The Exchange Rate as an Industrial Policy*

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March 21, 2024

Abstract

We study the role of exchange rates in industrial policy. We construct an open-economy macroeconomic framework with production externalities and show that the desirability of these policies critically depends on the dynamic patterns of externalities. When they are stronger in earlier stages of development, economies converging to the technological frontier can improve welfare by intervening in foreign exchange markets, keeping the exchange rate undervalued, and speeding up the transition; economies that are not converging to the technological frontier are better off by not using the exchange rate as an industrial policy tool. Capital-flow mobility and labor market dynamism play a central role in the effectiveness of these policies. We also discuss the role of capital controls as an industrial policy tool and use our framework to interpret historical experiences.

Keywords: Exchange rates, industrial policy, imperfect financial markets, growth take-off, capital controls

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This paper previously circulated under the title "Capital Controls as an Industrial Policy." We thank Oleg Itskhoki, Matteo Maggiori, Steve Redding, Richard Rogerson, Jesse Schreger, and seminar participants at various institutions for useful comments and suggestions. Shi Hu provided excellent research assistance.

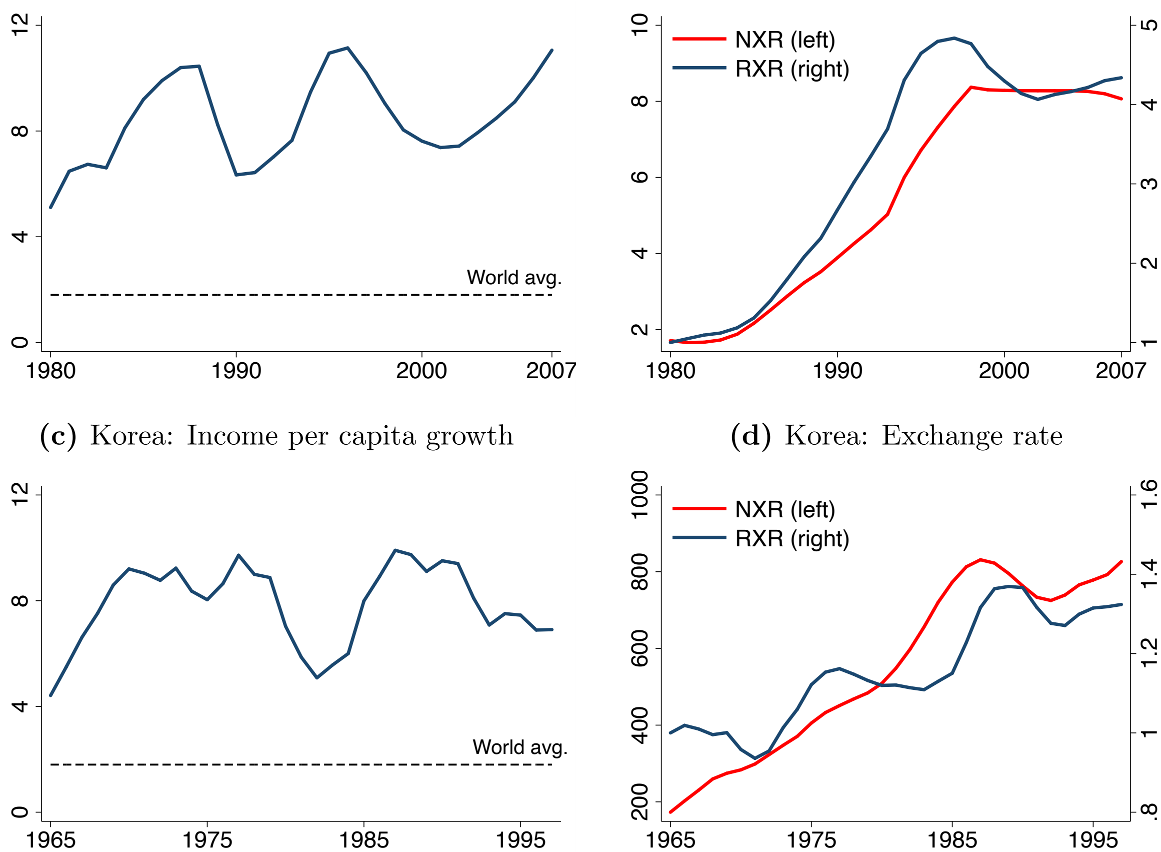
1. Introduction

A long-held view in policy circles is that the exchange rate can be used as a tool to foster development. The broad idea is that maintaining a depreciated exchange rate can stimulate the growth of strategic sectors by enhancing their competitiveness, and this helps speed growth processes. This narrative is based on salient examples of emerging-market economies' prolonged growth, which include the case of South Korea from the 1960s until the 1990s and the recent case of China from the 1980s until the 2010s. These economies experienced decades of high growth rates in per capita output that were on average more than three times higher than the global growth rate. These processes were accompanied by significant depreciations of the nominal and real exchange rates (see Figure 1).

In this paper, we ask how the exchange rate can be used as an industrial policy. To do so, we develop an open-economy macroeconomic framework with Marshallian production externalities and imperfect capital mobility. We show that the desirability of exchange-rate industrial policies critically depends on the dynamic patterns of externalities. When externalities are stronger in earlier stages of development, economies converging to the technological frontier can benefit from foreign exchange interventions aimed at keeping the currency undervalued at early stages of the transition, increasing labor supply, and directing resources to the tradable sector. On the contrary, in economies that are not converging, either because they are stagnating or because they are at the technological frontier, exchange rate industrial policies reduce welfare. While these economies may feature externalities, foreign exchange interventions are not the right tool to address them to the extent that externalities do not exhibit a dynamic pattern. Our framework also highlights the role of imperfect capital mobility and labor market dynamism as important features that determine the effectiveness of these policies.

The paper begins by constructing a theoretical framework to study the role of exchange rates as an industrial policy. The model embeds production externalities in a canonical open-economy framework with tradable and nontradable goods. Externalities exhibit a dynamic pattern that depend on the development stage of the economy, with stronger spillovers in

Figure 1: The Macroeconomic Effects of Exchange Rate Industrial Policy



(a) China: Income per capita growth

(b) China: Exchange rate

(c) Korea: Income per capita growth

(d) Korea: Exchange rate

*N*otes: Panels (a) and (c) show the 5-year moving average of the annual growth rate of per capita GDP. Panels (b) and (d) show the 5-year moving average of the nominal exchange rate per USD and multilateral real exchange rate (expressed as domestic currency per units of a basket of foreign currencies). Data sources: BIS, OECD, World Bank.

economies that are further from the technological frontier (see, e.g., Redding, 1999). The model also features imperfect financial markets, which allow the government to influence the path of the real exchange rate through foreign exchange interventions (see, e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021a).

The competitive equilibrium in the economy features an inefficiently slow speed of convergence to the technological frontier. Individual agents do not internalize that the social returns of labor are larger than private ones, and especially so in the early stages of development. Hence, the economy features inefficiently low levels of labor during the transition.

In addition, if production externalities are stronger in the tradable sector (as in Krugman, 1987), the competitive equilibrium features a distorted marginal rate of transformation between goods, with too little production of tradable goods. In this environment, a government with time-varying, sector-specific labor subsidies can attain the first-best allocation.

In the absence of these fiscal tools, the government can still use exchange rate policies to exploit the dynamic pattern of externalities and move the economy closer to its first-best allocation. The optimal "exchange rate industrial policy" features a depreciated exchange rate during early stages of the transition, which is attained with currency market interventions and the accumulation of international reserves. This policy affects the dynamic path of allocations through two channels. First, by purchasing foreign currency the return on saving in local currency increases, which stimulates savings, decreases consumption, and increases labor supply. Second, the depreciated exchange rate induces firms to redirect labor to the tradable sector.

Our theoretical framework emphasizes the role of dynamic externalities in providing a rationale for exchange rate industrial policies. When the government purchases foreign currency, it depreciates the current exchange rate at the expense of appreciating it in the future. Therefore, this type of policy is undesirable in economies that feature externalities that are not dynamic. This includes economies that are at the technological frontier, or those that are stagnant and not converging to it.

We also use our framework to study what determines the effectiveness of the use of exchange rate as an industrial policy. A necessary ingredient for this policy to work is to have imperfect capital mobility, so that foreign exchange interventions can affect the exchange rate and the macroeconomy. The easier it is for foreign intermediaries to engage in carry trades and exploit differential rates of return, the less effective and desirable is the use of the exchange rate as an industrial policy. We also show that allocations under the optimal exchange rate industrial policy can be implemented with time-varying taxes on debt. Therefore, our framework provides a rationale for the use of capital controls as an industrial policy.

