi_{b^*,t+1}}}_{\text{Discipline Effect}}.$$

The disciplining effect here depends on the sensitivity of optimal inflation to debt in foreign currency. Insofar the optimal level of inflation does not depend on the level of outstanding debt in foreign currency, the optimal choice of debt in foreign currency is undistorted, as characterized by the standard Euler equation.

We can characterize the equilibrium debt position in local currency for the particular case in which there are no inflation costs. This is summarized in the following proposition.

PROPOSITION **2** (Market Shutdown). *Consider the Markov perfect equilibrium (Definition 2). If $l(\pi) = 0$ for all π, the equilibrium features a market shutdown for debt in local currency: $q(\boldsymbol{s})b'(\boldsymbol{s}) = 0$.*

Proof. See Appendix E. □

The intuition behind this result is simple and relies on the lack of commitment regarding monetary policy. If the government does not have any cost of inflating debt, it will optimally choose to completely dilute any positive debt in local currency by choosing an arbitrarily large level of inflation. Foreign investors anticipate this and are unwilling to lend in local currency (i.e. $q(\mathbf{s}) = 0$). In this case of dilution through inflation it is with monetary policy that the government misbehaves and with debt policy that it disciplines this misbehavior.

Now we analyze the dilution through real exchange rate channel by focusing, for simplicity, in the case of a constant given monetary policy $\Pi(\mathbf{s}, b^{*'}, b^{'}, \epsilon^{\pi}) = \pi$, which would correspond to real debt denominated in non-tradable goods. Another way to dilute the value of repayment of debt in local currency is to depreciate the real exchange rate ex-post. This can be done through postponing tradable consumption by reducing debt issuance for the following period at the expense of distorting inter-temporal consumption decisions. This would call for lower levels of debt ex-post. However, this effect is distortionary ex-ante since the benefits of dilution are offset by lower debt prices offered by foreign investors that anticipate this behavior. The government takes this into account and affects its debt decisions to discipline the future incentives to dilute debt. Interestingly, debt choices are both the instruments used to dilute debt through real exchange rate depreciation and the instruments used to discipline the temptation to do so in the future. This can be seen from modified Euler equation for debt in local currency

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1} \underbrace{\left(1 + r_{c,t}\hat{b}_t\right)}_{\text{Dilution through RXR}} \underbrace{\frac{1}{1 + \frac{\hat{b}_{t+1} r_{c,t+1} \frac{1}{R} \frac{\partial (r(\pi_{t+2}, c_{Tt+2})\hat{b}_{t+2} + b^*_{t+2})}{\partial \hat{b}_{t+1}}}{r(\pi_{t+1}, c_{Tt+1})}}}_{\text{Discipline Effect}}.$$

The Euler equation features a component $\left(1 + r_{c,t}\hat{b}_t\right)$ that captures the dilution through real exchange rate and acts as a perceived higher interest rate when there is a positive outstanding debt in local currency. The presence of this term is due to the fact that raising one additional unit of good with debt increases tradable consumption by $1/\left(1 + r_{c,t}\hat{b}_t\right)$ which is less than one. This is because of the effect of consumption on debt repayment: higher tradable consumption increases the nominal exchange rate by $r_c(\pi_t, c_{Tt})$ (through an increase in the real exchange rate) which in turn increases the value of debt repayment and decreases tradable consumption. The Euler equation also features a component that captures disciplining role of debt current choices on future incentives to manipulate the payoff of debt in local currency through future debt choices. The disciplining effect is present in the case in which future choices of debt in local currency are non-zero, and depends on the elasticity of the payoff of debt in local currency to the real exchange rate and the extent to which current choices of debt in local currency affect future debt choices.

A similar expression characterizes the optimal choice of debt in foreign currency

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1} \underbrace{\left(1 + r_{c,t}\hat{b}_t\right)}_{\text{Dilution through RXR}} \underbrace{\frac{1}{1 + \hat{b}_{t+1} r_{c,t+1} \frac{1}{R} \frac{\partial(r(c_{Tt+2},\pi_{t+2})\hat{b}_{t+2} + b^*_{t+2})}{\partial b^*_{t+1}}}}_{\text{Discipline Effect}}.$$

The dilution effect is the same as the one for debt in local currency. The discipline effect now depends on the extent to which current choices of debt in foreign currency affect future debt choices. Both effects are present insofar outstanding debt in local currency and current choices of debt in local currency are different from zero.

In summary, while debt in local currency has useful hedging properties if the nominal exchange rate correlates negatively with income, it also gives perverse incentives to dilute its repayment through inflation or future debt choices. While the former characteristic points toward a higher debt in local currency, the latter calls for a substitution of debt in local currency for debt in foreign currency.

5. Quantitative Analysis

In this section we calibrate the model to match the salient features of the average economy studied in section 2 and evaluate the quantitative results in the light of the data on the currency composition of sovereign external debt. We then use our quantified model to offer potential explanations for the recent increase in the share of debt in local currency.

5.1. *Calibration*

To better understand the quantitative properties of the model we calibrate two economies. The first calibration corresponds to a "debt problem", in which the government only chooses debt, as in Definition 3, with no costs of inflation and the additional assumption (11) that the inflation policy is such that the exchange rate is exogenous and not affected by debt choices. This specification is aimed at quantifying the hedging properties of debt in local currency if one takes as given the joint dynamics of exchange rates and income observed in the data. The second and main calibration corresponds to a "full model", in which the government chooses debt and inflation, as in Definition 2. We maintain the calibration of the two economies as close as possible to be able to make comparisons across the model specifications.

One period corresponds to one year. The period utility function is assumed to be given by:

$$u(c_{T,t}, c_{N,t}) - l(\pi_t) = \frac{\left(c_{T,t}^{a} c_{N,t}^{1-a}\right)^{1-\sigma}}{1-\sigma} - \frac{\psi}{2}\left(\pi_t - \pi^*\right)^2,$$

where π^* is a target inflation rate. The parameter values are summarized in Table 3. The risk aversion coefficient σ is set to 5 in both specifications, which is within the upper values considered in the quantitative macroeconomics literature and in the lower values considered in the finance literature. We also consider alternative degrees of risk aversion in Appendix D.1. We set equal shares for tradables and non-tradables in the consumption aggregator to gain computational efficiency when solving the model.[13] The discount factor is calibrated to target an average stock of external public debt of 22% of annual GDP. The calibrated value is $\beta = 0.96$ for both specifications. The international risk-free interest rate is set to 1.04.[14] The value of δ, the decay rate of bonds, is chosen so that the duration of bonds is 4 years, which is in line with the average duration in emerging economies bonds reported in Cruces and Trebesch (2013). In Appendix D.1 we study how our results vary with the choice of this parameter value.

We normalize the level of the non-tradable endowment y_N to one. The exogenous stochastic processes differ over specifications. In the full model the only source of exogenous fluctuations is the level of tradable endowment, which we assume follows an AR(1) process in logs

$$\log y_{T,t} = \rho_{y_T} \log y_{T,t-1} + \epsilon_t^{y}, \tag{14}$$

where $\epsilon_t^y \sim N(0, \sigma_{y_T}^2)$. This process is estimated with annual data on tradable GDP for the period 1990-2014 for the panel of countries analyzed in section 2 that have available data. The data of tradable GDP was detrended using a log-linear trend. This process is estimated to be persistent. The estimation results are shown in Table 3.

For the debt problem, we assume that the shocks to the inflation policy ϵ^π are such that the inverse of the detrended nominal exchange rate, $\hat{e}_t^{-1} = \left(\frac{e_t}{P_{t-1}}\right)^{-1}$, follows the path observed in the data. In this model the exogenous state is thus given by $\mathbf{s}_t^x = (y_{Tt}, \hat{e}_t^{-1})$ which is assumed

[13]Once equilibrium prices are substituted into the budget constraint, the budget constraint becomes the following non-linear function of tradable consumption

$$c_T = y_T - b^* - \hat{b}\frac{1}{\pi}c_T^{1-a} + q^*\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[\frac{1}{\pi'}{c_T'}^{1-a}\left(1+\delta q'\right)\right]\left(\hat{b}' - \delta\hat{b}\frac{1}{\pi}\right),$$

where $\hat{b} = \frac{b}{P_{-1}/a}$ is the de-trended debt in local currency. The advantage of $a = 0.5$ is that c_T can be obtained in closed-form solution from this equation, which reduces by an order of magnitude the time to solve the model.

[14]In both specifications we have that $\beta < R^{-1}$.

Table 3. Model Calibration

Parameter		Debt Problem	Full Model	Source/Target
Preferences				
Risk aversion coefficient	σ	5.00	5.00	
Tradable share in utility	a	0.50	0.50	
Discount factor	β	0.96	0.96	Avg. external debt (22% of GDP)
Target of inflation	π^*	-	1.084	Avg. inflation (8.7%)
Cost of inflation	ψ	-	7.08	Lucas (2000), Burstein & Hellwig (2008)
Endowments and Interest Rate				
Risk free interest rate	R	1.04	1.04	
Decay rate of bonds	δ	0.76	0.76	Cruces and Trebesch (2013)
Non-tradable endowment	y_N	1.00	1.00	Normalization
Autocorrelation of y_T	ρ_{y_T}	0.69	0.69	Estimation, data tradable output
Std. dev. of y_T	σ_{y_T}	0.05	0.05	Estimation, data tradable output

to follow a first-order VAR process in logs,

$$\log \mathbf{s}_t^x = \Phi^0 + \Phi^1 \log \mathbf{s}_{t-1}^x + \varepsilon_t^x, \tag{15}$$

where $\varepsilon_t^x \sim N(0, \Omega)$. We set the constraint that $\phi_{12}^1 = 0$ which implies that lagged inverse of exchange rate does not affect tradable GDP.[15] This joint process is estimated with annual data on tradable GDP and nominal exchange rate for the panel of countries analyzed in section 2 for the 1990-2014 period. We first compute $\hat{e}^{-1}$ by dividing the nominal exchange rate vis-a-vis the US dollar by the lagged level of aggregate CPI and then taking the inverse of this ratio. We then detrend this variable using a log-linear trend. Further details on the computation of variables and the output of the estimation are provided in Appendix B. Both processes are estimated to be persistent. Additionally, output and the inverse of the nominal exchange rate are positively correlated as reflected by the positive covariance between their respective innovations. This positive correlation is consistent with the evidence shown in Table A3.

[15]We impose this constraint to obtain the same process for tradable GDP in the debt problem and the full model. The constraint is not numerically relevant: if we do not impose this constraint, the estimate is close to zero and not statistically significant.

The other set of parameters that differ across specifications are the ones related to the disutility over inflation. In the debt problem inflation costs are assumed to be zero ($\psi = 0$). In the full-model the target rate of inflation is calibrated to deliver an average rate of inflation in line with the observed average inflation of 8.7%. Its calibrated value is 1.084. For the full model, we calibrate the value of ψ, the parameter that governs the disutility of excess inflation, to obtain a direct welfare cost of inflation that is in line with that estimated in Lucas (2000). In that paper, inflation affects real money balances, and a permanent increase of 1% in annual inflation has associated a welfare loss of 0.1% in permanent consumption. These estimates are in line with those in Burstein and Hellwig (2008) that study a richer model with costs of inflation steaming from money balances and menu costs. What we do is compute a permanent increase of 1% in annual inflation and calculate the equivalent drop in permanent consumption associated to it. Then we calibrate ψ such that this drop in consumption is the same as in that paper. The value of ψ that yield such welfare loss is 7.1. Later, we analyze the quantitative relevance of this and other key parameters.

5.2. *Model Predictions and Data*

We analyze the model's quantitative performance by comparing moments from the model's ergodic set with moments of the data. To compute the model's moments we simulate the exogenous stochastic processes of each economy (tradable income in the full model, and tradable income and the inverse of the exchange rate in debt problem) for 100,000 periods and trace the evolution of several macroeconomic variables. The moments are computed by eliminating the first 10,000 observations. The moments from the data correspond the average of countries analyzed in section 2.

5.2.1. *Predictions of Debt Problem*

Table 4 compares the moments regarding debt positions in the model with their data counterparts. We first analyze the moments from the debt problem in the light of the data moments. In the data the average level of external sovereign debt is 22% of GDP, of which 25% is denominated in local currency. In the debt problem the total level of sovereign debt is the same as in the data since it is a target of the calibration. However, the debt problem predicts a share of debt in local currency of 91%, significantly higher than the one observed in the data. The high levels of debt in local currency are due to the strong negative correlation between output and exchange rate shown in the data. This co-movement makes debt in local currency an attractive security for smoothing tradable consumption. We interpret this result as indicating the presence of certain frictions that prevent governments from exploiting the hedging properties of debt in foreign and local currency.

The cyclical component of the implicit inflation rate in the debt problem can be backed out using equation (11).[16] The debt problem does not match the volatilities of the macro variables. The standard deviation of debt, its currency composition and inflation are overestimated in the debt problem specification. The standard deviation of GDP and the exchange rate match their data counterparts since they are mostly determined by the exogenous processes estimated for the shocks.

The debt problem matches the countercyclicality of debt but predicts countercyclical inflation which is at odds with the data. Finally, this specification is unable to replicate the strong procyclicality of the share of debt in local currency. While in the data the correlation between output and the difference between debt in local and foreign currency as a share of GDP is 50%, its model counterpart is 8%. This suggests that the variation of the hedging properties of debt in local currency over the business cycle cannot account for a large share of the cyclical variation of the currency composition of debt observed in the data.

5.2.2. *Predictions of Full Model*

We now focus on the case of the full model economy. This specification introduces the incentive problems associated to debt dilution and endogenizes the determination of the nominal exchange rate in equilibrium. The model moments are reported in the third column of Table 4. As in the debt problem, the average level of total debt matches its data counterpart since it is a target of the calibration. However, unlike the debt problem, the full model approximates better the currency composition of debt. The model predicts a 10% average share of debt in local currency which is closer to the 25% observed in the data. This result suggests that the government's lack of commitment regarding debt issuance policy and monetary policy is quantitatively relevant in determining the optimal currency composition of government debt. Nevertheless, the model underestimates the observed share of debt in local currency.[17]

[16] Its average level is undetermined since we feed in data on the cyclical component of the inverse of the detrended exchange rate. We therefore set the average rate to match the data counterpart.

[17] In the context of our model, this underestimation could be due to an underestimation of inflation costs in our baseline calibration or higher degrees of risk aversion of the borrowers relative to the those of the lenders. We explore the quantitative implications of alternative parametrizations in the next subsection. Alternatively, it could also be due other unmodeled attractive features of debt in local currency such as the flexibility to reduce the country's exposure to self-fulfilling crises, as argued in Aguiar et al. (2015).

Table 4. Data and Model Moments

Moment	Data	Debt Problem	Full Model	Monetary Shocks
Average Levels				
Debt	22.0%	22.2%	22.4%	21.4%
Share of Debt in LC	24.7%	90.8%	9.6%	16.4%
Inflation	8.7%	8.7%	8.7%	8.7%
Standard deviation				
Debt	4.2%	5.5%	3.1%	3.1%
Share of Debt in LC	2.3%	24.4%	3.0%	3.6%
Inflation	3.9%	22.6%	0.1%	0.4%
Exchange Rate	13.2%	14.4%	0.5%	0.7%
Real Exchange Rate	12.6%	3.8%	0.4%	0.5%
GDP	3.3%	2.3%	2.2%	2.2%
Correlations with GDP				
Debt	-55.5%	-31.0%	-50.6%	-51.9%
Share of Debt in LC	50.0%	8.0%	34.1%	30.6%
Inflation	7.4%	-23.6%	2.7%	0.4%
Exchange Rate	-50.6%	-28.0%	-72.9%	-65.5%
Real Exchange Rate	-54.1%	4.0%	-77.3%	-84.4%

Notes: The column *Data* refers to average moments for the sample countries detailed in Table 1, using annual data, for the period 1990-2014. For countries in which the 1990 data for a given variable was not available, we consider the earliest data available for that variable. Debt refers to external debt over GDP; the average share of debt in LC refers to the ratio between debt in local currency to total debt; to compute the standard deviation and correlation with GDP of the share of debt in local currency, we measure this variable as the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP. Average inflation was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B. The last three columns refer to the moments of simulations of alternative models. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100).

The full model also approximates well the standard deviation of debt, its currency composition and GDP. However, inflation and the real exchange rate, are significantly less volatile than in the data. This is partly due to the simplicity of the model. Later in this section we argue that a better fit for inflation can be obtained in an enhanced version of the model with monetary shocks.

The model correctly approximates the counter-cyclicality of the nominal exchange rate. In the model the correlation between GDP and the nominal exchange rate is -73%, compared to -51% in the data. This feature is relevant in shaping the hedging properties of debt in local currency since the government will find attractive to issue debt in local currency to the extent that the nominal exchange rate is negatively correlated with tradable income. In this version of the model the nominal exchange rate is endogenously determined. Importantly, like in the data (see Table A3), the nominal and real exchange rate have a strong positive comovement. The real exchange rate is countercyclical in the model: given that markets are incomplete, tradable consumption co-varies positively with tradable endowment, which in turn delivers a procyclical relative price of non-tradables. This feature highlights the relevance of including an endogenous real exchange rate in our theoretical framework with tradable and non-tradable goods.

The full model also generates, as in the data, a procyclical share of debt in local currency. Quantitatively, the model predicts a correlation between output and the difference between debt in local and foreign currency as a share of GDP of 34% close to the 50% observed in the data.[18] The cyclical behavior of the share of debt in local currency is related to the cyclical properties of the incentive problems associated to this type of debt. For a given level of debt in local currency the incentives to dilute debt via inflation differ with the realization of the endowment. In periods of low realizations of the endowment the marginal utility of consumption is high and so is the marginal benefit of saving resources for consumption by reducing debt repayments. This increases the attractiveness of diluting debt in local currency through inflation. Given that the temptation to engage in costly inflation is higher in bad times the government optimally chooses to tilt its currency composition to foreign currency to mitigate the higher incentives to dilute debt. The cyclicality of the share of debt in local currency is consistent with the dynamic response of debt in both currencies to a shock to tradable output, shown in Figure A.5. Finally, in the model inflation is acyclical, as in the data. The reason is that the changes in the incentives to dilute over the cycle are offset by the procyclicality of the share of debt in local currency.

[18]Note that this data moment (correlation between GDP, and the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP) differs from that reported in Table 1 (correlation between GDP and the share of debt in local currency). We focus on this moment because in model simulations the total level of debt occasionally takes values close to zero.

5.3. *The Role of Inflation Costs*

The strength of the incentive problems are partially determined by the costs of inflation. In this section we analyze how the main results of the full model change as we consider different costs of inflation. In particular, we solve the full model for a specification with low inflation costs ($\psi = 3.5$, half of the baseline value), high inflation costs ($\psi = 14.2$, twice the baseline value) and infinite inflation costs ($\psi = \infty$), and each time re-calibrate the discount factor and target inflation to match observed average levels of debt and inflation. The rest of the parameters are set to the values of the baseline calibration.

Results are shown in Table 5. In the case with low inflation costs the average share of debt in local currency drops to only 1.4% compared to the 9.6% in the baseline calibration. In the case with high inflation costs the share increases to 17.0%. All the other moments remain close to the values in the baseline calibration. This implies that the costs of engaging in distortionary inflation are key in determining average currency composition of sovereign debt. Our baseline calibration is computed with inflation costs taken from studies that estimate losses from money balances and inefficiencies coming from menu costs. To the extent that these costs should also incorporate reputational costs for deviating from pre-set targets, then our baseline estimates would underestimate the optimal share of debt in local currency.

We also solve our model for the case of infinite inflation costs, or equivalently, a model in which monetary policy is given by $\pi = \pi^*$. This parametrization is interesting as it reflects the case in which the government can only dilute debt in local currency by engaging in real exchange rate depreciation. As shown in the last column of Table 5, the average share of debt in local currency in this specification is 40%, more than double than that in the baseline specification, and higher than that observed in the data.

5.4. *Debt Currency Composition during Episodes of Prolonged Expansion and Disinflation*

As mentioned in section 2, the last decade was characterized by an increase in the share of debt in local currency in emerging economies. In this section we analyze in depth these recent dynamics through the lens of our model. To understand this episode it is first important to put it into context. During the same period, these economies also experienced a prolonged expansion of economic activity and a significant reduction in inflation, as documented in Figure 1 for the average emerging economy. Moreover, expansion and disinflation were generalized across countries, observed in more than 70% of the countries of our sample. We are interested in studying whether the increase in the share of debt in local currency observed over the last

TABLE 5. Model Moments and Inflation Costs

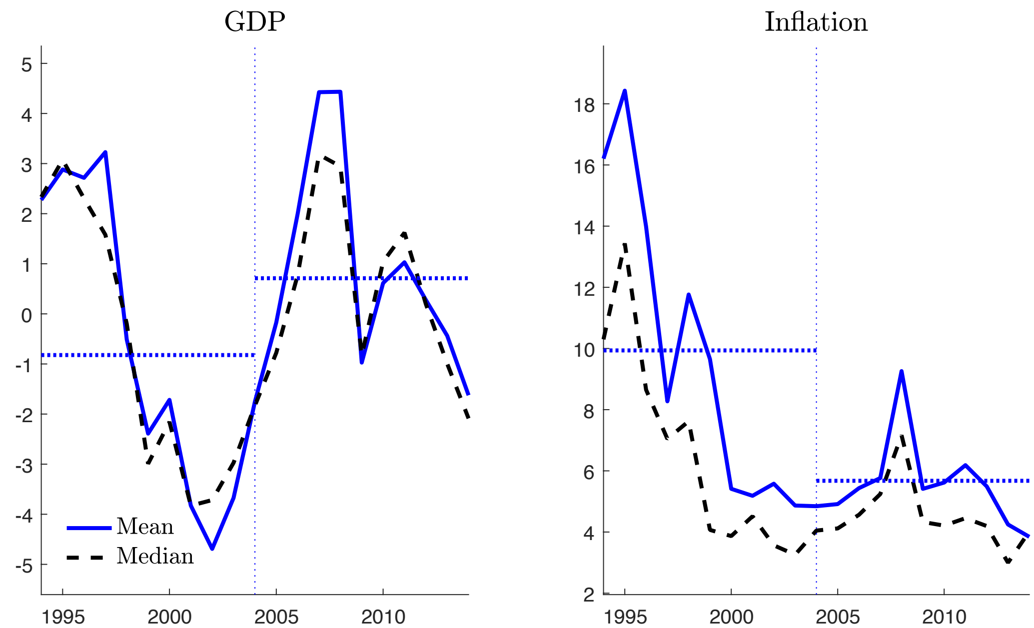
Moment	Baseline ($\psi = 7.1$)	Low Inf. Cost ($\psi = 3.5$)	High Inf. Cost ($\psi = 14.2$)	RXR Dilution ($\psi = \infty$)
Average Levels				
Debt	22.4%	22.1%	22.1%	21.9%
Share of Debt in LC	9.6%	1.4%	17.0%	39.8%
Inflation	8.7%	8.7%	8.7%	8.7%
Standard deviation				
Debt	3.1%	3.1%	3.2%	3.3%
Share of Debt in LC	3.0%	2.7%	4.6%	5.2%
Inflation	0.1%	0.1%	0.1%	0.0%
Exchange Rate	0.5%	0.5%	0.5%	0.4%
GDP	2.2%	2.2%	2.2%	2.2%
Correlations with GDP				
Debt	-50.6%	-51.6%	-50.1%	-50.6%
Share of Debt in LC	34.1%	41.3%	25.8%	-18.6%
Inflation	2.7%	6.7%	1.7%	0.2%
Exchange Rate	-72.9%	-71.1%	-71.3%	-66.2%

Notes: The different columns refer to model simulations under different parameters of the cost of inflation, ψ, with the discount factor and the target inflation rate recalibrated to match the observed average levels of inflation and debt. The remaining parameters are set at the values of the baseline calibration. Debt refers to external debt over GDP; the average share of debt in LC refers to the ratio between debt in local currency to total debt; to compute the standard deviation and correlation with GDP of the share of debt in local currency, we measure this variable as the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100). For data counterpart of these moments, see Table 4.

decade can be related to the prolonged expansion of economic activity and the reduction of inflation.

A limitation of our quantitative model in this task it that, as shown in Table 4, the volatility of inflation is too low relative to the data, implying that our simulated economy cannot generate periods of disinflation of the magnitude observed during the last decade. One potential reason

FIGURE 1. Expansion and Disinflation in Emerging Economies over the Last Decade



Notes: GDP in percent deviation of a log-linear trend and annual inflation rate, for the sample countries detailed in Table 1. *Mean inflation* was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B.

for why the model predicts a low volatility of inflation is that fluctuations in the model are only driven by endowment shocks, while a large body literature associates the transition from high-inflation to stabilization in emerging economies during our period of analysis to changes in monetary policy regimes (see for example Sargent et al. (2009) and Walsh (2009)). To capture these changes in monetary regimes, in this section we extend the model to incorporate shocks to the cost of inflation. In particular, in this extended version of the model the utility cost of inflation is assumed to be given by:

$$l(\pi; s_M) = \frac{1}{2}\psi(s_M)\left(\pi_t - \pi^*(s_M)\right)^2, \tag{16}$$

where $s_M \in \{l, h\}$ follows a first-order Markov process. It is assumed that $\pi^*(s_M = l) < \pi^*(s_M = h)$ and $\psi(s_M = l) > \psi(s_M = h)$, meaning that $s_M = l$ is a "low-inflation" regime, with a low target inflation and high costs of deviating from the target, and $s_M = h$ a "high-inflation" regime with a high target inflation rate and low costs of deviating from the target.

To calibrate the monetary shock, we assume that $\pi^*(s_M = l) = \pi^* - \delta_{\pi^*}$, $\pi^*(s_M = h) = \pi^* + \delta_{\pi^*}$ and that $\psi(s_M = l) = \psi + \delta_\psi$, $\psi(s_M = h) = \psi - \delta_\psi$, where $\{\pi^*, \psi\}$ are determined as in the baseline calibration. We associate the "high inflation" regime with the 1994-2003 period and the "low inflation" regime with the 2004-2014 period. We set $\delta_{\pi^*} = 0.02$ to match the average inflation rates in both periods. We set $\delta_{\psi} = 2.7$ to match the ratio of the standard deviation of the inflation rate in both periods. Finally, we assume a symmetric transition probability for s_M with the probability of remaining in the same state given by $p = 0.99$, so that the regimes are near permanent. The rest of the parameters are set as in the baseline calibration with the discount factor β recalibrated to target the average level of external public debt.

The moments of the simulated economy with monetary shocks are reported in Table 4. As in the baseline full model the simulated economy with monetary shocks displays predictions for debt and for the currency composition of debt in line with those observed in the data. Equipped with this extended model, we can now study periods of expansion and disinflation similar to those observed over the last decade. To do this, we simulate the calibrated version of the model for 500,000 periods and identify periods of expansion and disinflation as those non-overlapping periods of 20 years ending in time t, in which the average GDP in the decade ending in period t is at least Δ_y percentage points higher than the average for the decade ending in period $t-10$, and the average inflation for the decade ending in time t is at least Δ_π percentage points lower than the average for the decade ending in period $t-10$. The dynamics of the variables of interest are then averaged across all identified episodes. We choose the values of the parameters $\{\Delta_y, \Delta_\pi\}$ to match the average increase in output (relative to a log-linear trend) and the average decrease of inflation observed during the last decade, reported in the first column of Table 6. In addition, to disentangle the relative strength of these two processes, we also identify separately episodes of expansion and disinflation, conditioning only on the increase in GDP in the former, and only on the fall of inflation in the latter.