Another important ingredient determining the effectiveness of exchange rate industrial policies is the dynamism of labor markets and the sectoral composition of dynamic externalities. These policies are most effective in environments with highly elastic labor supply in which changes in the exchange rate create large rellocation of labor and production. Additionally, the policies are most desirable when dynamic externalities are present in sectors that can more easily attract additional labor as they become more competitive.

Finally, we use our framework to interpret historical experiences. The Asian growth miracles are often referred to as emblematic examples of export-led growth in the context of undervalued currencies. Through the lens of our model these economies appear to meet the central ingredients required for effective exchange rate industrial policies: a process of convergence to the technological frontier, initially underdeveloped financial markets, and "demographic dividends" that imply a highly elastic labor supply to the tradable sector. The salient characteristics of the Asian examples appear in contrast with those from Latin American experiences, which are often referenced as failures of these types of policies. Most Latin American economies did not experience convergence processes, featured larger costs to sectoral reallocation of labor and a capital account that was relatively more open.

Related Literature. Our paper is related to several strands of the literature. First, the paper builds on the new generation of macroeconomic models of the exchange rate and imperfect financial markets, surveyed by Maggiori (2022). These models have been used to study exchange rate dynamics, their connection with the macroeconomy, and the effectiveness of foreign exchange interventions (see, for example, Gabaix and Maggiori, 2015; Fanelli and Straub, 2021; Itskhoki and Mukhin, 2021a,b, 2023). We contribute to this literature by studying exchange rate market interventions in economies with production externalities.

Second, the paper contributes to the literature that studies the role of exchange rates for economic development (see, for example, Hirschman, 1958; Rodrik, 1986; Krugman, 1987; Baldwin and Krugman, 1989; Rodrik, 2008; Aghion, Bacchetta, Ranciere and Rogoff, 2009). Our work builds on the literature showing that maintaining an undervalued exchange rate and managing capital inflows can be desirable in the presence of production externalities in the tradable sector (see, for example, Michaud and Rothert, 2014; Korinek and Serven, 2016;

Guzman, Ocampo and Stiglitz, 2018; Benigno, Fornaro and Wolf, 2022).[1] We complement this literature by providing a theoretical framework that can be used to assess when these policies are desirable and when they are not. Our conclusions, highlighting the role of dynamic externalities as a necessary condition, echo those in Itskhoki and Moll (2019) in which dynamic patterns of externalities endogenously emerge as a consequence of financial frictions.

Our paper is also related to the literature that studies how capital flows to fast-growing developing economies (see, e.g., Lucas, 1990; Alfaro, Kalemli-Ozcan and Volosovych, 2008; Aguiar and Amador, 2011; Gourinchas and Jeanne, 2013, among others). Closest to our paper, a rising literature studies China's integration into international capital markets. This literature has highlighted the central role of exchange rate policy and capital controls in the process of international integration (see, for example, Song, Storesletten and Zilibotti, 2011; Jeanne, 2013; Song, Storesletten and Zilibotti, 2014; Farhi and Maggiori, 2019; Bahaj and Reis, 2020; Clayton, Dos Santos, Maggiori and Schreger, 2022). We contribute to this literature by showing that growth processes without capital inflows can result from the use of exchange rates and capital controls as industrial policies to redirect resources to strategic sectors.

Finally, our paper is also related to the literature that studies industrial policy. Notable contributions in the area of international trade include Redding (1999); Melitz (2005); Bartelme, Costinot, Donaldson and Rodriguez-Clare (2019) and Gaubert, Itskhoki and Vogler (2021). Harrison and Rodríguez-Clare (2010) provide a survey of this literature. Other applications have been studied in the context of network economies (Liu, 2019); economies with financial frictions (Itskhoki and Moll, 2019); and the financial sector (Farhi and Tirole, 2021). Most of this work advocates industrial policies that take the form of import tariffs, taxes or subsidies to sectoral production, and direct financial interventions. Our work com-

[1] Michaud and Rothert (2014) study how policies that impose borrowing constraints on households can correct learning-by-doing externalities. Benigno *et al.* (2022) develop a model with knowledge spillovers and firms' financing frictions to explain emerging economies with fast growth, current account surpluses, and reserve accumulation. A related literature studies the connection between international financial markets, technology spillovers, and growth. Recent contributions in this area include Alberola and Benigno (2017); Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2017); Queralto (2020); Ates and Saffie (2021).

plements this literature by focusing on exchange rate policies as a tool for industrial policy.

The rest of the paper is organized as follows. Section 2 presents the model economy and Section 3 characterizes the optimal exchange rate industrial policy. Section 4 analyzes model extensions and the use of capital controls as an industrial policy. Section 5 concludes.

2. Theoretical Framework

We consider a canonical small-open-economy (SOE) model with tradable and nontradable goods. There are three type of agents in the domestic economy: households, firms, and the government. We enrich this setting to include dynamic production externalities and segmented asset markets. The rest of the world trades tradable goods and an external asset with the domestic economy.

We study the optimal exchange rate policy when the economy experiences a growth process and externalities dissipate as the economy transitions to the technological frontier.

2.1. Environment

Households. The environment is deterministic and time is infinite, discrete, and denoted by $t = 0, 1, ...$ The representative household has preferences over an infinite stream of consumption C_t and labor L_t:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{{C_t}^{1-\sigma}}{1-\sigma} - \phi L_t \right]. \tag{1}$$

The consumption good is a composite aggregator of tradable C_{Tt} and nontradable C_{Nt} consumption,

$$C_t = \left[\omega^{\frac{1}{\eta}} (C_{Tt})^{1-\frac{1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} (C_{Nt})^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $\omega \in (0,1)$ is the weight on the tradable good and $\eta > 0$ is the elasticity of substitution between tradable and nontradable consumption. Households receive their income from labor and profits from domestic firms. They can save or borrow using a domestic currency bond. Their budget constraint expressed in domestic currency is given by

$$P_{Tt}C_{Tt} + P_{Nt}C_{Nt} + B_{t+1} = W_t L_t + \Pi_t + T_t + R_t B_t, \tag{3}$$

where P_{Tt}, P_{Nt} are the prices of tradables and nontradables; B_{t+1} are the bonds purchased in t that mature in $t+1$; R_t is the domestic currency interest rate; W_t is the nominal wage; Π_t are the profits from firms in the tradable and nontradable sectors; and T_t are transfers from the government.

The household's problem is to choose allocations $\{C_t, C_{Tt}, C_{Nt}, L_t, B_{t+1}\}_{t=0}^{\infty}$ that maximize utility (1), subject to the aggregation technology (2); the sequence of budget constraints (3), given a sequence of prices, profits and transfers; and an initial level of bonds B_0. The first-order conditions that characterize the solution to the household's problem are

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} = p_t \left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}, \tag{4}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_t}{P_{Tt}} = \phi, \tag{5}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R_{t+1} \frac{P_{Tt}}{P_{Tt+1}} \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{6}$$

where $p_t \equiv P_{Nt}/P_{Tt}$ is the relative price of nontradable goods. The first equation relates the marginal utility of consuming tradables and nontradables to its relative price. The second equation equates the marginal disutility of supplying labor to the product of the real wage in tradable goods and the marginal utility of consuming tradables. The last equation is the Euler equation, in which the relevant interest rate is the real interest rate of the bond in local currency.

Firms. There is a representative firm in each sector. The firm in sector $i = T, N$ employs labor l_{it} and produces goods according to the following decreasing-returns-to-scale production technology:

$$y_{it} = A_t L_{it}^{\gamma_{it}} l_{it}^{\alpha}. \tag{7}$$

The firm's productivity $Z_{it} \equiv A_t L_{it}^{\gamma_{it}}$ is the product of an exogenous and an endogenous component. The exogenous component A_t evolves according to

$$A_t = \rho\varphi\overline{A} + (1-\rho)A_{t-1}, \tag{8}$$

for $t \geq 1$, where $\overline{A}$ is the technological frontier; $\varphi \in (0,1)$ is the distance to the frontier in the steady-state; $A_0 \leq \varphi\overline{A}$ is the initial productivity; and $\rho \in (0,1)$ governs the speed of convergence. The endogenous component, $L_{it}^{\gamma_{it}}$, captures the Marshallian production externalities by which aggregate sectoral labor, L_{it}, increases the productivity of firms working in that sector. In equilibrium, $L_{it} = l_{it}$ given the representative firm assumption. These production externalities can arise due to learning-by-doing, knowledge spillovers, or labor pooling (Lucas, 1988; Krugman, 1992). Given our purposes, we remain agnostic about which are the fundamental reasons that give rise to the externalities. We assume that the externalities are sector-dependent and a function of the distance to the technological frontier, i.e., $\gamma_{it} = \Gamma_i(\overline{A}/A_t)$, and make the following assumption regarding the relative strength and dynamics of these sectoral externalities.

Assumption 1. *Suppose that* $(1-\alpha) > \Gamma_T(\overline{A}/A_t) \geq \Gamma_N(\overline{A}/A_t) = 0$, *and* Γ_T *is increasing in* $\overline{A}/A_t$.

The first condition assumes that externalities are only present in the tradable sector, and is common in the literature on industrial policy in open economies (see, e.g., Krugman, 1987). The logic for this assumption is that learning-by-doing and knowledge spillovers are more likely to be present in exporting sectors such as manufacturing and less so in the nontradable sectors of developing economies, which prior to growth take-off are more concentrated in local agricultural sectors. In Appendix B.1, we relax this assumption and characterize optimal exchange rate industrial policy when the economy features externalities in the nontradable sector that could be stronger or weaker than those in the tradable sector.

The second condition assumes that externalities are stronger the further the economy is from the technological frontier. This captures the idea that externalities are larger in the initial growth phase of a sector, when the role of learning and knowledge acquisition is more relevant. See Redding (1999), Melitz (2005) and Itskhoki and Moll (2019) for examples of papers that study industrial policies in economies with dynamic externalities that dissipate as sectors grow. Finally, note that we are not imposing any assumption on the level of externalities once the economy reaches its frontier. It could be that economies at the frontier feature a permanently positive externality.

Firms choose labor to maximize their profits, which are given by $\Pi_{it} = P_{it} A_t l_{it}^{\alpha} L_{it}^{\gamma_{it}} - W_t l_{it}$, which gives rise to the following aggregate labor demand:

$$\alpha A_t L_{it}^{\alpha+\gamma_{it}-1} = W_t / P_{it}. \tag{9}$$

Government. The government manages a portfolio of bonds in local and foreign currency and lump-sum transfers its proceedings to households. Its budget constraint is given by

$$F_{t+1} + \mathcal{E}_t F_{t+1}^* + T_t = R_t F_t + \mathcal{E}_t R^* F_t^*, \tag{10}$$

where F_{t+1} and F_{t+1}^* are the local and foreign currency bonds purchased in period t, respectively; R^* is the foreign currency interest rate; and $\mathcal{E}_t$ is the nominal exchange rate expressed as domestic currency per unit of foreign currency.

Rest of the world. The rest of the world exchanges tradable goods and foreign currency bonds with the government of the small open economy, and provides a perfectly elastic supply of funds at interest rate R^*. Financial markets are segmented and the rest of the world cannot trade domestic currency bonds. Finally, we assume that the law of one price holds for tradable goods and normalize the foreign currency price of tradables, so that

$P_{Tt} = \mathcal{E}_t$.

Competitive equilibrium. We can now define a competitive equilibrium for given government policies.

***Definition* 1** (Competitive equilibrium)**.** *Given initial asset positions F_0, F_0^*, a competitive equilibrium is a sequence of private allocations $\{C_t, C_{Tt}, C_{Nt}, L_t, B_{t+1}, L_{Tt}, L_{Nt}\}_{t=0}^{\infty}$, prices $\{P_{Tt}, P_{Nt}, W_t, \mathcal{E}_t, R_t\}_{t=0}^{\infty}$, and government policies $\left\{F_{t+1}, F_{t+1}^*, T_t\right\}_{t=0}^{\infty}$ such that:*

1. *Allocations solve the households' and firms' problem, given prices;*
2. *Government policies satisfy the government budget constraint;*
3. *Markets clear:*

$$L_t = L_{Tt} + L_{Nt}, \tag{11}$$

$$C_{Nt} = A_t L_{Nt}^{\alpha}, \tag{12}$$

$$F_{t+1} + B_{t+1} = 0. \tag{13}$$

Equations (11), (12), and (13) are the market-clearing conditions for labor, nontradable goods, and the local currency bond. Due to financial market segmentation, households and the government need to take opposing asset positions in local currency.

We now derive the equations that characterize the competitive equilibrium allocations. These will serve as implementability conditions for the optimal policy problem. Combining (4), (5), (9), and (11), we obtain

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}, \tag{14}$$

$$\frac{\phi\left(L_{Tt}+L_{Nt}\right)^{\nu}}{\left(\omega/C_{Tt}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}. \tag{15}$$

The first equation equates the marginal rate of substitution between tradable and nontradable goods to their private marginal rate of transformation. The second equation equates the marginal rate of substitution between tradables and labor with the private marginal product of labor. Finally, competitive equilibrium allocations are also characterized by the market-clearing condition for nontradables (12), and the balance of payments condition (or tradable goods market clearing),

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*, \tag{16}$$

which states that net imports should be financed with external debt. Note that the household's Euler equation is not an implementability condition and is used to pin down the local currency interest rate R_t.

3. Exchange Rate Industrial Policy

This section characterizes the optimal exchange rate industrial policy (XR-IP). We begin by characterizing the first-best allocation, which serves as a useful benchmark.

3.1. First-best allocation

***Definition* 2** (First best)**.** *A first-best allocation is the allocation* $\tilde{\mathbf{x}}_t \equiv \left\{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_{Tt}, \tilde{L}_{Nt}, A_t\right\}$ *that maximizes utility* (1)*, subject to the consumption aggregator definition* (2)*, the balance of payments condition* (16)*, and market-clearing conditions for labor* (11) *and nontradable goods* (12)*.*

The first-order conditions that characterize the first-best allocation are

$$\left(\frac{1-\omega}{\omega}\frac{\tilde{C}_{Tt}}{\tilde{C}_{Nt}}\right)^{\frac{1}{\eta}} = \frac{(\alpha+\gamma_{Tt})}{\alpha}\frac{\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}}{\tilde{L}_{Nt}^{\alpha-1}}, \tag{17}$$

$$\frac{\phi}{\left(\omega/\tilde{C}_{Tt}\right)^{\frac{1}{\eta}}\tilde{C}_t^{\frac{1}{\eta}-\sigma}} = (\alpha+\gamma_{Tt})A_t\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{18}$$

$$\left(\frac{\omega}{\tilde{C}_{Tt}}\right)^{\frac{1}{\eta}}\tilde{C}_t^{\frac{1}{\eta}-\sigma} = \beta R^*\left(\frac{\omega}{\tilde{C}_{Tt+1}}\right)^{\frac{1}{\eta}}\tilde{C}_{t+1}^{\frac{1}{\eta}-\sigma}. \tag{19}$$

The first equation equates the marginal rate of substitution between tradable and nontradable goods to their social marginal rate of transformation. The second equation equates the marginal rate of substitution between tradables and labor to the social marginal product of labor. The last equation is the Euler equation that equates the intertemporal marginal rate of substitution to the foreign currency interest rate.

The social marginal rate of transformation and the social marginal product of labor are higher than their private counterparts due to production externalities in the tradable sector. These differences introduce wedges in the intratemporal allocation of labor and consumption in the competitive equilibrium, relative to the first-best allocation, that cannot be undone with foreign exchange (FX) intervention. The following proposition formalizes this result.

***Proposition* 1** (Impossibility result)**.** *The first-best allocation is not attainable with FX intervention.*

We include all proofs in Appendix A. FX intervention affects the intertemporal margin of consumption by affecting the path of the exchange rate and the rate of return of domestic savings. This policy cannot attain the first-best allocation because the wedges introduced by the production externality affect the intratemporal allocation of consumption and labor. On the other hand, as the next proposition states, fiscal policy can attain the first-best allocation through time- and sector-specific labor subsidies.

***Proposition* 2.** *The first-best allocation is attainable with FX intervention and the following time-varying labor subsidies to the tradable sector:*

$$\tau_{Tt}^{L} = \frac{\gamma_{Tt}}{\alpha + \gamma_{Tt}}.$$

This is a familiar result from the macro-public finance literature. Labor subsidies undo the wedges between the social and private marginal rates of transformation, and the FX intervention is such that the returns on saving in local and foreign currency are equal. While this is the most desirable policy from a social perspective, it may be difficult to implement from a political economy perspective.

3.2. Optimal exchange rate industrial policy

We now study the optimal exchange rate policy as a second-best policy. The optimal exchange rate policy consists of a government policy that maximizes the lifetime utility of households subject to the implementability conditions that characterize a competitive equilibrium. We formally define this problem below.