Table 6 reports the change in GDP, inflation, the share of debt in local currency -measured as the difference between the share of debt in local and foreign currency-, and the ratio of external public debt to GDP, for the simulated episodes (second column), and compares them to the dedollarization episode observed in the data for the average emerging economy over the last decade (first column). The model predicts that episodes of expansion and disinflation entail a significant increase in the share of debt in local currency, of more than 80% of that observed in the data (14.0% in the data, compared to 11.6% in the model). In addition, the model predicts significant deleveraging during these episodes, consistent but smaller than the reduction in external public debt observed in the data (12.1% in the data, compared to 7.7% in the model).

Table 6. Debt Currency Composition During Episodes of Prolonged Expansion and Disinflation: Data and Model

	Data	Model		
	2004-2014 vs. 1994-2003	Expansion & disinflation episodes	Expansion episodes	Disinflation episodes
Δ *Targeted variables*				
GDP	1.5%	1.5%	1.6%	0.1%
Inflation	-4.3%	-4.2%	-0.1%	-4.1%
Δ *Untargeted variables*				
Share of Debt in LC	14.0%	11.6%	5.3%	2.8%
Debt	-12.1%	-7.7%	-6.2%	0.5%

Notes: The column *Data 2004-2014 vs. 1994-2003* refers, for each variable of interest, to the difference between the average 2004-2014 and the average 1994-2003, for the sample of countries detailed in Table 1. *GDP* refers to percent deviations of GDP from a log-linear trend; *inflation* to the annual percent increase in the price level; *share of debt in LC* refers to the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP; *debt* refers to external debt over GDP. For the share of debt in local currency, given that the data starts in 2004 (as detailed in section 1) we consider the change between 2004 and 2014 instead of the change in the average of the two decades. For countries in which the 2004 data was not available, the difference is taken from the earliest data available. The columns under *Model* refer, for each variable of interest, to the change of a given variable under simulated episodes defined in section 5.4.

We then disentangle the strength of each driving factor in explaining the increase in the share of debt in local currency in the model. This is shown in the last two columns of Table 6, that report the change in the simulated data in periods of economic expansions only and disinflation only. The third column of Table 6 shows that, of the increase in the share of debt in local currency in the model, roughly half is explained by the prolonged expansion alone: the share of debt in local currency increases by 5.3% in these episodes. As previously discussed, higher levels of output lead to lower marginal benefits of debt dilution, which make the government more willing to shift the currency composition of its debt towards local currency (see Figure A.5). The fourth column of Table 6 shows that around one-fourth of the decrease of the predicted dedollarization can be explained by the change in the monetary regime alone: the share of debt in local currency increases by 2.8% in periods of disinflation only. Faced with higher inflation costs, the government can optimally choose to issue larger amounts of debt in local currency, given that higher inflation costs reduce the incentive problems associated with diluting debt in local currency via inflation. The behavior of the currency composition of debt during these episodes is consistent with the dynamic response of debt in both currencies to a positive shock to tradable output and to a change in the monetary regime from a regime of high inflation costs to one of low inflation costs (see Figure A.5). The rest of the predicted dedollarization can be explained by the interaction between expansion and monetary regime: in episodes of expansion and disinflation the share of debt in local currency increases by 3.5% more than in episodes of only expansion and only disinflation combined.

6. Alternative Debt Instruments

In this section we discuss the use of alternative debt instruments that can serve as substitutes of debt in local currency.

6.1. *Inflation-Linked Debt*

An often proposed way to overcome the incentive problems associated to debt in local currency is to issue inflation-linked debt denominated in local currency. This is in fact a security commonly observed in several countries including the US and emerging economies. We show that this security partially overcomes the incentive problems associated to dilution. While diluting debt repayment through inflation is no longer possible, the government can still dilute its repayment value through the manipulation of the real exchange rate.

Consider the case of one-period inflation-linked debt in local currency. This is a security issued in period t that pays π_{t+1} units of local currency in $t+1$, where π_t is the gross inflation rate. The risk-neutral price expressed in local currency of this debt security is

$$q_t^\pi = \frac{1}{R} e_t \mathbb{E}\left[\frac{\pi_{t+1}}{e_{t+1}}\right].$$

Let b_t^π be the stock of inflation-linked debt in local currency with which the government enters the period. The resource constraint, expressed in local currency, of an economy in which the government can issue debt in foreign currency as well as inflation-linked debt in local currency is given by

$$e_t c_{T,t} + e_t b_t^* + b_t^\pi \pi_t = e_t y_{T,t} + e_t q_t^* b_{t+1}^* + q_t^\pi b_{t+1}^\pi,$$

where we already imposed the law of one price for tradable goods and market clearing for non-tradable goods. Substituting for the expressions for debt prices and using the fact that $e_t = \pi_t C_{C_T,t} P_{t-1}$ we can express the resource constraint in terms of tradable goods as

$$c_{T,t} + b_t^* + b_t^\pi \frac{1}{C_{C_T,t} P_{t-1}} = y_{T,t} + \frac{1}{R} b_{t+1}^* + \frac{1}{R} \mathbb{E}\left[\frac{1}{C_{C_T,t+1} P_t}\right] b_{t+1}^\pi, \tag{17}$$

From this expression we can already see that the repayment of inflation-linked debt in local currency no longer depends on the inflation rate, but still depends on the real exchange rate. Therefore, its repayment value can be diluted by affecting the real exchange rate ex-post with future debt choices as explained in section 4.2. Given that $C_{C_T,t}$ is decreasing in tradable consumption, the presence of positive inflation-linked debt gives rise to an incentive to reduce future debt issuance, postpone tradable consumption and keep the real exchange rate depreciated.

Finally, the hedging properties of inflation-linked debt may be different from those of debt in local currency. As seen in (17), whether inflation-linked debt is a useful security for hedging purposes depends on the co-movement between the real exchange rate and GDP, which in the data is of the order of -54%.

6.2. *Other linked debt.*

Another example of debt instrument that has been used are GDP-linked bonds. This debt instrument can overcome the time inconsistency problem in the case of endowment economies, but no so in the case of economies with production. In the later the government has incentives to cool down the level of economic activity to repay less debt.

Finally, another possibility is to issue debt denominated in the currency of another country that has strong co-movements with the local currency. This would completely eliminate the incentive problems but at the expense of exposing the economy to additional sources of risk.

7. Conclusion

This paper focuses attention on three stylized facts regarding the currency denomination of sovereign external debt in emerging economies. First, the currency denomination of sovereign debt is tilted towards foreign currency. Second, the share of debt in local currency has increased significantly over the past decade. Third, the share of debt denominated in local currency is procyclical.

We develop a model of optimal choice of debt denominated in foreign and local currency in which the government also controls monetary policy. The choice of the currency composition of debt is governed by an incentive-hedging trade-off associated to debt in local currency. We calibrate the model for a representative emerging economy and find that the model can account for the main three stylized facts highlighted at the beginning. According to our model, the gradual dissipation of the 'original sin' is, to a high extent, linked to the prolonged expansion and stabilization of inflation that emerging economies experienced in the past decade.

Finally, we leave for future research studying the problem of private borrowing by currency and exploring feedback effects from debt positions to economic activity.

References

Aguiar, M. and M. Amador (2014): *Sovereign Debt*, North-Holland, 647–87.

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2013): "Crisis and Commitment: Inflation Credibility and the Vulnerability to Sovereign Debt Crises," Manuscript, Harvard University.

——— (2015): "Coordination and Crisis in Monetary Unions," *The Quarterly Journal of Economics*, 130, 1727–1779.

Aguiar, M., M. Amador, H. Hopenhayn, and I. Werning (2016): "Take the Short Route: Equilibrium Default and Maturity," Manuscript, Princeton University.

Aguiar, M. and G. Gopinath (2006): "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 69, 64–83.

Araujo, A., M. Leon, and R. Santos (2013): "Welfare analysis of currency regimes with defaultable debts," *Journal of International Economics*, 89, 143–153.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 98, 690–712.

Arellano, C. and A. Ramanarayanan (2012): "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120, 187–232.

Arslanalp, S. and T. Tsuda (2014): "Tracking Global Demand for Emerging Market Sovereign Debt," IMF Working Paper 14/39.

Barro, R. J. and D. B. Gordon (1983): "Rules, discretion and reputation in a model of monetary policy," *Journal of Monetary Economics*, 12, 101–121.

Bianchi, J., J. C. Hatchondo, and L. Martinez (2018): "International Reserves and Roll-Over Risk," Forthcoming at *The American Economic Review*.

Bocola, L. and A. Dovis (2016): "Self-Fulfilling Debt Crises: A Quantitative Analysis," Manuscript, University of Pennsylvania.

Bohn, H. (1990): "A positive theory of foreign currency debt," *Journal of International Economics*, 29, 273–292.

Broner, F. and J. Ventura (2011): "Globalization and Risk Sharing," *The Review of Economic Studies, Vol. 78, No. 1.*

Broner, F. A., G. Lorenzoni, and S. L. Schmukler (2013): "Why do emerging economies borrow short term?" *Journal of the European Economic Association*, 11, 67–100.

Burstein, A., M. Eichenbaum, and S. Rebelo (2005): "Large Devaluations and the Real Exchange Rate," *Journal of Political Economy*, 113, 742–784.

Burstein, A. and C. Hellwig (2008): "Welfare costs of inflation in a menu cost model," *The American Economic Review*, 438–443.

Calvo, G. A. (1987): "On the Time Consistency of Optimal Policy in a Monetary Economy," *Econometrica*, 46, 1411–1428.

——— (1988): "Servicing the public debt: The role of expectations," *The American Economic Review*, 647–661.

Calvo, G. A. and E. G. Mendoza (1996): "Mexico's balance-of-payments crisis: a chronicle of a death foretold," *Journal of International Economics*, 41, 235–264.

Chari, V. V., A. Dovis, and P. J. Kehoe (2015): "Rethinking Optimal Currency Areas," Manuscript, University of Minnesota.

Chari, V. V. and P. J. Kehoe (1990): "Sustainable plans," *The Journal of Political Economy*, 98, 783.

——— (1993): "Sustainable Plans and Debt," *Journal of Economic Theory*, 61, 230–261.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *The American Economic Review*, 102, 2674–2699.

Cochrane, J. H. (2001): "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69, 69–116.

Cole, H. and T. Kehoe (2000): "Self-Fulfilling Debt Crises," *The Review of Economic Studies*, 67, 91–116.

Cruces, J. J. and C. Trebesch (2013): "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117.

Da-Rocha, J.-M., E.-L. Giménez, and F.-X. Lores (2013): "Self-fulfilling crises with default and devaluation," *Economic Theory*, 53, 499–535.

Diaz-Gimenez, J., G. Giovanetti, R. Marimon, and P. Teles (2008): "Nominal debt as a burden on monetary policy," *Review of Economic Dynamics*, 11, 493–514.

Du, W., C. Pflueger, and J. Schreger (2017): "Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy," Manuscript, Princeton University.

Du, W. and J. Schreger (2015): "Sovereign Risk, Currency Risk, and Corporate Balance Sheets," Manuscript, Harvard University.

Duffie, D., W. Fleming, and H. M. Soner (1997): "Hedging in incomplete markets with HARA utility," *Journal of Economic Dynamics and Control*, 21, 753–782.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 48, 289–309.

Eichengreen, B. and R. Hausmann (2010): *Other People's Money: Debt Denomination and Financial Instability in Emerging Market Economies*, University of Chicago Press.

Eichengreen, B., R. Hausmann, and U. Panizza (2002): " Original sin: the pain, the mystery, and the road to redemption," In IADB Conference 'Currency and Maturity Matchmaking: Redeeming Debt from Original Sin.'.

Engel, C. and J. Park (2018): "Debauchery and Original Sin: The Currency Composition of Sovereign Debt," Manuscript, University of Wisconsin Madison.

Fanelli, S. (2018): "Monetary Policy, Capital Controls, and International Portfolios," Working paper.

Farhi, E. and J. Tirole (2012): "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *The American Economic Review*, 102, 60–93.

Fischer, S. (1983): "Welfare Aspects of Government Issue of Indexed Bonds," in *Inflation, Debt, and Indexation*, ed. by R. Dornbusch and M. H. Simonsen, M.I.T. Press, 247–266.

Hanke, S. H. and N. E. Krus (2013): "World Hyperinflations," *The Handbook of Major Events in Economic History, Randall Parker and Robert Whaples, eds., Routledge Publishing, Summer.*

Hausmann, R. and U. Panizza (2003): "On the determinants of Original Sin: an empirical investigation," *Journal of international Money and Finance*, 22, 957–990.

IMF and T. World Bank (2016): "Development of Local Currency Bond Markets," Staff Note, https://www.imf.org/external/np/g20/pdf/2016/121416.pdf.

Korinek, A. (2009): "Insurance Properties of Local and Foreign Currency Bonds in Small Open Economies," Manuscript, Johns Hopkins University.

Kydland, F. E. and E. C. Prescott (1977): "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 85, 473.

Leeper, E. M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27, 129–147.

Lucas, R. E. (2000): "Inflation and Welfare," *Econometrica*, 68, 247–274.

Nickell, S. (1981): "Biases in Dynamic Models with Fixed Effects," *Econometrica*, 1417–1426.

Nuño, G. and C. Thomas (2015): "Monetary Policy and Sovereign Debt Vulnerability," Manuscript, Banco de España.

Perez, D. J. (2015): "Sovereign Debt, Domestic Banks and the Provision of Public Liquidity," Manuscript, New York University.