***Definition* 3.** *An optimal exchange rate industrial policy is a government policy that solves the following problem:*

$$\max_{\{C_{it},L_{it},F^*_{t+1}\}_{t\geq 0}^{i=T,N}} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \phi L_t\right] \quad \textit{subject to} \tag{P1}$$

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}},$$

$$\frac{\phi}{(\omega/C_{Tt})^{\frac{1}{\eta}}\, C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1},$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*,$$

the consumption aggregator definition (2)*, and the market-clearing conditions for nontradable goods* (12)*.*

This problem is characterized by the following modified Euler equation

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R^* \frac{\theta(\mathbf{x}_{t+1},\gamma_{Tt+1})}{\theta(\mathbf{x}_t,\gamma_{Tt})} \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{20}$$

where $\theta(\mathbf{x}_t,\gamma_{Tt})$ is a function that depends on the allocations of the economy, $\mathbf{x}_t \equiv \{C_{Tt}, C_{Nt}, L_{Tt}, L_{Nt}, A_t\}$, and the strength of the externality at a given time period t. We provide an expression for this function in Appendix A.3.

We contrast the allocations under the optimal exchange rate industrial policy with a benchmark allocation that corresponds to a competitive equilibrium in which the government is a "passive" agent, in the sense that it intermediates capital flows as if households have direct access to saving and borrowing at the foreign currency interest rate, but does take into account the effect of its foreign exchange rate interventions on production externalities. We formalize this benchmark notion as follows:

***Definition* 4.** *A laissez-faire competitive equilibrium is a competitive equilibrium with an associated government policy in which UIP holds, i.e.,* $R_{t+1} = R^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$.

Quadratic-linear approximation to policy problem. To provide a tractable analytical characterization of the optimal policy, we first make the following parametric assumptions:

ASSUMPTION 2. *Suppose* $\sigma = \eta = 1$ *and* $\beta R^* = 1$.

The first condition corresponds to the Cole and Obstfeld (1991) preference parameterization. Under this parameterization the equilibrium non-tradable allocations are independent of inter-temporal considerations and given by

$$C_{Nt} = A_t \left[\frac{\alpha(1-\omega)}{\phi}\right]^{\alpha}, \tag{21}$$

$$L_{Nt} = \frac{\alpha(1-\omega)}{\phi}. \tag{22}$$

Furthermore, in Appendix A.4, we show that the optimal policy problem can be approximated by a quadratic-linear problem in terms of log-deviations from the first best allocation.[2] In particular, we can approximate to a second order the welfare loss from the first best allocation as

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + \omega(\alpha+\gamma)\right] x_t^2\right], \tag{23}$$

where $z_t = \log(C_{Tt}) - \log(\tilde{C}_{Tt})$ and $x_t = \log(L_{Tt}) - \log(\tilde{L}_{Tt})$ are the log deviations from the first-best tradable consumption and labor, respectively; and γ is weighted function of the path of γ_{Tt}, with the expression provided in Appendix A.4. Furthermore, we can also

[2] See Itskhoki and Mukhin (2023) for a description of this approximation approach to a general class of policy problems.

approximate to a first order the implementability conditions:

$$z_t = \psi_t + (\alpha + \gamma - 1)x_t, \tag{24}$$

$$\sum_{t=0}^{\infty} \beta^t \left(z_t - (\alpha + \gamma)x_t\right) = 0, \tag{25}$$

where $\psi_t \equiv \log \alpha - \log(\alpha + \gamma_{Tt}) \leq 0$. The first equation equates the marginal rate of substitution between tradables and labor with the private marginal product of labor. The second equation corresponds to the intertemporal budget constraint of the economy. With this simplified problem, we can characterize the optimal policy.

***Lemma* 1.** *The allocations that solve the policy problem* (P1) *can be approximated with those that maximize* (23) *subject to* (24) *and* (25).

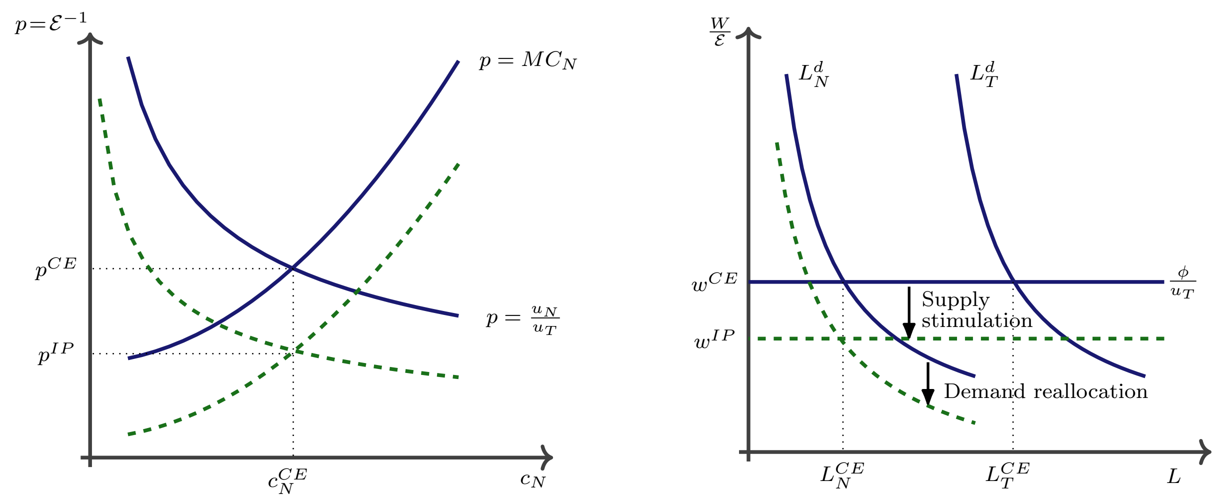
Henceforth, all the results refer to the solution to the apppoximate quadratic-linear problem. This problem highlights the trade-offs faced by the planner. Ideally, the planner would like to set tradable consumption and labor to their first-best levels every period. However, private choices determine a static relationship between them. Therefore, the planner can only choose the path of net savings in the economy that determines a path for labor and consumption given private choices.

Converging economies. We begin by characterizing the optimal policy in economies experiencing transitional dynamics, converging to a steady-state productivity closer to the technological frontier. These economies exhibit a path of production externalities in the tradable sector that are stronger in the early phase of the growth process.

Prior to characterizing the optimal policy it is useful to explain the macroeconomic effects of increasing aggregate savings in the initial period (see Figure 2). Higher savings reduce current tradable consumption, which affects current allocations through two channels. First, lower tradable consumption induces a stimulation of aggregate labor supply. Second, a lower tradable consumption generates a reallocation of labor demand. The depressed aggregate demand reduces the demand for tradable and nontradable goods, depreciates the exchange

Figure 2: The Macroeconomic Effects of Exchange Rate Industrial Policy in Initial Periods

(a) Non-tradable Goods Market **(b)** Labor Market



*N*otes: This figure shows the allocations of the laissez-faire competitive equilibrium (blue line) and the optimal exchange rate industrial policy (green dotted line) in the nontradable goods market and the labor market in $t = 0$.

rate, and lowers labor demand from the nontradable sector. For the parameterization under Assumption 2, the positive supply stimulation and negative demand reallocation effects for the nontradable labor market cancel out, which implies the same level of nontradable production as in the laissez-faire equilibrium. In the tradable sector, both effects contribute to higher labor.

In the case of converging economies, the optimal policy features high saving rates in the initial periods, when the tradable production externalities are stronger, to induce more tradable labor. The following proposition characterizes the optimal policy:

***Proposition* 3** (XR-IP in converging economies)**.** *Suppose that the economy starts below its steady-state level of productivity (i.e., $A_0 < \varphi\overline{A}$), which implies a decreasing path of externalities in tradable production (i.e., γ_{Tt} decreasing in t). The optimal exchange rate industrial policy in these economies implies $\exists\ \bar{t} > 0$ such that:*

$$\mathcal{E}_t^{IP} > \mathcal{E}_t^{CE}, \quad L_{Tt}^{IP} > L_{Tt}^{CE}, \quad C_{Tt}^{IP} < C_{Tt}^{CE} \qquad\qquad \textit{if } t < \bar{t},$$

with opposite inequalities if $t > \bar{t}$. This is achieved with trade balance surpluses, $TB_t^{IP} > TB_t^{CE}$ if $t < \bar{t}$, and $F_{t+1}^{*IP} > F_{t+1}^{*CE}$ for all t. Furthermore, $\tilde{L}_{Tt} > L_{Tt}^{IP}$ for all t.

Figure 3 shows the dynamic path of variables in the optimal policy. The economy features lower tradable consumption, higher tradable labor and a depreciated exchange rate relative to the laissez-faire competitive equilibrium exchange rate in the initial periods when the externality is stronger. The depreciated exchange rate is attained with currency market interventions and the accumulation of international reserves. By generating a current account surplus and accumulating international reserves, the economy generates a net creditor position that implies a current account deficit, larger tradable consumption, lower tradable production, and a more appreciated exchange rate in future periods when the production externality dissipates.

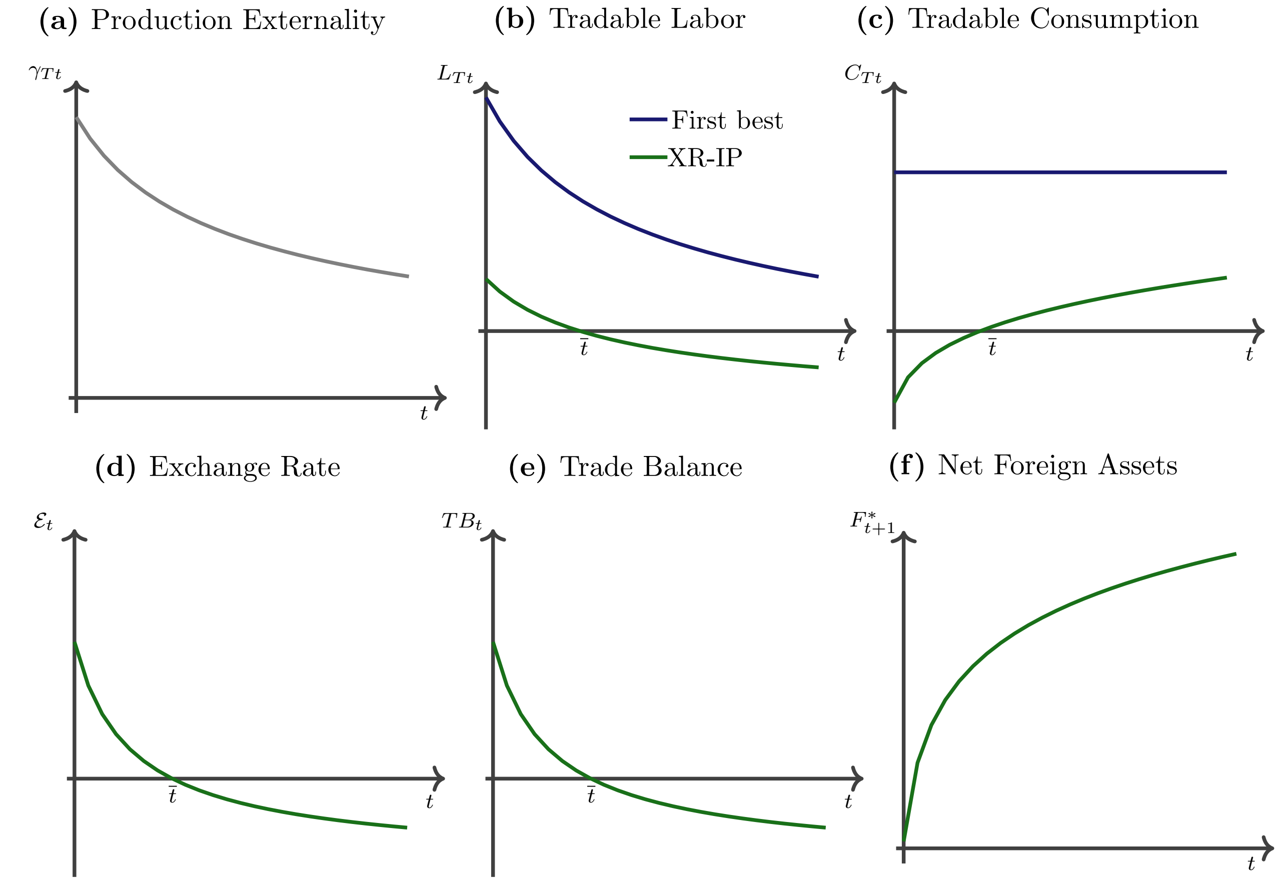
Figure 3 also shows that tradable labor in the optimal policy is always below its first-best level. This is because attaining the same labor allocation as in the first-best has associated a large distortion on the inter-temporal consumption margin that makes this allocation suboptimal.

Economies not in transition. We now characterize the optimal exchange rate industrial policy in economies that are not converging to the technological frontier or that are at the technological frontier. In this case, given that externalities do not exhibit a dynamic pattern, there is no role for this type of policy because any exchange rate depreciation the government induces in early periods is associated with an appreciation in later periods when the trade account balance is reversed. The following proposition formalizes this result.

***Proposition* 4** (XR-IP in economies not in transition)**.** *Consider an economy that is not converging to the technological frontier or that is at the technological frontier (i.e., $A_0 = \varphi \overline{A}$). Allocations from the optimal exchange rate industrial policy coincide with the laissez-faire competitive equilibrium.*

Note that this result holds even if economies feature permanent production externalities. When these externalities are constant, FX intervention is not the right tool to address them.

Figure 3: Exchange Rate Industrial Policy Dynamics



*N*otes: This figure shows the dynamics of the allocations of the first best (blue line) and the optimal exchange rate industrial policy (green line) in deviations from those of the laissez-faire competitive equilibrium.

This result also indicates that, in our framework, a necessary ingredient for exchange-rate industrial policies that depart from the laissez-faire competitive equilibrium to be welfare enhancing is that the economy is converging to the technological frontier so that externalities exhibit a dynamic pattern.

4. On the Efficiency of Exchange Rate Industrial Policy

In this section we study what determines the effectiveness of the use of exchange rate as an industrial policy and relate the theoretical insights to different historical experiences. We focus on the role of international capital mobility and labor market characteristics.

4.1. International capital mobility

Consider an extension of the baseline model in which households can trade in international capital markets that operate imperfectly as in Gabaix and Maggiori (2015). Suppose that there is a unit measure of foreign financial intermediaries that engages in carry trade by buying and selling bonds in different currencies. Their aggregate balance sheet is $Q^*_{t+1} = -Q_{t+1}/\mathcal{E}_t$, where Q^*_{t+1} and Q_{t+1} are bonds purchased in period t in foreign and local currency, respectively. Their demand for local currency assets is given by

$$Q_{t+1} = \frac{1}{\Gamma}\left[\mathcal{E}_t - \frac{R^*}{R_{t+1}}\mathcal{E}_{t+1}\right], \tag{26}$$

where $\Gamma \geq 0$ is a measure of intermediaries risk bearing capacity.[3] When $\Gamma = 0$ there is free capital mobility and the equilibrium features the interest rate parity condition. When $\Gamma \to \infty$ no intermediation is possible and this model collapses to the baseline model. The market clearing condition for domestic currency bonds is $F_{t+1} + B_{t+1} + Q_{t+1} = 0$, and the balance of payments condition is

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F^*_t - F^*_{t+1} + \frac{Q_{t+1}}{\mathcal{E}_t} - R_t \frac{Q_t}{\mathcal{E}_t}.$$

In this setup we show the following result.

***Proposition* 5** (XR-IP with international capital mobility)**.** *Consider the economy with international intermediaries in the initial period. Suppose that the economy starts below its steady-state level of productivity and converges to it in the next period (i.e., $A_0 < \varphi\overline{A}$ and $\rho = 1$). The optimal exchange rate industrial policy ("IP-B") implies*

$$\mathcal{E}_0^{IP} > \mathcal{E}_0^{IP-B} > \mathcal{E}_0^{CE}, \quad L_{T0}^{IP} > L_{T0}^{IP-B} > L_{T0}^{CE}, \quad C_{T0}^{IP} < C_{T0}^{IP-B} < C_{T0}^{CE}, \quad CA_0^{IP} > CA_0^{IP-B} > CA_0^{CE}.$$

In an economy with intermediaries the social planner faces an additional cost of distorting inter-temporal consumption choices. Doing so opens a wedge between the returns

[3] This demand arises from an optimization problem of intermediaries that maximize next period's profits subject to an incentive compatibility constraint. See Gabaix and Maggiori (2015) for further details.

in local and foreign currency, which intermediaries exploit with carry trades and extract positive profits from the economy. The optimal response of the planner to this environment is to reduce the strength of the exchange rate interventions.

In the extreme of a completely open capital account, foreign exchange interventions become ineffective as households undo them by trading with the rest of the world in a frictionless way. This gives rise to a motive for why governments may want to regulate the capital account, which we explore next.

Capital Controls as an Industrial Policy. Consider a simplified version of the model in which there are only foreign currency bonds that households trade with the rest of the world. Suppose further that the government has access to a capital control policy in the form of a time-specific tax on households' savings/borrowing, τ_t^B. The following result shows an equivalence between the allocations attained with the optimal XR-IP policy in the baseline model and those in this economy with optimal capital controls.