Persson, M., T. Persson, and L. E. O. Svensson (1987): "Time Consistency of Fiscal and Monetary Policy," *Econometrica*, 55, 1419–1430.

Phelan, C. (2006): "Public trust and government betrayal," *Journal of Economic Theory*, 130, 27–43.

Pitchford, R. and M. L. J. Wright (2012): "Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment," *The Review of Economic Studies*, 79, 812–837.

Rogoff, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *The Quarterly Journal of Economics*, 100, 1169–1189.

Sargent, T., N. Williams, and T. Zha (2009): "The Conquest of South American Inflation," *Journal of Political Economy*, 117, 211–256.

Sims, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

Sunder-Plassmann, L. (2014): "Inflation, Default and the Denomination of Sovereign Debt," Manuscript, University of Minnesota.

Svensson, L. E. O. (1985): "Money and Asset Prices in a Cash-in-Advance Economy," *Journal of Political Economy*, 93, 919–944.

Viceira, L. M. (2001): "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income," *The Journal of Finance*, 56, 433–470.

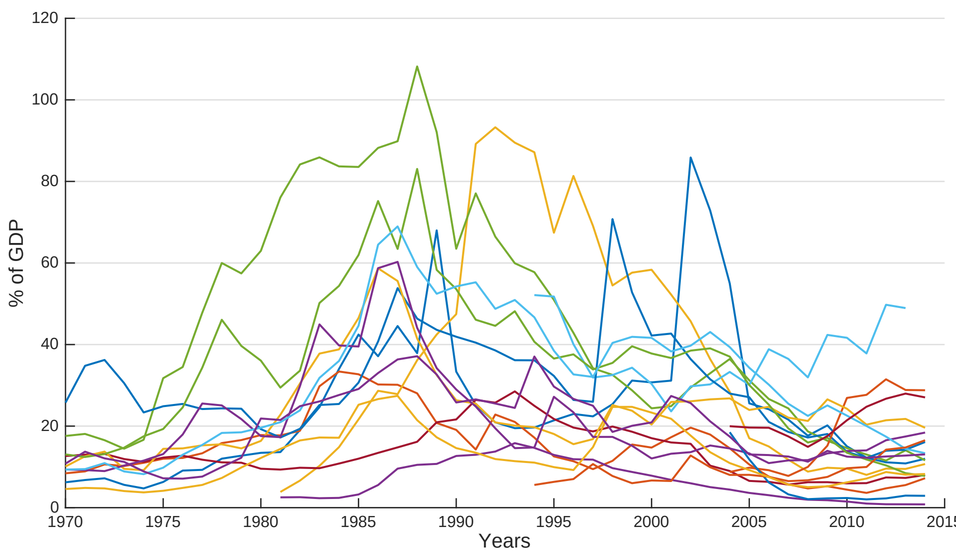
Walsh, C. E. (2009): "Inflation targeting: what have we learned?" *International Finance*, 2, 195–233.

Yue, V. Z. (2010): "Sovereign Default and Debt Renegotiation," *Journal of International Economics*, 80, 176–187.

Appendix A. Additional Figures and Tables (For Online Publication)

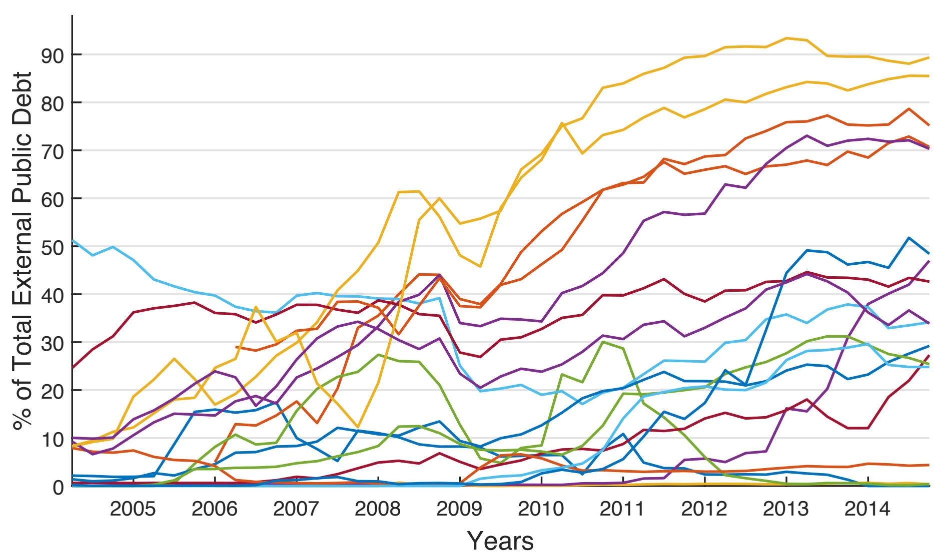
A.1. *Additional Figures*

Figure A.1. Evolution of Total External Public Debt



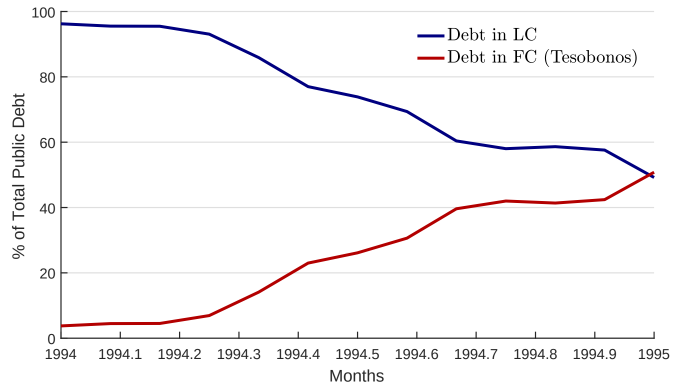
Notes: Based on WDI data. Stock of external public debt as a percentage of annual GDP for the countries in the sample.

Figure A.2. Evolution of the Currency Composition of Sovereign External Debt



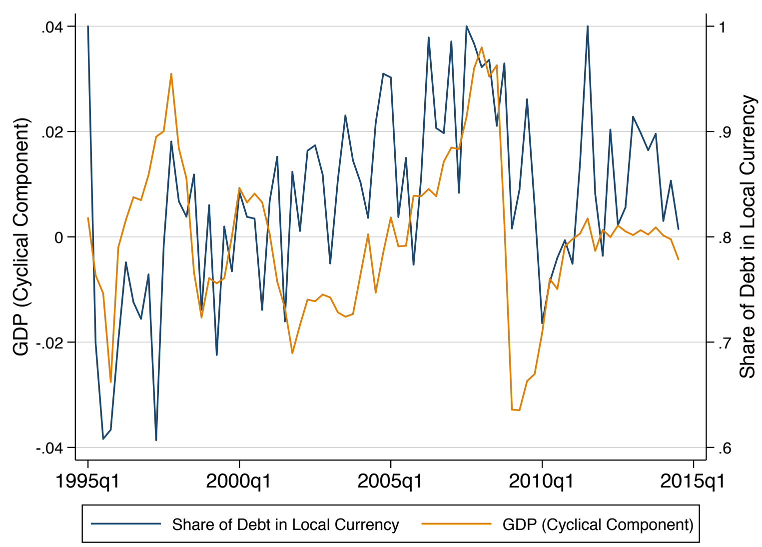
Notes: Based on Arslanalp & Tsuda (2014). Share of sovereign external debt in local currency as a fraction of total sovereign external debt for the countries in the sample.

FIGURE A.3. Currency Composition of Sovereign Debt: The Case of Mexico During the Tequila Crisis



Notes: This figures shows the share of total public debt in local and foreign currency. It is based on data from Banxico. Local currency debt includes Ajustabonos, CETES and Bondes. Foreign-currency-denominated debt includes Tesobonos.

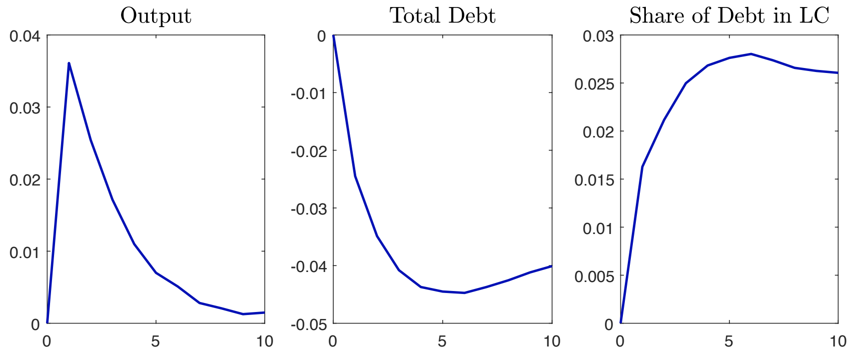
FIGURE A.4. Issuance Data: Cyclicality of Currency Denomination



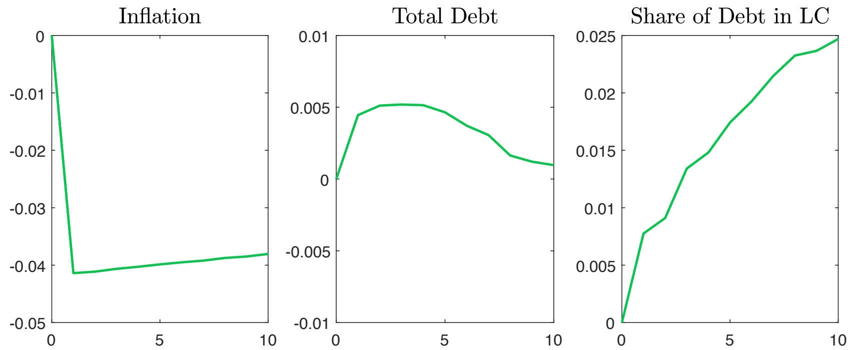
Notes: The share of issuance in local currency is computed as the sum of the face value of bonds in local currency divided by the sum of the face value of total bond issued in a given quarter by a given country. The blue line plots the average share of issuance in local currency across all countries in our sample. The orange line plots the average cyclical component of real GDP. Trend GDP is computed with an HP filter.

FIGURE A.5. Impulse Response Functions

(A) Response to a shock to y_T



(B) Response to a shock to the inflation regime



Notes: The top panel plots the response of variables to a one-standard-deviation shock to tradable endowment (y_T) in the calibrated model with monetary shocks. For details on the model calibration, see section 5.4. Variables are expressed in deviations from their value before the shock hits ($t = 0$). The response of output is expressed in percentage points. The response of debt is expressed in percent of GDP. The share of debt is measured as the difference between debt in local and foreign currency expressed as percent of GDP. The bottom panel plots the response of variables to a change from the 'high inflation costs' regime to the 'low inflation costs' regime. The response of inflation is expressed in percentage points. In both panels, the graphs display the the average response of variables to exogenous shocks starting from different states in the ergodic distribution. The impulse response functions were computed in the model with monetary shocks.

A.2. *Additional Tables*

TABLE A1. Cyclicality of the Currency Composition of Sovereign Debt

	Correlation with Output				
	Debt in LC	Debt in FC	Share of Debt in LC		
Country	HP-trend Filtering	HP-trend Filtering	HP-trend Filtering	Linear-trend Filtering	Share of LC debt at constant XR
Argentina	19%	-18%	24%	26%	17%
Brazil	38%	-64%	72%	88%	47%
Bulgaria	55%	-47%	53%	35%	54%
China	-21%	2%	-23%	-83%	-22%
Egypt	27%	-5%	31%	66%	30%
Hungary	31%	-58%	50%	10%	44%
India	6%	-69%	15%	-1%	7%
Indonesia	17%	-39%	27%	15%	9%
Lithuania	-55%	-43%	-70%	-63%	-71%
Malaysia	57%	-11%	44%	3%	37%
Mexico	53%	-51%	59%	49%	44%
Peru	63%	-63%	64%	8%	64%
Philippines	-16%	-53%	-7%	9%	-7%
Poland	-20%	-34%	5%	-21%	-27%
Russia	22%	-48%	28%	-25%	30%
South Africa	41%	13%	16%	-15%	22%
Thailand	30%	-9%	-11%	-16%	-13%
Turkey	58%	-40%	62%	68%	58%
Average	22%	-35%	24%	9%	18%
Median	28%	-42%	27%	9%	26%
Std. Dev.	33%	25%	36%	44%	35%

Notes: Debt in LC and debt in FC refer, respectively, to debt denominated in local currency and in foreign currency over GDP. The share of debt in LC refers to the share of external public debt denominated in local currency. The correlations between output and debt in LC, debt in FC, and the share of debt in LC refer to the correlations between the cyclical component of real GDP and the cyclical component debt in LC, debt in FC, and the share of debt in LC. In the first three columns variables are detrended using the HP filter. In the fourth column they are detrended with a linear trend. The last column computes the same moment as the third column but with the share of debt in local currency measured at constant exchange rates of 2006.Q1. The share of debt in local currency and the correlation with GDP is computed for the period 2004-2014, when data by currency becomes available.