***Proposition* 6** (XR-IP with capital controls)**.** *Consider a model variant in which households can save or borrow in foreign currency and the government can impose a capital control. The allocations of the optimal exchange rate policy can be attained by imposing the following time-varying capital control:*

$$\tau_t = \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} - 1.$$

This equivalence result emerges because in both economies the government can control the inter-temporal allocation of consumption by means of FX intervention in the baseline model, and capital controls in this model. Therefore, the allocations under both optimal policies coincide. This result echoes those of Farhi, Gopinath and Itskhoki (2014), who show that exchange rate devaluations can be replicated with a combination of fiscal tools.

4.2. Labor market supply

Consider now a variant of the baseline model with elastic sector-specific labor supply, where preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \phi_T \frac{L_{Tt}^{1+\nu}}{1+\nu} - \phi_N \frac{L_{Nt}^{1+\nu}}{1+\nu} \right],$$

where $\phi_i > 0$ for $i = T, N$, and ν^{-1} is the Frisch elasticity of sector-specific labor supply. In this setup we show the following result.[4]

Proposition 7 (Labor supply elasticity)**.** *Consider a version of the model with elastic labor supply in the tradable and nontradable sectors. The optimal exchange rate industrial policy implies the following initial allocations for the optimal policy*

$$\frac{\mathcal{E}_0^{IP}}{\mathcal{E}_0^{CE}} > 1, \quad \frac{L_{T0}^{IP}}{L_{T0}^{CE}} > 1, \quad \frac{C_{T0}^{CE}}{C_{T0}^{IP}} > 1, \quad \frac{CA_0^{IP}}{CA_0^{CE}} > 1, \quad \frac{F_1^{*IP}}{F_1^{*CE}} > 1,$$

and all these ratios are increasing in the elasticity of labor supply, ν^{-1}.

This proposition states two results. First, that the same type of policy carries through to this more general setup. Second, the strength of the optimal intervention and its effects on exchange rate depreciation in the initial periods are stronger the more elastic the labor supply is. When labor supply is very elastic, the government can more effectively induce larger labor in the tradable sector, and exploit the production externality in the initial periods of convergence, when it is stronger.

4.3. Multiple sectors

In practice the tradable sector is composed of multiple sectors that may have different externalities. In this section we consider a variant of the model two tradable sectors, $j = 1, 2$.

[4]See Berger, Herkenhoff and Mongey (2022) for an example of similar preferences in the context of firm-specific labor supply. In Appendix B.2 we also study the case of fixed aggregate labor supply, and show that the optimal exchange rate industrial policy shares the same characteristics as in the baseline model. In this model, the optimal policy does not feature a supply stimulation channel, and only affects allocation through a reallocation of sectoral labor demand.

Sectors differ in the externality patterns, γ_{Tjt}, and in their labor supply. Preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \phi_{T1} \frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi_{T2} \frac{L_{T2t}^{1+\nu_2}}{1+\nu_2} - \phi_N \frac{L_{Nt}^{1+\nu_N}}{1+\nu_N} \right], \tag{27}$$

where tradable consumption is an aggregator of tradable sector varieties $j = 1, 2$

$$C_{Tt} = C_{T1t}^{1/2} C_{T2t}^{1/2}. \tag{28}$$

In this environment, we show the following sufficient statistic result. Define $\epsilon_t = \log(\mathcal{E}_t) - \log(\tilde{\mathcal{E}}_t)$ as the log deviation from the first-best exchange rate, and $\psi_{jt} = \log \alpha - \log(\alpha + \gamma_{Tjt}) \leq 0$.

***Proposition* 8** (Multiple sectors)**.** *In both the single-tradable-sector model and the multiple-tradable-sector model the optimal exchange rate industrial policy follows the same law of motion:*

$$(1 + D)\epsilon_t + D\psi_t = (1 + D)\epsilon_{t+1} - D\psi_{t+1},$$

where $D > 0$ and ψ_{t+1} are model-specific. In the single-tradable-sector model $D = \frac{(\alpha+\gamma)^2+(1+\nu)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)^2}$ and $\psi_t = \log \alpha - \log(\alpha + \gamma_{Tt})$. In the multiple-tradable-sector model $D = \left[\frac{1}{2}D_1 + \frac{1}{2}D_2\right]$, and $\psi_t = \frac{D_1}{D_1+D_2}\psi_{1t} + \frac{D_2}{D_1+D_2}\psi_{2t}$, where the sector weights are $D_j = \frac{(\alpha+\gamma)^2+(1+\nu_j)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu_j)^2}$ for $j = 1, 2$.

This proposition shows that the exchange rate policy in the model with multiple tradable sectors behaves similarly to the baseline model. It approximately follows the path of a weighted average of the production externalities of both tradable sectors. In addition, the optimal policy places a greater weight on sectors which have more elastic labor supply, as stronger externalities in these sectors can be exploited more by the policy.

4.4. A discussion of historical experiences

Our framework can be used to interpret historical experiences on the use of exchange rate market intervention to foster economic growth. The emblematic cases of these policies are the Asian growth miracles and more recently the Chinese growth process (see, e.g., Page, 1994; Song *et al.*, 2014). Through the lens of our model these economies arguably met the necessary condition for the desirability of these policies which is to be experiencing a convergence process. In addition, they also featured two characteristics that in our model make these policies more effective and desirable. First, the policies were conducted in environments with capital account interventions and initially underdeveloped financial markets, which made FX interventions more effective in affecting exchange rates. Second, the economies featured easiness in the reallocation of labor across sectors. In the salient case of China, there was a significant labor migration from the local rural sector to the urban manufacturing sector (see, Cai, 2016, for a discussion of the demographic dividend in China).

The salient characteristics of the Asian examples appear in contrast with those from Latin American experiences, which are often referenced as failures of these types of policies. Most Latin American economies did not experience convergence processes, featured larger costs to sectoral reallocation of labor and a capital account that was relatively more open. Through the lens of our model, this configuration makes exchange rate industrial policy less effective and desirable.

5. Conclusion

In this paper, we studied how exchange rate policies can be used to speed development. These types of policies arise in economies characterized by production externalities and that are converging toward the technological frontier. In the early stages of the transition, governments can optimally intervene in exchange rate markets and keep currencies undervalued, thereby increasing labor supply and redirecting resources to the tradable sector. Our paper also highlights the limits of exchange rate industrial policies. Since these policies imply persistently influenced real exchange rates, they cannot plausibly be implemented through monetary policy. They are also less effective in economies that are highly integrated into capital markets or feature large heterogeneity of production externalities among tradable sectors.

Although our analysis has concentrated on policies from the perspective of individual economies, our framework can be extended to study interactions in the global economy. An interesting application in this regard is the idea of "currency wars," which was introduced during China's take-off. Our framework could be used to study the extent to which these global dynamics can arise as a result of multiple economies trying to exploit the dynamic patterns of production externalities and their implications for geoeconomics, as highlighted in Clayton, Maggiori and Schreger (2023). We leave the study of these global interactions for future research.

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A. Theoretical Appendix

A.1. Proof of Proposition 1

Given the initial foreign currency asset position F_0^*, the conditions that characterize the competitive equilibrium allocation $\{C_{Tt}, C_{Nt}, L_{Tt}, L_{Nt}, F_{t+1}^*\}_{t=0}^{\infty}$ are

$$\left(\frac{(1-\omega)}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}, \tag{A.1}$$

$$\frac{\phi\left(L_{Tt}+L_{Nt}\right)^{\nu}}{\left(\omega/C_{Tt}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.2}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R_{t+1} \frac{L_{Nt}^{\alpha-1}/L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt+1}^{\alpha-1}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}} \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.3}$$

$$C_{Nt} = A_t L_{Nt}^{\alpha}, \tag{A.4}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*, \tag{A.5}$$

where in equation (A.3) we substitute $P_{Tt} = \frac{L_{Nt}^{\alpha-1}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}}$ into (4) after normalizing $P_{Nt} \equiv 1$ without loss of generality, and combine firms' labor demand from (9).

The conditions that characterize the first-best allocation $\{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_{Tt}, \tilde{L}_{Nt}, \tilde{F}_{t+1}^*\}_{t=0}^{\infty}$ are

$$\left(\frac{(1-\omega)}{\omega}\frac{\tilde{C}_{Tt}}{\tilde{C}_{Nt}}\right)^{\frac{1}{\eta}} = \left(\frac{\alpha+\gamma_{Tt}}{\alpha}\right)\frac{\tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}}{\tilde{L}_{Nt}^{\alpha-1}}, \tag{A.6}$$

$$\frac{\phi\left(\tilde{L}_{Tt}+\tilde{L}_{Nt}\right)^{\nu}}{\left(\omega/\tilde{C}_{Tt}\right)^{\frac{1}{\eta}} \tilde{C}_t^{\frac{1}{\eta}-\sigma}} = (\alpha+\gamma_{Tt}) A_t \tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.7}$$

$$\left(\frac{\omega}{\tilde{C}_{Tt}}\right)^{\frac{1}{\eta}} \tilde{C}_t^{\frac{1}{\eta}-\sigma} = \beta R^* \left(\frac{\omega}{\tilde{C}_{Tt+1}}\right)^{\frac{1}{\eta}} \tilde{C}_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.8}$$

$$\tilde{C}_{Nt} = A_t \tilde{L}_{Nt}^{\alpha}, \tag{A.9}$$

$$\tilde{C}_{Tt} - A_t \tilde{L}_{Tt}^{\alpha+\gamma_{Tt}} = R^* \tilde{F}_t^* - \tilde{F}_{t+1}^*. \tag{A.10}$$

Observe that satisfying the first-best intertemporal optimality condition (A.8) in the competitive equilibrium (A.3) requires government foreign exchange intervention $\{F^*_{t+1}\}_{t=0}^{\infty}$ such that $R_{t+1} = R^* \frac{L_{Nt+1}^{\alpha-1}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}}{L_{Nt}^{\alpha-1}/L_{Tt}^{\alpha+\gamma_{Tt}-1}}$ for all t. Further, equations (A.6)–(A.7) for the first best and (A.1)–(A.2) in the competitive equilibrium only concide if $\gamma_{Tt} = 0$ for all t. Therefore, in the presence of production externalities, $\gamma_{Tt} > 0$ for some t; then the first-best allocation cannot be achieved in the competitive equilibrium for any $\{F^*_t\}_{t=0}^{\infty}$.

A.2. Proof of Proposition 2

With tradable and nontradable sector-specific labor subsidies τ^L_{it}, the firm problem for each sector $i \in \{T, N\}$ is

$$\max_{l_{it}} \quad \pi_{it} = P_{it} A_t l_{it}^{\alpha} L_{it}^{\gamma_{it}} - (1 - \tau^L_{it}) W_t l_{it}. \tag{A.11}$$

The firm profit maximization conditions for labor demand in each sector l_T and l_N give

$$\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} = (1 - \tau^L_{Tt}) \frac{W_t}{P_{Tt}}, \tag{A.12}$$

$$\alpha A_t L_{Nt}^{\alpha-1} = (1 - \tau^L_{Nt}) \frac{W_t}{P_{Nt}}. \tag{A.13}$$

The government budget constraint is

$$F_{t+1} + \mathcal{E}_t F^*_{t+1} + T_t + \tau^L_{Tt} W_t L_{Tt} + \tau^L_{Nt} W_t L_{Nt} = R_t F_t + \mathcal{E}_t R^* F^*_t, \tag{A.14}$$

which, combined with the household budget constraint and firms' profits, gives the balance of payments condition (16).

Given τ^L_{Tt}, τ^L_{Nt}, the conditions that characterize the competitive equilibrium are

$$\left(\frac{(1-\omega)}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{(1-\tau^L_{Nt})}{(1-\tau^L_{Tt})}\frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}, \tag{A.15}$$

$$\frac{\phi\left(L_{Tt}+L_{Nt}\right)^{\nu}}{\left(\omega/C_{Tt}\right)^{\frac{1}{\eta}}C_t^{\frac{1}{\eta}-\sigma}} = \frac{1}{(1-\tau^L_{Tt})}\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.16}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R_{t+1}\frac{L_{Nt}^{\alpha-1}/L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt+1}^{\alpha-1}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.17}$$

$$C_{Nt} = A_t L_{Nt}^{\alpha}, \tag{A.18}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*. \tag{A.19}$$

Setting

$$\tau^L_{Tt} = \frac{\gamma_{Tt}}{\alpha+\gamma_{Tt}}, \tag{A.20}$$

$$\tau^L_{Nt} = 0, \tag{A.21}$$

gives identical conditions to the first best (A.6)–(A.7) for the competitive equilibrium. Government policies $\left\{F_{t+1}, F^*_{t+1}, T_t\right\}_{t=0}^{\infty}$ can then be used to equate $R_{t+1} = R^*\frac{L_{Nt}^{\alpha-1}/L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt+1}^{\alpha-1}/L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}}$ for all t so that (A.17) in the competitive equilibrium is equivalent to the first-best condition (A.8).

A.3. Optimal Exchange Rate Industrial Policy

In this section we solve the optimal exchange rate industrial policy problem (P1). After substituting the nontradable goods market-clearing condition $C_{Nt} = A_t L_{Nt}^{\alpha}$, the Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{1}{(1+\nu)} \left(L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}} \right)^{1+\nu}$$
$$+ \zeta_t \left[\frac{(C_{Nt}/A_t)^{\frac{\alpha-1}{\alpha}}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} - \left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} \right]$$
$$+ \xi_t \left[\phi \frac{\left(L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}} \right)^{\nu}}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} - \left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \right]$$
$$+ \lambda_t \left[R^* F_t^* - F_{t+1}^* + A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt} \right] \Bigg\}. \quad \text{(A.22)}$$

The first-order conditions for $C_{Tt}, C_{Nt}, L_{Tt}, F_{t+1}^*$ are

$$\left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} + \zeta_t \frac{1}{\eta} \left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} \frac{1}{C_{Tt}} + \xi_t \left[\frac{1}{\eta} \left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} \frac{1}{C_{Tt}} C_t^{\frac{1}{\eta}-\sigma} - \left(\frac{1}{\eta} - \sigma \right) \left(\frac{\omega}{C_{Tt}} \right)^{\frac{2}{\eta}} C_t^{\frac{2}{\eta}-\sigma-1} \right] = \lambda_t, \quad \text{(A.23)}$$

$$\left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} - \phi (L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}})^{\nu} \frac{1}{\alpha} C_{Nt}^{\frac{1}{\alpha}-1} (1/A_t)^{\frac{1}{\alpha}}$$
$$+ \zeta_t \left[- \frac{(C_{Nt}/A_t)^{\frac{\alpha-1}{\alpha}}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}} \frac{1}{\eta} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} \frac{1}{C_{Nt}} + \frac{(\alpha-1)}{\alpha} \frac{C_{Nt}^{\frac{-1}{\alpha}} (1/A_t)^{\frac{\alpha-1}{\alpha}}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} \right]$$
$$+ \xi_t \left[\phi \nu \frac{(L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}})^{\nu-1}}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} \frac{1}{\alpha} C_{Nt}^{\frac{1}{\alpha}-1} (1/A_t)^{\frac{1}{\alpha}} - \left(\frac{1}{\eta} - \sigma \right) \left(\frac{\omega}{C_{Tt}} \right)^{\frac{1}{\eta}} C_t^{\frac{2}{\eta}-\sigma-1} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} \right] = 0, \quad \text{(A.24)}$$

$$- \phi (L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}})^{\nu} + \zeta_t \left[-(\alpha + \gamma_{Tt} - 1) \frac{(C_{Nt}/A_t)^{\frac{\alpha-1}{\alpha}}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}} \left(\frac{1-\omega}{C_{Nt}} \right)^{\frac{1}{\eta}} \frac{1}{L_{Tt}} \right]$$
$$+ \xi_t \left[\phi \nu \frac{(L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}})^{\nu-1}}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} - (\alpha + \gamma_{Tt} - 1) \phi \frac{(L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}})^{\nu}}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} \frac{1}{L_{Tt}} \right] = -\lambda_t (\alpha + \gamma_t) A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \quad \text{(A.25)}$$

$$\lambda_t = \beta R^* \lambda_{t+1}. \quad \text{(A.26)}$$

Combining these expressions gives

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \frac{\theta_{t+1}(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta_t(\mathbf{x}_t, \gamma_{Tt})} \beta R^* \left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.27}$$