Table A2. Facts on Public and Private External Debt

	Average Debt		Corr. Debt & GDP		Corr. Public
Country	Public	Private	Public	Private	& Private Debt
Argentina	28%	9%	-84%	-76%	92%
Brazil	12%	10%	-77%	-44%	55%
Bulgaria	43%	29%	-44%	66%	-34%
China	7%	2%	50%	-33%	-33%
Egypt	34%	0%	-65%	19%	-0%
Hungary	38%	44%	1%	-14%	11%
India	14%	5%	-49%	-28%	-42%
Indonesia	30%	16%	-71%	-36%	88%
Lithuania	19%	n.a.	-62%	n.a.	n.a.
Malaysia	23%	12%	-89%	-11%	14%
Mexico	17%	5%	-69%	21%	12%
Peru	31%	7%	-73%	-20%	30%
Philippines	35%	11%	-53%	-3%	43%
Poland	22%	n.a.	-66%	n.a.	n.a.
Russia	5%	n.a.	-56%	n.a.	n.a.
South Africa	9%	8%	-35%	-25%	15%
Thailand	12%	17%	-85%	-17%	49%
Turkey	20%	12%	-70%	-6%	5%
Average	22%	12%	-55%	-14%	20%
Median	21%	10%	-66%	-17%	14%
Std. Dev.	11%	11%	34%	33%	41%

Notes: The averages of external public and private debt are computed over the period 1990-2014. The correlation between output and the public (private) external debt refers to the correlation between the cyclical component of real GDP and the cyclical component the public (private) external debt to GDP ratio. Both variables are detrended using the HP filter. Data source: WDI.

Table A3. GDP and Exchange Rates in Emerging Economies

Country	$\rho_{y,e}$	$\rho_{y,rxr}$	$\rho_{e,rxr}$
Argentina	-61%	-79%	90%
Brazil	-81%	-83%	95%
Bulgaria	-63%	-62%	94%
China	39%	-21%	47%
Egypt	-66%	-66%	93%
Hungary	-57%	-43%	90%
India	-65%	-78%	83%
Indonesia	-85%	-72%	97%
Lithuania	-64%	-17%	41%
Malaysia	-77%	-78%	99%
Mexico	-48%	-69%	82%
Peru	2%	-58%	49%
Philippines	-30%	-16%	97%
Poland	-18%	-25%	74%
Russia	-58%	-64%	96%
South Africa	-21%	-6%	98%
Thailand	-87%	-84%	97%
Turkey	-53%	-53%	100%
Average	-50%	-54%	85%
Median	-59%	-63%	94%
Std. Dev.	33%	26%	19%

Notes: $\rho_{y,e}$ refers to the correlation coefficient between the cyclical component of real GDP and the cyclical component of the nominal exchange rate, measured as units of local currency per unit of foreign currency. $\rho_{y,rxr}$ refers to the correlation coefficient between the cyclical component of real GDP and the cyclical component of the real exchange rate, measured as the ratio of the US CPI index expressed in local currency to the domestic CPI index. $\rho_{e,rxr}$ refers to the correlation coefficient between the cyclical component of the nominal exchange rate and the real exchange rate. Correlations are computed for the period 1990-2014. Data is at the annual frequency, trends are computed with HP filter. Data source: WDI.

Appendix B. Data Sources and Estimation (For Online Publication)

B.1. *Data sources*

The data on sovereign debt comes from two sources. Annual data on sovereign external debt by country for the period 1990-2014 comes from WDI. Quarterly data on sovereign external debt in foreign and local currency by country for the period 2004-2014 comes from Arslanalp and Tsuda (2014). The authors collect data on central government debt by currency denomination. They estimate foreign and domestic holdings of this debt issued in global and local markets. The definition of foreign and domestic investors they use follows the residency principle of external debt statistics. For this they use data from the IMF, BIS and national sources. In order to reach the final estimates the authors make certain assumptions in cases in which the necessary data was unavailable.[19] These assumptions are stated in the paper.

Data on bond issuance comes from Bloomberg. We collected data on all bonds issuances recorded in the Bloomberg terminal for all countries in our sample during the period 1990-2014. For each bond we have data on the institutional name of the debtor, its face value, maturity, date of issuance and currency of denomination. We do not know the residence of who purchased the bonds, nor the market of issuance. Therefore, our data can include bonds that were purchased by domestic creditors. To address this issue we exclude all bonds with very short maturities, with very small face values and those issued by central banks. These bonds are likely to be acquired by domestic investors (IMF and World Bank (2016)). Specifically, we exclude all bonds that satisfy any of the following three conditions: 1. the word "bank" or "banco" appears in the name of the issuing institution, 2. the face value is less than 0.1% of GDP and 3. the maturity is less than 6 months. As we argue next, our main results are invariant to this filtering procedure. We also excluded bonds associated to debt restructuring and bank recapitalizations (Argentina, Russia and Indonesia). Once we filter out these observations we are left with a total of 14,745 bonds. We then compute the total issuance as the sum of the face value of all the bonds in a given country and year, expressed as a percentage of GDP. The share of issuance in local currency is computed as the sum of the face value of all issued bonds denominated in local currency divided by total bond issuance.

We check whether the micro-data on bond issuances aggregated at the country level is consistent with the aggregate data on debt levels. We do so by comparing at the country level the average levels of debt with the average levels of debt issuances, and the average share of

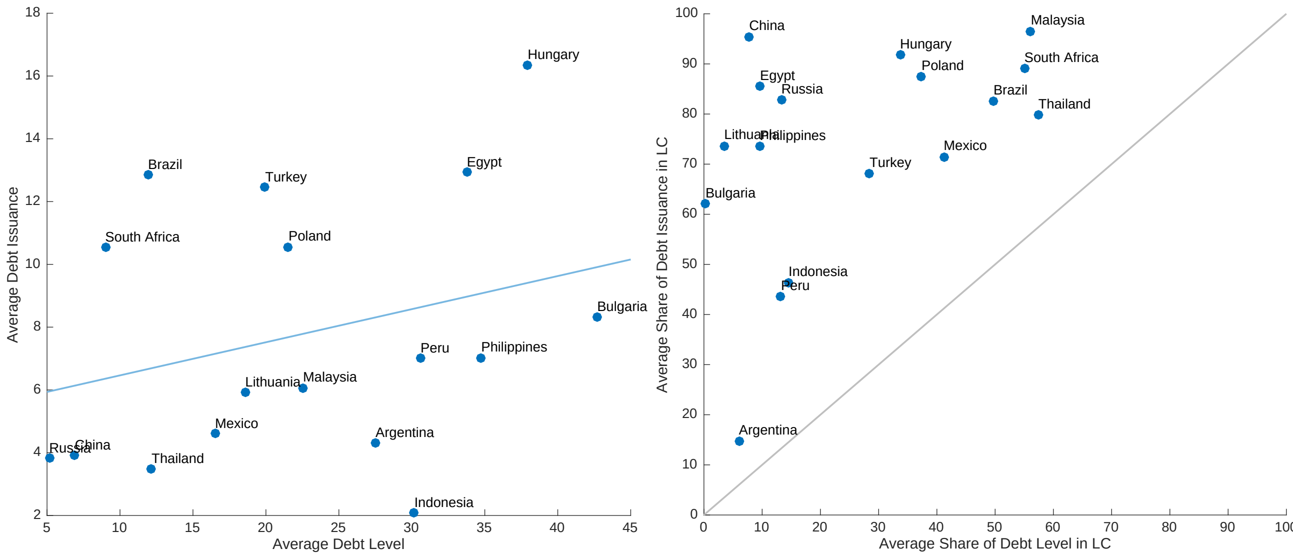
[19]For example, to estimate foreign bank holdings of public debt the authors use data on BIS about foreign bank holdings of non-bank debt and multiply it by the share of non-bank debt that corresponds to public debt.

debt levels in local currency with the average share debt issuance in local currency. The cross-country correlation between the average level of debt and the average level of debt issuance for the same period of time is 30% (see Figure B.1a). The cross-country correlation between the share of debt levels in local currency and the share of debt issuance in local currency is 49% (see Figure B.1b). Additionally, the average share of debt issuance in local currency is larger than the average share of debt levels in local currency for all countries. This is consistent with the fact that the share of debt levels in local currency has grown over time (given that debt issuance is one component of the change in debt levels). Overall, this evidence is suggestive of rough consistency between the two datasets. However, there are some discrepancies in specific countries. For example, in Bulgaria, while the share of debt levels in local currency is near zero, the average share of debt issuance in local currency is around 60%. This inconsistency may be due to imprecisions in our method for filtering out bond issuances that are purchased by domestic investors and/or imprecisions in the estimates in debt levels, which in fact are acknowledged in Arslanalp and Tsuda (2014) for the case of Bulgaria.[20] Since both datasets are constructed based on assumptions and proxies we view them as complements.

[20]For certain years in their sample, data on the sum of external debt holdings by investors collected from several sources exceeded the total level of external debt reported in IMF data. In those years, the authors 'calculated alternative measures of foreign bank holdings using other emerging markets as a benchmark'.

Figure B.1. Data on Debt Levels and Issuance: A Comparison

(a) Debt and Issuance: Levels (b) Debt and Issuance: Share of Debt in LC



Notes: Panel A compares data on debt levels and on debt issuance. The horizontal axis shows the average level of sovereign external debt measured as a % of GDP by country for the period 1990-2014. The source of this data is WDI. The vertical axis shows the average annual sovereign debt issuance as a % of GDP by country for the period 1990-2014. The data was computed from micro-data on bond issuance. The blue line is the best linear fit. Panel B compares data on the share of debt in local currency for debt levels and for debt issuance. The horizontal axis shows the average share of sovereign external debt levels in local currency by country for the period 1990-2014. This data is based on Arslanalp & Tsuda (2014). The vertical axis shows the average share of sovereign debt issuance in local currency by country for the period 1990-2104. The data was computed from micro-data on bond issuance. The green line is the 45 degree line.

Finally, we also assess the robustness of the analysis of the issuance data by computing the same analysis with the data without filtering out bonds with small maturities and small face values. Results, shown in Table B1, indicate that the main stylized facts are robust to the analysis of unfiltered data.

Data on GDP at an annual frequency comes from WDI. Data on real GDP at a quarterly frequency was obtained from national sources and IMF. Two measures of tradable output were constructed and considered. The first one is the sum of agriculture and industry value added. The second is industrial production. We used the one that had data availability for the longest time period for each country.

Table B1. Facts on Sovereign Debt Issuance by Currency: Unfiltered Data

Country	Annual Issuance Average (% of GDP)	Share of Issuance in LC Avg. 90-03 (% of Issuance)	Share of Issuance in LC Avg. 04-14 (% of Issuance)	Share of Issuance in LC Correlation with Output
Argentina	5.2%	7%	38%	-17%
Brazil	13.6%	66%	99%	43%
Bulgaria	62.8%	54%	82%	26%
China	6.4%	85%	100%	-5%
Egypt	24.4%	98%	97%	5%
Hungary	22.5%	96%	93%	6%
Indonesia	1.6%	15%	69%	-18%
Lithuania	8.4%	88%	73%	53%
Malaysia	7.0%	95%	100%	39%
Mexico	9.2%	66%	97%	19%
Peru	97.1%	72%	77%	1%
Philippines	13.3%	76%	94%	-3%
Poland	11.6%	79%	88%	24%
Russia	6.4%	64%	94%	45%
South Africa	14.3%	95%	98%	16%
Thailand	7.7%	76%	99%	-71%
Turkey	14.9%	54%	93%	38%
Average	19.2%	70%	88%	12%
Median	11.6%	76%	94%	16%
Std. Dev.	24.4%	26%	16%	31%

Notes: The average annual issuance is computed over the period 1990-2014. It is computed as the sum of the face value of all bonds issued in a given year as a percentage of GDP and then averaged across years. The high values of Bulgaria and Peru in average issuance reflect valuation effects during years of hyperinflation. The share of issuance in LC refers to the share of bond issuance denominated in local currency. The correlation between output and the share of debt in LC refers to the correlation between the cyclical component of real GDP and the cyclical component the share of bond issuance denominated in local currency. Both variables are detrended using the HP filter. These data contains all bond issuances without filtering out bonds with low face values and low maturities. See Appendix B.1 for details on the issuance data.

Data on inflation refers to $\pi_t = \frac{P_t}{P_{t-1}}$ where P_t is the country's CPI and was obtained from WDI. A number of countries of our sample experienced hyperinflation during the 1980s and 1990s (for a detail of hyperinflation episodes see, for example, Hanke and Krus (2013)). We excluded data on inflation above 100 percent so that moments on inflation are not influenced by these extreme episodes. Data on nominal exchange rates refers to units of local currency per U.S. dollar and was obtained from Bloomberg. The real exchange rate was also computed vis-a-vis the US dollar, i.e. $RER_t = \frac{e_t P_t^*}{P_t}$, where P_t^* is the US CPI and P_t is the country's CPI.

B.2. *Estimation of exogenous processes*

In the full model of section 5 we assume that $y_{T,t}$ follows an AR(1) process in logs as specified in (14). This process is estimated with annual data on the cyclical component of tradable output for the period 1990-2014 for all countries in our sample. Since we have country level data, we need to estimate a dynamic panel version of (14) with country-fixed effects. We estimate this panel with OLS. These estimates may be subject to the bias identified in Nickell (1981). To assess whether this bias is important in our estimates, we also estimate individual processes for each country (which are not subject to any bias) and compare the estimated parameters from the dynamic panel with the histogram of estimates from country-specific regressions (see Figure B.2). As can be seen in the first two panels, the estimates of ρ_{y_T} and $\sigma^2_{y_T}$ are very close to the average estimates from country-specific regressions.

In the debt problem we assume that $(y_{T,t}, \hat{e}_t^{-1})$ follow a first-order VAR process in logs as specified in (15). Data for $\hat{e}_t^{-1}$ is obtained by computing the cyclical component of the inverse of the ratio between the nominal exchange rate and the lagged CPI, also for the period 1990-2014. Cyclical components were computed by estimating a log-linear trend.

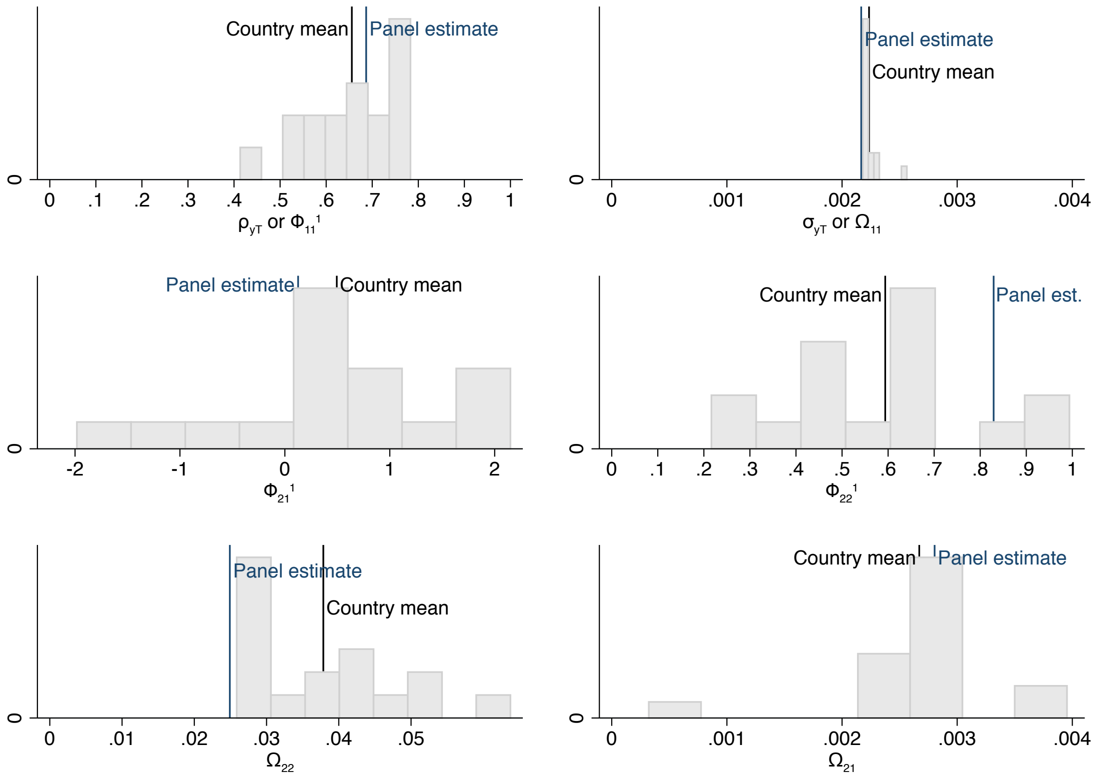
The following pooled-OLS estimates were obtained (including in the estimation country fixed effects):

$$\widehat{\Phi}^1 = \begin{bmatrix} .687 & .000 \\ .126 & .829 \end{bmatrix}, \ \widehat{\Omega} = \begin{bmatrix} .0021 & .0028 \\ .0028 & .0249 \end{bmatrix}.$$

Since the estimation procedure is pooled OLS, the same process for y_T is estimated in both the debt problem and full model. Consistent with the results in Table A3, exhibits an implicit negative relationship between the nominal exchange rate and output, as reflected by the positive entries on the off-diagonal elements of the covariance matrix $\widehat{\Omega}$.

We also perform country-specific VAR estimations and compare the estimated coefficients with those estimated in the panel VAR. The auto-regressive component of the inverse of the

Figure B.2. Country-Specific Exogenous Processes: Histogram of Estimated Parameters



Notes: These plots show the histograms of the each estimated coefficient in country-specific versions of the VAR (15). The average coefficient and the panel estimate are included in each graph. All regressions are estimated with OLS. The panel regression is estimated with pooled OLS. The first two panels also correspond to the country-specific estimation for the AR process for y_T (14).

exchange rate and the covariance of innovations are close the average of their country-specific estimates.

Appendix C. Detrended Problem and Numerical Solution (For Online Publication)

C.1. *Stationary problem*

Define $\hat{b} = \frac{b}{P_{-1}}$. Using (9), the resource constraint can be re-expressed as

$$\begin{aligned} c_T &= y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) P_{-1}\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[r\left(P', \frac{c_T'}{y_N}\right)(1+\delta q')\right]\left(\hat{b}' - \delta\frac{\hat{b}}{\pi}\right)P, \\ &= y_T - b^* - r\left(\pi, \frac{c_T}{y_N}\right)\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[r\left(\pi', \frac{c_T'}{y_N}\right)(1+\delta q')\right]\left(\hat{b}' - \delta\frac{\hat{b}}{\pi}\right). \end{aligned} \tag{18}$$

Similarly, we can de-trend the recursive expression for the price of debt in local currency

$$\begin{aligned} q &= \frac{1}{r\left(P, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[r\left(P', \frac{c_T'}{y_N}\right)(1+\delta q')\right], \\ &= \frac{1}{r\left(1, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[r\left(\pi', \frac{c_T'}{y_N}\right)(1+\delta q')\right]. \end{aligned}$$

Define $\hat{\mathbf{s}} = (b^*, \hat{b}, y)$ as the de-trended state. We can then write the de-trended version of problem (P1) as

$$V(\hat{\mathbf{s}}) = \max_{b^{*\prime}, \hat{b}', \pi, c_T} u(C(c_T, y_N)) - l(\pi) + \beta\mathbb{E}\left[V(\hat{\mathbf{s}}')\right] \tag{P1'}$$

subject to

$$c_T = y_T - b^* - r\left(\pi, \frac{c_T}{y_N}\right)\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}'\right)\right]\left(\hat{b}' - \delta\frac{\hat{b}}{\pi}\right).$$

The government takes as given $\mathcal{X}(\hat{\mathbf{s}})$ when solving the problem. In equilibrium this object solves the following fixed point

$$\mathcal{X}(\hat{\mathbf{s}}) = r\left(\pi, \frac{c_T}{y_N}\right)(1 + \delta q(\hat{\mathbf{s}})),$$

where the price of local currency is determined by the following recursive equation

$$q(\hat{\mathbf{s}}) = \frac{1}{r\left(1, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}'\right)\right]. \tag{19}$$

Similarly, we can then write the de-trended version of problem (P2) as

$$V(\hat{\mathbf{s}}, \epsilon^{\pi}) = \max_{b^{*\prime}, \hat{b}^{\prime}, \pi, c_T} u(C(c_T, y_N)) - l(\Pi(\hat{\mathbf{s}}, \epsilon^{\pi}, b^{*'}, b^{'})) + \beta \mathbb{E}\left[V(\hat{\mathbf{s}}^{\prime})\right] \tag{P2'}$$

subject to

$$c_T = y_T - b^* - r\left(\Pi(\hat{\mathbf{s}}, \epsilon^{\pi}, b^{*'}, b^{'}, \frac{c_T}{y_N}\right)\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right]\left(\hat{b}^{\prime} - \delta\frac{\hat{b}}{\pi}\right).$$

Governments take $\mathcal{X}\left(\hat{\mathbf{s}}\right)$ as given. In equilibrium, it solves the following recursive structure

$$\mathcal{X}\left(\hat{\mathbf{s}}\right) = r\left(\Pi(\hat{\mathbf{s}}, \epsilon^{\pi}, b^{*'}, b^{'}), \frac{c_T}{y_N}\right)(1 + \delta q(\hat{\mathbf{s}})),$$

where the price of local currency is determined by the following recursive equation

$$q(\hat{\mathbf{s}}) = \frac{1}{r\left(1, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right].$$

C.2. *Solution Method*

We solve for equilibrium using a global numerical method that combines value function iteration and policy function iteration. Solving the model implies finding policy functions $\{b^{*\prime}(\hat{\mathbf{s}}), \hat{b}^{\prime}(\hat{\mathbf{s}}), \pi(\hat{\mathbf{s}})\}$ that solve (P1'). The algorithm to solve for the policies numerically follows these steps:

(1) Generate a discrete grid for variable x state space $G_x = x_1, x_2, ..., x_{N_x}$, for $x = y_T, b^*, \hat{b}$. The total aggregate state space is given by $S = G_{y_T} \times G_{b^*} \times G_{\hat{b}}$.

(2) Conjecture a multi-dimensional object $EQ(b^{*\prime}, \hat{b}^{\prime}, y_T)$ as a guess for $\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right]$, which is the expectation term associated to the price of debt in local currency.

(3) Solve for tradable consumption $c_T(\hat{\mathbf{s}}, b^{*\prime}, \hat{b}^{\prime}, \pi)$ using the resource constraint (18). This is a non-linear equation in consumption since it also appears in the exchange rate expression.

(4) Solve problem (P1') using value function iteration method. To achieve numerical accuracy we solve in finer grids and use numerical optimizers. Once the maximum was over the finer grids was identified, we use a numerical optimizer routine to find the maximum in a continuous neighborhood around the initially identified point. We use quadrature methods to compute all expectations and piecewise linear interpolation to interpolate policies outside the grids.

(5) Compute $q(\hat{\mathbf{s}}, b^{*\prime}, \hat{b}^{\prime})$ using (19). Then compute $\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right]$.

(6) If $\sup_{\hat{\mathbf{s}}}\left\|EQ(b^{*\prime}, \hat{b}^{\prime}, y_T) - \mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right]\right\| < \epsilon$, for small ϵ stop. Otherwise, update the guess EQ and start from the first point again.

Appendix D. Robustness and Extensions (For Online Publication)

D.1. *The Role of Debt Maturity and Risk Aversion*

In this section we study how our results are affected by the degree of risk aversion of households and the maturity of debt. The degree of risk aversion can affect the trade-off associated to the currency composition of debt since it determines how valuable are the hedging properties of debt in local currency. We solve the model with a lower degree of risk aversion and find that, consistent with the above-mentioned argument, the share of debt in local currency is smaller than in the baseline model (see Table D1).

Table D1. Sensitivity Analysis: Risk Aversion and Bond Maturity

Moment	Data	Baseline ($\sigma = 5,\ \delta = 0.76$)	Low Risk Av. ($\sigma = 2,\ \delta = 0.76$)	Short Term Debt ($\sigma = 5,\ \delta = 0$)
Average Levels				
Debt	22.0%	22.4%	22.7%	19.8%
Share of Debt in LC	24.7%	9.6%	5.4%	6.0%
Inflation	8.7%	8.7%	8.7%	8.7%
Standard deviation				
Debt	4.2%	3.1%	3.0%	2.9%
Share of Debt in LC	2.3%	3.0%	2.8%	2.8%
Inflation	3.9%	0.1%	0.1%	0.1%
Exchange Rate	13.2%	0.5%	0.5%	0.7%
GDP	3.3%	2.2%	2.2%	2.2%
Correlations with GDP				
Debt	-55.5%	-50.6%	-62.4%	-56.3%
Share of Debt in LC	50.0%	34.1%	47.7%	43.2%
Inflation	7.4%	2.7%	5.5%	-2.5%
Exchange Rate	-50.6%	-72.9%	-74.7%	-75.9%

Notes: The column *Data* refers to average moments for the sample countries detailed in Table 1, for the period 1990-2014. Debt refers to external debt over GDP; the average share of debt in LC refers to the ratio between debt in local currency to total debt; to compute the standard deviation and correlation with GDP of the share of debt in local currency, we measure this variable as the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP. The column *Baseline* refers to the simulations of the baseline model. The columns *Low Risk Av.* and *Short Term Debt* refer to the simulations of the of models with $\sigma = 2$ and $\delta = 0$, respectively. In these two alternative calibrations, we leave all remaining parameters as in the baseline model with the exception of the discount factor that is recalibrated to match the level of total debt and the inflation cost parameter that is recalibrated to deliver the targeted welfare costs of inflation.

The maturity of debt can affect the quantitative relevance of incentive problems since it determines the extent to which the government can spread the inflation costs of dilution over time (Cochrane (2001)), and can also affect the level of debt in local currency necessary to attain certain degree of hedging.[21] We solve the model with short-term debt and find that the share of debt in local currency is lower than in the baseline model (see Table D1), pointing to the fact that the reduction in the scale of debt in local currency to attain certain hedging predominates over the inability of the government to smooth its inflationary indiscipline over time.

D.2. *Models with Detailed Inflation Costs*

In this section we show that models that feature cash-in-advance constraints or money in the utility function can give rise inflation losses featured as a decreasing function in the household's preferences. We also show that under certain functional form assumptions these inflation losses show up as a separable function, similar to our baseline specification. We embed the cash-in-advance and money in the utility function specifications in the context of our model of currency composition of sovereign debt to maintain the parallelism with our baseline model as close as possible.

D.2.1. *Cash-in-advance model*

Consider a variant of our baseline economy in which there are three goods: tradable and non-tradable goods, and the cash good. The cash good is produced with labor with production function $c_{Xt} = n_t$, where c_{Xt} denotes household's consumption of the cash good and n_t the labor supplied by households. We adopt the Svensson (1985) timing convention by which money brought from the previous period can only be used to purchase the cash good. This specification gives rise to inefficiencies due to realized inflation. Consuming the cash good yields utility $v(c_{Xt})$ which is increasing and concave, and labor entails disutility $g(n_t) = \chi n_t$. The household problem now involves choices of money holdings and labor in addition to intra-temporal consumption:

$$\max_{c_{Tt},c_{Nt},c_{Xt},n_t,M_t} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, c_{Nt})) + v\left(c_{Xt}\right) - \chi n_t\right]$$

subject to

$$p_{Xt}c_{Xt} + e_t c_{Tt} + p_{Nt}c_{Nt} + M_t = p_{Xt}n_t + e_t y_{Tt} + p_{Nt}y_{Nt} + T_t + M_{t-1} \tag{20}$$

$$p_{Xt}c_{Xt} \leq M_{t-1} \tag{21}$$

[21] The flow repayment of debt (as opposed to the stock) is what determines the scale of the hedging. Hence, the necessary stock of domestic debt to attain a given degree of hedging is increasing in the maturity of debt.

given M_0. We implicitly imposed that wages are equal to prices using zero-profits for firms in the cash good sector. The first order conditions of this problem are given by:

$$\begin{aligned} u'(c_t) &= \lambda_t P_t, \\ v'(c_{Xt}) &= (\lambda_t + \mu_t) p_{Xt}, \\ \chi &= \lambda_t p_{Xt}, \\ \lambda_t &= \mathbb{E}[\beta(\lambda_{t+1} + \mu_{t+1})], \end{aligned}$$

where $c_t = C(c_{Tt}, c_{Nt})$ is the consumption aggregator, P_t is the ideal price index, and λ_t and μ_t are the Lagrange multipliers of the budget constraint (20) and the cash-in-advance constraint (21). We focus on the case in which the cash-in-advance constraint is binding. Define the surplus utility for the cash good as $\tilde{v}(c_{Xt}) \equiv v(c_{Xt}) - \chi c_{Xt}$. Imposing market clearing in the cash good and the non-tradable good we can express the household utility as

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, y_N)) + \tilde{v}\left(\frac{m_{t-1}}{\pi_t}\frac{u'(c_t)}{\chi}\right)\right],$$

where $m_t \equiv \frac{M_t}{P_t}$ are the real balances and their law of motion satisfies

$$\mathbb{E}\left[\beta\tilde{v}'\left(\frac{m_t}{\pi_{t+1}}\frac{u'(c_{t+1})}{\chi}\right)\frac{1}{p_{Xt}}\right] = \frac{u'(c_t)}{P_t} - \mathbb{E}\left[\beta\frac{u'(c_{t+1})}{P_{t+1}}\right].$$

This expression is decreasing in π_t. Hence, inflation shows up as a loss in the utility function.

If we further assume that $v(c_{xt}) = \nu\log(c_{xt}) + \chi c_{xt}$ then we get the following expression for utility in which inflation enters as a separable negative cost (like in our baseline model)

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, y_N)) - \nu\log(\pi_t) + \nu\log\left(\frac{m_{t-1}u'(c_t)}{\chi}\right)\right]$$

where the law of motion of m_t satisfies

$$m_t = \frac{\beta\nu}{u'(c_t) - \mathbb{E}\left[\beta\frac{u'(c_{t+1})}{\pi_{t+1}}\right]}.$$

D.2.2. *Model with Preferences for Real Money Balances*

Consider another variant of our baseline economy in which households have preferences for real money balances, with the equivalent timing as the cash-in-advance model. The household problem now involves choices of money holdings and consumption:

$$\max_{c_{Tt}, c_{Nt}, M_t} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, c_{Nt})) + v\left(\frac{M_{t-1}}{P_t}\right)\right]$$

subject to

$$p_{Xt}c_{Xt} + e_t c_{Tt} + M_t = e_t y_{Tt} + p_{Nt} y_{Nt} + T_t + M_{t-1} \tag{22}$$

given M_0, where $v(\cdot)$ is an increasing and concave function. It follows directly from the definition of real balances that utility is decreasing in realized inflation π_t. Additionally, for the particular case of $v\left(\frac{M_{t-1}}{P_t}\right) = \nu \log\left(\frac{M_{t-1}}{P_t}\right)$ we can show that realized inflation enters utility in the following separable decreasing function

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, y_N)) - \nu \log(\pi_t) + \nu \log(m_{t-1})\right]$$

where the law of motion of m_t satisfies

$$m_t = \frac{\beta\nu}{u'(c_t) - \mathbb{E}\left[\beta \frac{u'(c_{t+1})}{\pi_{t+1}}\right]}.$$

Note that the model with cash in advance and money in the utility function are symmetric with the only difference that in the cash in advance model real balances are defined over the price of the cash good instead of the ideal price index.

D.2.3. *Quantitative Analysis of the Model with Preferences for Real Balances*

In this section we calibrate and analyze the quantitative predictions of the model with money in the utility function. We find that this model delivers quantitative results that are in line with those of the baseline model.

We adopt the log specification for utility associated to real balances. The recursive problem of the government is given by

$$V(b^*, b, m_{-1}, y_T) = \max_{b^{*\prime}, b', \pi, c_T} u(C(c_T, y_N) + \nu \log(m_{-1}) - \nu \log(\pi) + \beta \mathbb{E}\left[V(\mathbf{s}')\right]$$

subject to

$$c_T = y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) b + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \tilde{q}\left(b' - \delta b\right),$$

$$\tilde{q} = \frac{1}{R}\mathbb{E}\left[\mathcal{R}(\mathbf{s}') + \delta \tilde{q}(\mathbf{s}')\right],$$

$$m = \frac{\beta\nu}{u'(c) - \mathbb{E}\left[\beta \frac{u'(c')}{\pi'}\right]}$$

$$P = \pi P_{-1}.$$

There are two main differences with the baseline specification. First, the losses of inflation are concave here and convex in the baseline specification. Second, there is an extra term $\nu \log(m_{-1})$

that enters utility which the government takes into account when choosing consumption. Next we argue that the presence of this new term is not important quantitatively for the optimal choices of the government.

To calibrate the model we set the same parameter values as in the baseline specification with the exception of β, which we calibrate to match the total level of external public debt. The calibrated value of the discount factor is $\beta = 0.96$. The new parameter is ν which governs the preferences for real balances. We follow a symmetric approach as in our baseline model and calibrate ν to obtain the same welfare loss of inflation as in the baseline model. That is, we calibrate ν so that a increase in inflation of 1% has associated a loss of 0.1% in consumption equivalent terms. The calibrated value is $\nu = 0.002$.

We then simulate data from this model and compare the moments from the simulated data with those of the data and the baseline model. Results are shown in the third column of Table D2. The main quantitative results of the model with money in the utility function are in line with those of the baseline model. In particular, the share of debt in local currency is 9% compared to 10% in the baseline model. The remaining moments are also similar, including the rate of inflation. This finding suggest that we do not loose generality by focusing on the baseline model in which inflation costs enter in a reduced-form way.

D.3. *Model with Costs of Currency Depreciation*

Consider a variation of the baseline model in which the losses come from fluctuations in the nominal exchange rate, $l(\Delta e_t)$, where $\Delta e_t \equiv \frac{e_t}{e_{t-1}}$. As in section 4.2, we focus on the case of short-term debt ($\delta = 0$). Without loss of generality we set $y_N = 1$, and assume y_T is constant. The recursive government problem can be expressed as:

$$V(\hat{b}_t, b^*_t) = \max_{\hat{b}_{t+1}, b^*_{t+1}, c_t, \Delta e_t} u(C(c_{Tt}, 1)) - l(\Delta e_t) + \beta V(\hat{b}_{t+1}, b^*_{t+1})$$

subject to

$$y_T - c_{Tt} + \frac{1}{R} r(\mathcal{D}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1} - r(\Delta e_t)\hat{b}_t + \frac{1}{R} b^*_{t+1} - b^*_t = 0,$$

where $r(\Delta e_t) = \frac{1}{\Delta e_t}$ is the repayment function (which now only depends on the depreciation rate), and $\mathcal{D}(\hat{b}_{t+1}, b^*_{t+1})$ is the expected nominal currency depreciation. The first-order conditions of this problem are given by:

$$[c_t]: \qquad u'(c_t)C_{c_T,t} = \lambda_t,$$

$$[\pi_t]: \qquad -l'(\Delta e_t) = \lambda_t r'(\Delta e_t)\hat{b}_t,$$

$$[\hat{b}_{t+1}]: \qquad \beta V_{\hat{b}}(\hat{b}_{t+1}, b^*_{t+1}) = -\lambda_t \frac{1}{R}\frac{\partial r(\mathcal{D}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}},$$

$$[b^*_{t+1}]: \qquad \beta V_{b^*}(\hat{b}_{t+1}, b^*_{t+1}) = -\lambda_t \frac{1}{R}\left[1 + \frac{\partial r(\mathcal{D}(\hat{b}_{t+1}, b^*_{t+1}))}{\partial b^*_{t+1}}\hat{b}_{t+1}\right].$$

Envelope conditions:

$$[\hat{b}_t]: \qquad V_{\hat{b}}(\hat{b}_t, b^*_t) = -\lambda_t r(\Delta e_t),$$

$$[b^*_t]: \qquad V_{b^*}(\hat{b}_t, b^*_t) = -\lambda_t.$$

Comparing these optimality conditions with those of the model with inflation costs (baseline model) we can see that the trade-off are similar. In particular the optimal depreciation is countercyclical and the optimal choice of debt take into account disciplining effects. In addition, in the particular case of an infinite cost of depreciation the nominal exchange rate is fixed and the both assets are payoff-equivalent, which leads to portfolio indeterminancy.

D.4. *Model with Outright Default*

In this section we extend our baseline model to allow for outright default. We assume that every period the government can choose to repay or default on its debt. We assume that the default decision applies to all debt regardless of its currency of denomination. We follow most quantitative models of default and assume that following a default the government faces a stochastic number of periods during which it is excluded from credit markets. It regains access to credit markets with probability $\theta \in (0, 1)$ every period. We follow Arellano (2008) and assume that in those periods in which the country is in autarky the level of tradable output is given by[22]

$$y^{def}_{Tt} = \begin{cases} \hat{y} & \text{if } y_{Tt} > \xi\mathbb{E}[y_T] \\ y & \text{if } y_{Tt} \leq \xi\mathbb{E}[y_T]. \end{cases}$$

[22]The assumption that a default triggers a drop in y_T results in a real exchange rate depreciation after a default, a common feature of the data.

Denote $\mathbf{s} = \{b^*, b, y_T, P_{-1}\}$ the aggregate state, and $q(\mathbf{s}, b^{*\prime}, b)$ and $q(\mathbf{s}, b^{*\prime}, b)$ the price schedules of debt in foreign and local currency, respectively, both expressed in foreign currency. The government's problem written in recursive form is given by:

$$V(\mathbf{s}) = \max_{\iota \in \{0,1\}} \iota V^r(\mathbf{s}) + (1-\iota) V^a(y_T, P_{-1}).$$

The value of repaying is given by:

$$\begin{aligned}
V^r(\mathbf{s}) &= \max_{b^{*\prime}, b', \pi, c_T} \{u\left(C(c_T, y_N)\right) - l\left(\pi\right)\} + \beta \mathbb{E}\left[V(\mathbf{s}')\right] \\
&\quad \text{subject to} \\
c_T &= y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) b + q^*(\mathbf{s}, b^{*\prime}, b)\left(b^{*\prime} - \delta b^*\right) + \tilde{q}(\mathbf{s}, b^{*\prime}, b)\left(b' - \delta b\right), \\
P &= \pi P_{-1}.
\end{aligned}$$

The value of default (or being in autarky) is given by:

$$V^d(y_T, P_{-1}) = u\left(C(y_T^{def}, y_N)\right) - l\left(\pi^*\right) + \beta \mathbb{E}\left[\theta V(0, 0, y_T', P) + (1-\theta) V^d(y_T', P)\right]$$

Note that we are assuming that the government re-enters with zero debt to markets. We are already imposing that the optimal inflation rate while in autarky is π^*. This is because there are no incentives to incur in costly inflation since there is no debt to dilute. Finally, in equilibrium risk-neutral investors obtain an expected return of R and debt prices satisfy the following recursive expressions

$$\begin{aligned}
q^*(\mathbf{s}, b^{*\prime}, b) &= \frac{1}{R} \mathbb{E}\left[1 + \delta \mathcal{Q}^*(\mathbf{s}')\right], \\
\tilde{q}(\mathbf{s}, b^{*\prime}, b) &= \frac{1}{R} \mathbb{E}\left[\mathcal{R}(\mathbf{s}') + \delta \tilde{\mathcal{Q}}(\mathbf{s}')\right],
\end{aligned}$$

where $\mathcal{R}(\mathbf{s}), \mathcal{Q}^*(\mathbf{s}), \tilde{\mathcal{Q}}(\mathbf{s})$ are the inverse of the nominal exchange rate and prices of debt in foreign and local currency evaluated in the optimal policies.

We calibrate the model using the same functional forms and parameter values as in the baseline model. This specification introduces two new parameters: the re-entry probability θ and the parameter that governs the output cost of default ξ. We follow Chatterjee and Eyigungor (2012) and set $\theta = 0.15$ so that the average length of the exclusion period is 6.5 years. We calibrate β and ξ so that we match the average level of total debt and obtain a frequency of default of 3.5%, which is in the range of frequencies targeted in quantitative models of default. The calibrated values are $\beta = 0.94$ and $\xi = 0.82$.

Table D2 shows the moments associated to the model with default, compared to the moments from the data and from the baseline model. The main quantitative results remain in the model with default. The average share of debt in local currency is 14.6%, which is close to the 9.6% share in the baseline model and the 24.7% in the data. The remaining models are also very close to those of the baseline model.

Table D2. Models with Real Money Balances and Default: Results

Moment	Data	Baseline Model	Model with Money	Model with Outright Default
Average Levels				
Debt	22.0%	22.4%	22.6%	22.1%
Share of Debt in LC	24.7%	9.6%	8.9%	14.6%
Inflation	8.7%	8.7%	7.3%	8.7%
Standard deviation				
Debt	4.2%	3.1%	3.1%	4.0%
Share of Debt in LC	2.3%	3.0%	2.9%	10.5%
Inflation	3.9%	0.1%	0.5%	0.4%
Exchange Rate	13.2%	0.5%	0.7%	1.7%
Real Exchange Rate	12.6%	0.4%	0.5%	1.5%
GDP	3.3%	2.2%	2.2%	2.3%
Correlations with GDP				
Debt	-55.5%	-50.6%	-51.2%	-59.8%
Share of Debt in LC	50.0%	34.1%	42.1%	11.8%
Inflation	7.4%	2.7%	4.0%	-11.0%
Exchange Rate	-50.6%	-72.9%	-53.0%	-68.3%
Real Exchange Rate	-54.1%	-77.3%	-76.2%	-76.0%

Notes: The column *Data* refers to average moments for the sample countries detailed in Table 1, using annual data, for the period 1990-2014. For countries in which the 1990 data for a given variable was not available, we consider the earliest data available for that variable. Debt refers to external debt over GDP; the average share of debt in LC refers to the ratio between debt in local currency to total debt; to compute the standard deviation and correlation with GDP of the share of debt in local currency, we measure this variable as the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP. Average inflation was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B. The column Baseline Model reports moments of the simulations of the baseline model. The column Model with Money reports moments from the model with money in the utility function. The final column reports the moments from the model with outright default. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100).

Appendix E. Omitted Proofs and Results (For Online Publication)

Proof of Proposition 1

Define the aggregate exogenous state as $\mathbf{s}_t^x = (y_{Tt}, \hat{e}_t^{-1})$. The government problem (P2) for the case of $l(\pi) = 0$ and short-term debt can be expressed recursively as

$$V(b^*, \hat{b}, \mathbf{s}^x) = \max_{b^{*\prime}, \hat{b}^\prime, c_T} u(C(c_T, y_N)) + \beta \mathbb{E}\left[V(b^{*\prime}, \hat{b}^\prime, \mathbf{s}^{x\prime})\right]$$

$$s.t.$$

$$c_T = y_T - b^* - \hat{e}^{-1}\hat{b} + q^* b^{*\prime} + \hat{e}^{-1} q \hat{b}^\prime, \tag{23}$$

$$b^* + \hat{e}^{-1}\hat{b} \leq \overline{b},$$

$$\Pr(\mathbf{s}^{x\prime} | \mathbf{s}^x) = g(y_T, \hat{e}^{-1}, y_T^\prime, \hat{e}^{-1\prime}).$$

The second restriction implies a borrowing constraint on total debt which prevents Ponzi games. We assume $\overline{b}$ is large enough such that this constraint is not binding and thus we can ignore this constraint. The optimality conditions that are necessary and sufficient are given by

$$u^\prime(c_t) C_{c_T,t} q^* = \beta \mathbb{E}\left[u^\prime(c_{t+1}) C_{c_T,t+1}\right] \tag{24}$$

$$u^\prime(c_t) C_{c_T,t} \hat{e}_t^{-1} q = \beta \mathbb{E}\left[u^\prime(c_{t+1}) C_{c_T,t+1} \hat{e}_{t+1}^{-1}\right]. \tag{25}$$

Now we derive equation (13) from (24), (25) and the definition of debt prices. From the definition of covariance we have

$$\mathbb{E}\left[u^\prime(c_{t+1}) C_{c_T,t+1} \hat{e}_{t+1}^{-1}\right] = \mathbb{COV}\left(u^\prime(c_{t+1}) C_{c_T,t+1}, \hat{e}_{t+1}^{-1}\right) + \mathbb{E}\left[u^\prime(c_{t+1}) C_{c_T,t+1}\right] \mathbb{E}\left[\hat{e}_{t+1}^{-1}\right].$$

Using (25) and (7) we get

$$u^\prime(c_t) C_{c_T,t} \mathbb{E}\left[\hat{e}_{t+1}^{-1}\right] = \beta R \left[\mathbb{COV}\left(u^\prime(c_{t+1}) C_{c_T,t+1}, \hat{e}_{t+1}^{-1}\right) + \mathbb{E}\left[u^\prime(c_{t+1}) C_{c_T,t+1}\right] \mathbb{E}\left[\hat{e}_{t+1}^{-1}\right]\right].$$

Using (24) this last equation is equivalent to

$$\mathbb{COV}\left(u^\prime(c_{t+1}) C_{c_T,t+1}, \hat{e}_{t+1}^{-1}\right) = 0. \tag{26}$$

To prove the proposition we assume that the optimal controls $b^{*\prime}, c_T, c_N$ satisfy (23)-(24) and we find $b^\prime$ that satisfies (26).

Consider the case in which $\rho(y_T, \hat{e}^{-1}) = 1$. Then there exist $\gamma_0, \gamma_1 > 0$ such that $\hat{e}^{-1} = \gamma_0 + \gamma_1 y_T$, with $\gamma_1 = \left(var(\hat{e}^{-1})/var(y_T)\right)^{1/2}$. Substituting this equation into (23) and we get

$$c_T = y_T - b^* - (\gamma_0 + \gamma_1 y_T)\hat{b} + q^* b^{*\prime} + q^* \mathbb{E}\left[\gamma_0 + \gamma_1 y_T^\prime | y_T\right] \hat{b}^\prime. \tag{27}$$

Note that the conditional expectation does not depend on y_T due to the iid assumption. From (27) we can see that by setting $\hat{b}' = \gamma_1^{-1} > 0$ we make c_T independent of the realization of y_T. Note now that since y_N is deterministic the only source of randomness of $u'(c_{t+1})C_{c_T,t+1}$ comes from c_T, which we just showed is deterministic. Hence, it follows that (26) is satisfied.

The proof for the case of $\rho(y_T, \hat{e}^{-1}) = -1$ is analogous with the difference that now we can write $\hat{e}^{-1} = \gamma_0 - \gamma_1 y_T$, for some γ_0 and $\gamma_1 = (var(\hat{e}^{-1})/var(y_T))^{1/2}$.

Now consider the case of $\rho(y_T, \hat{e}^{-1}) = 0$. We show that for $b = 0$ (26) is satisfied. Using (23) and (7) we have that

$$c_T = y_T - b^* - \hat{e}^{-1}\hat{b} + q^* b^{*\prime} + q^* \mathbb{E}\left[\hat{e}^{-1\prime}\right]\hat{b}'$$

Since $\hat{e}^{-1}$ is i.i.d. over time then c_T is independent of $\hat{e}^{-1}$ if $b = 0$, which implies that (26) is satisfied.

Proof of Proposition 2

First note that the $\pi^*(\mathbf{s}) = \infty$ for any $\mathbf{s}$ with $b > 0$. Since in equilibrium foreign lenders must obtain an expected return of R, then in equilibrium it must be the case that $q(\mathbf{s}) = 0$ whenever $b'(\mathbf{s}) > 0$. For states in which $b'(\mathbf{s}) = 0$ then the result follows trivially.

Derivation of Euler Equations

This subsection derives the Euler equations shown in section 4. We assume there is no uncertainty and $\delta = 0$, and derive one generalized Euler equation for each type of debt, that embeds the Euler equations for the case of dilution through inflation and the case of dilution through real exchange rate.

Define $\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \hat{\mathcal{B}}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})$ the expected consumption, inflation and debt policies in local and foreign currency, respectively. In an equilibrium, these expectations are consistent with optimal policies. Without loss of generality we set $y_N = 1$, and assume y_T is constant. The recursive government problem (P1') can be expressed as:

$$V(\hat{b}_t, b^*_t) = \max_{\hat{b}_{t+1}, b^*_{t+1}, c_t, \pi_t} u(C(c_{Tt}, 1)) - l(\pi_t) + \beta V(\hat{b}_{t+1}, b^*_{t+1})$$

subject to

$$y_T - c_{Tt} + \frac{1}{R} r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1} - r(\pi_t, c_{Tt})\hat{b}_t + \frac{1}{R} b^*_{t+1} - b^*_t = 0,$$

The first-order conditions of this problem are given by:

$$[c_t]: \qquad u'(c_t)C_{c_T,t} = \lambda_t(1 + r_c(\pi_t, c_{Tt})\hat{b}_t),$$

$$[\pi_t]: \qquad -l'(\pi_t) = \lambda_t r_P(\pi_t, c_{Tt})\hat{b}_t,$$

$$[\hat{b}_{t+1}]: \qquad \beta V_{\hat{b}}(\hat{b}_{t+1}, b^*_{t+1}) = -\lambda_t \frac{1}{R}\frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}},$$

$$[b^*_{t+1}]: \qquad \beta V_{b^*}(\hat{b}_{t+1}, b^*_{t+1}) = -\lambda_t \frac{1}{R}\left[1 + \frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))}{\partial b^*_{t+1}}\hat{b}_{t+1}\right].$$

Envelope conditions:

$$[\hat{b}_t]: \qquad V_{\hat{b}}(\hat{b}_t, b^*_t) = -\lambda_t r(\pi_t, c_{Tt}),$$

$$[b^*_t]: \qquad V_{b^*}(\hat{b}_t, b^*_t) = -\lambda_t.$$

Combining these equations we get two Euler equations:

$$\frac{u'(c_t)C_{c_T,t}}{1 + r_c(\pi_t, c_{Tt})\hat{b}_t}\frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} = \beta R\frac{u'(c_{t+1})C_{c_T,t+1}}{1 + r_c(\pi_{t+1}, c_{Tt+1})\hat{b}_{t+1}}r(\pi_{t+1}, c_{Tt+1}), \tag{28}$$

$$\frac{u'(c_t)C_{c_T,t}}{1 + r_c(\pi_t, c_{Tt})\hat{b}_t}\left[1 + \frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))}{\partial b^*_{t+1}}\hat{b}_{t+1}\right] = \beta R\frac{u'(c_{t+1})C_{c_T,t+1}}{1 + r_c(\pi_{t+1}, c_{Tt+1})\hat{b}_{t+1}}. \tag{29}$$

The price sensitivity of debt can be calculated. Differentiating the resource constraint at $t+1$ with respect to $\hat{b}_{t+1}$ and b^*_{t+1}:

$$\begin{aligned}\frac{\partial \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1})}{\partial \hat{b}_{t+1}} =& \frac{1}{R}\frac{\partial r(\mathcal{C}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})), \mathcal{P}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})))\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}) + \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})}{\partial \hat{b}_{t+1}} \\ & - \frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}},\end{aligned} \tag{30}$$

$$\begin{aligned}\frac{\partial \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1})}{\partial b^*_{t+1}} =& \frac{1}{R}\frac{\partial r(\mathcal{C}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})), \mathcal{P}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})))\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}) + \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})}{\partial b^*_{t+1}} \\ & - \frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial b^*_{t+1}} - 1.\end{aligned} \tag{31}$$

To simplify notation denote

$$\frac{\partial r(c_{Tt+2}, \pi_{Tt+2})\hat{b}_{t+2} + b^*_{t+2}}{\partial \hat{b}_{t+1}} \equiv$$

$$\frac{\partial r(\mathcal{C}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})), \mathcal{P}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})))\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}) + \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})}{\partial \hat{b}_{t+1}}.$$

and an analogous expression for the derivative with respect to b^*_{t+1}. Applying the chain rule to the second term of the right hand side of equations (30) and (31) yields

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} =& r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{C}_b(\hat{b}_{t+1},b^*_{t+1})\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1}), \qquad (32)\\ \frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial b^*_{t+1}} =& \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{C}_{b^*}(\hat{b}_{t+1},b^*_{t+1})\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_{b^*}(\hat{b}_{t+1},b^*_{t+1}). \qquad (33)\end{aligned}$$

With these equations we can derive the Euler equations. First we derive the Euler equation for debt in local currency. Combining (30) and (32) we get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} =& r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{Tt+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial \hat{b}_{t+1}}\\ &-\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}}\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1}).\end{aligned}$$

We re-arrange the same equation to get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} =& \frac{1}{1+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))}\Big[r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{Tt+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial \hat{b}_{t+1}}\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1})\Big].\end{aligned}$$

Substituting this equation in (28) we get the modified Euler equation for debt in local currency

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1}\underbrace{\left(1+r_{c,t}\hat{b}_t\right)}_{\text{Dilution through RXR}}\underbrace{\frac{1}{1+\frac{\hat{b}_{t+1}\left(r_{c,t+1}\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{t+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial \hat{b}_{t+1}}+r_{P,t+1}\pi_b(\hat{b}_{t+1},b^*_{t+1})\right)}{r(\pi_{t+1},c_{Tt+1})}}}_{\text{Discipline Effect}}. \qquad (34)$$

Now we derive the Euler equation for debt in foreign currency. We follow a similar approach as before. Combining (31) and (33) we get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}), b^*_{t+1})\hat{b}_{t+1}}{\partial b^*_{t+1}} =& + \hat{b}_{t+1} r_c(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2}, \pi_{Tt+2})\hat{b}_{t+2} + b^*_{t+2}}{\partial b^*_{t+1}} \\ & - \hat{b}_{t+1} r_c(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}), b^*_{t+1})\hat{b}_{t+1}}{\partial b^*_{t+1}} \\ & - \hat{b}_{t+1} r_c(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1})) \\ & + \hat{b}_{t+1} r_P(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1}, b^*_{t+1}).\end{aligned}$$

We re-arrange the same equation to get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}), b^*_{t+1})\hat{b}_{t+1}}{\partial b^*_{t+1}} =& \frac{1}{1 + \hat{b}_{t+1} r_c(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))}\Big[- \hat{b}_{t+1} r_c(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1})) \\ & + \hat{b}_{t+1} r_c(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2}, \pi_{Tt+2})\hat{b}_{t+2} + b^*_{t+2}}{\partial b^*_{t+1}} \\ & + \hat{b}_{t+1} r_P(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\mathcal{P}_{b^*}(\hat{b}_{t+1}, b^*_{t+1})\Big].\end{aligned}$$

Substituting this equation in (29) we get the modified Euler equation for debt in foreign currency

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1} \underbrace{\left(1 + r_{c,t}\hat{b}_t\right)}_{\text{Dilution thr. RXR}} \underbrace{\frac{1}{1 + \hat{b}_{t+1}\left(r_{c,t+1}\frac{1}{R}\frac{\partial r(c_{Tt+2}, \pi_{t+2})\hat{b}_{t+2} + b^*_{t+2}}{\partial b^*_{t+1}} + r_{P,t+1}\pi_{b^*,t+1}\right)}}_{\text{Discipline Effect}}. \tag{35}$$