where

$$\begin{aligned}
\theta_t(\mathbf{x}_t, \gamma_{Tt}) \equiv 1 + \frac{M_t}{N_t} \Bigg\{ & \frac{1}{\eta}\frac{1}{C_{Tt}} - \left(\frac{1}{\eta} - \sigma\right)\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-1} + \left(\frac{\alpha}{\alpha+\eta-\alpha\eta}\right)\frac{(C_{Nt}/A_t)^{\frac{1}{\alpha}}}{C_{Tt}} \\
& \left[\nu \frac{1}{L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}}}\frac{1}{\alpha} - \left(\frac{1}{\eta} - \sigma\right)\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-1} L_{Tt}^{\alpha+\gamma_{Tt}-1} A_t\right]\Bigg\}, \qquad \text{(A.28)} \\
M_t \equiv \gamma_{Tt} A_t & \left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} \left(\frac{\omega}{C_{Tt}}\right)^{-\frac{1}{\eta}} (C_{Nt}/A_t)^{\frac{\alpha-1}{\alpha}}, \qquad \text{(A.29)} \\
N_t \equiv & -(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}\eta C_{Tt}\left[\left(\frac{1}{\eta}-\sigma\right)\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-1}\right] + \nu \frac{1}{L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}}} \\
& + Q_t \Bigg\{ \frac{1}{\eta}\frac{1}{C_{Tt}} - \left(\frac{1}{\eta} - \sigma\right)\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-1} + \left(\frac{\alpha}{\alpha+\eta-\alpha\eta}\right)\frac{(C_{Nt}/A_t)^{\frac{1}{\alpha}}}{C_{Tt}} \\
& \left[\nu \frac{1}{L_{Tt} + (C_{Nt}/A_t)^{\frac{1}{\alpha}}}\frac{1}{\alpha} - \left(\frac{1}{\eta} - \sigma\right)\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-1} L_{Tt}^{\alpha+\gamma_{Tt}-1} A_t\right]\Bigg\}, \qquad \text{(A.30)} \\
Q_t \equiv (\alpha + \gamma_{Tt}) A_t & \left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} \left(\frac{\omega}{C_{Tt}}\right)^{-\frac{1}{\eta}} (C_{Nt}/A_t)^{\frac{\alpha-1}{\alpha}} - (\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}\eta C_{Tt}. \qquad \text{(A.31)}
\end{aligned}$$

A.4. Proof of Lemma 1

We first show that under Assumption 2, the optimal exchange rate industrial policy (XR-IP) problem is independent of the nontradables block $\{C_{Nt}, L_{Nt}\}_{t=0}^{\infty}$. To see this for the more general case that allows for externalities in both the tradable and nontradable goods sectors we have the two constraints

$$\left(\frac{\omega}{C_{Tt}}\right) = \phi \frac{1}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}}, \tag{A.32}$$

$$\left(\frac{\omega}{C_{Tt}}\right) = \frac{\left(\frac{C_{Nt}}{A_t}\right)^{\frac{\alpha+\gamma_{Nt}-1}{\alpha+\gamma_{Nt}}}}{L_{Tt}^{\alpha+\gamma_{Tt}-1}} \left(\frac{1-\omega}{C_{Nt}}\right). \tag{A.33}$$

Combining these two equations gives

$$\frac{\phi}{\alpha A_t} = \left(\frac{C_{Nt}}{A_t}\right)^{\frac{\alpha+\gamma_{Nt}-1}{\alpha+\gamma_{Nt}}} \left(\frac{1-\omega}{C_{Nt}}\right), \tag{A.34}$$

$$\Rightarrow C_{Nt} = A_t \left[\frac{\alpha(1-\omega)}{\phi}\right]^{\alpha+\gamma_{Nt}}, \tag{A.35}$$

and the nontradable goods market-clearing condition $C_{Nt} = A_t L_{Nt}^{\alpha+\gamma_{Nt}}$ determines L_{Nt}, which shows the nontradables block $\{C_{Nt}, L_{Nt}\}_{t=0}^{\infty}$ is exogenous for this analytical case.

The XR-IP problem is then to solve for the tradables block C_{Tt}, L_{Tt}, and F_{t+1}^*

$$\max_{\{C_{Tt}, L_{Tt}, F_{t+1}^*\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[\omega \log C_{Tt} - \phi L_{Tt}\right] + constant$$

$$s.t. \quad \left(\frac{\omega}{C_{Tt}}\right) = \phi \frac{1}{\alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}}, \tag{A.36}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*, \tag{A.37}$$

$$F_0^* \text{ given.} \tag{A.38}$$

To derive the approximation of this XR-IP problem, we first define the reference balanced trade (BT) allocation $\{\overline{C}_T, \overline{L}_T\}$ by

$$\overline{C}_T = \overline{Y}_T = \overline{A}\ \overline{L}_T^{\alpha+\gamma}, \tag{A.39}$$

$$\frac{\phi}{(\omega/\overline{C}_T)} = (\alpha+\gamma)\overline{A}\ \overline{L}_T^{\alpha+\gamma-1}, \tag{A.40}$$

where $\overline{A}, \gamma$ are defined below. Therefore in the BT allocation

$$\overline{L}_T = \frac{\omega(\alpha+\gamma)}{\phi}. \tag{A.41}$$

To approximate the welfare function, we take a second-order approximation of the welfare function around the BT allocation

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega \log C_{Tt} - \phi L_{Tt}\right]. \tag{A.42}$$

A second-order Taylor expansion for the tradable consumption term around the BT

$$\omega \log C_{Tt} = \omega c_{Tt} + \omega \log \overline{C}_T, \tag{A.43}$$

where $c_{Tt} \equiv \log C_{Tt} - \log \overline{C}_T$, similarly for l_{Tt}, y_{Tt}. A second-order Taylor expansion for the tradable labor term around the BT

$$-\phi L_{Tt} = -\phi \overline{L}_T e^{l_{Tt}} = \; -\phi \overline{L}_T - \phi \overline{L}_T l_{Tt} - \frac{1}{2}\phi \overline{L}_T l_{Tt}^2 \tag{A.44}$$

Therefore, welfare in terms of deviations and ignoring terms independent of c_{Tt} and l_{Tt}

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega c_{Tt} - \phi \overline{L}_T (l_{Tt} - \frac{1}{2} l_{Tt}^2)\right]. \tag{A.45}$$

Welfare for the first best (FB) approximated around the BT

$$\tilde{\mathbb{W}}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega \tilde{c}_{Tt} - \phi \overline{L}_T (\tilde{l}_{Tt} - \frac{1}{2} \tilde{l}_{Tt}^2)\right]. \tag{A.46}$$

We now approximate the resource constraint relative to the BT

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^* \tag{A.47}$$

$$\frac{C_{Tt}}{\overline{C}_T} - \frac{A_t L_{Tt}^{\alpha+\gamma_{Tt}}}{\overline{A}\ \overline{L}_T^{\alpha+\gamma}} = R^* \frac{F_t^*}{\overline{Y}_T} - \frac{F_{t+1}^*}{\overline{Y}_T} \tag{A.48}$$

$$e^{c_{Tt}} - e^{a_t + (\alpha+\gamma_{Tt}) l_{Tt} + (\gamma_{Tt}-\gamma)\log \overline{L}_T} = R^* f_t^* - f_{t+1}^*, \tag{A.49}$$

where $a_t \equiv \log A_t - \log \overline{A}$, $f_t^* \equiv \frac{F_t^*}{\overline{Y}_T}$. A first-order approximation of the LHS around the BT

$$c_{Tt} - a_t - (\alpha+\gamma) l_{Tt} - (\gamma_{Tt} - \gamma)\log \overline{L}_T = R^* f_t^* - f_{t+1}^*. \tag{A.50}$$

For welfare, a second-order approximation of the LHS around the BT

$$\begin{aligned} c_{Tt} + \frac{1}{2}c_{Tt}^2 - a_t - \frac{1}{2}a_t^2 - (\alpha+\gamma) l_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 l_{Tt}^2 \\ -(\gamma_{Tt}-\gamma)\log \overline{L}_T - \frac{1}{2}(\gamma_{Tt}-\gamma)^2 (\log \overline{L}_T)^2 = R^* f_t^* - f_{t+1}^*. \end{aligned} \tag{A.51}$$

Iterating this forward and using the transversality condition $\lim_{s\to\infty} \beta^s f_{t+s}^* = 0$

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t c_{Tt} \\ &= -\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}c_{Tt}^2 - a_t - \frac{1}{2}a_t^2 - (\alpha+\gamma) l_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 l_{Tt}^2 - (\gamma_{Tt}-\gamma)\log \overline{L}_T - \frac{1}{2}(\gamma_{Tt}-\gamma)^2 (\log \overline{L}_T)^2\right] \\ &+ \frac{1}{\beta} f_0^*, \end{aligned} \tag{A.52}$$

and similarly for the FB allocation relative to the BT

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t \tilde{c}_{Tt} \\ &= -\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\tilde{c}_{Tt}^2 - a_t - \frac{1}{2}a_t^2 - (\alpha+\gamma) \tilde{l}_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 \tilde{l}_{Tt}^2 - (\gamma_{Tt}-\gamma)\log \overline{L}_T - \frac{1}{2}(\gamma_{Tt}-\gamma)^2 (\log \overline{L}_T)^2\right] \\ &+ \frac{1}{\beta} f_0^*. \end{aligned} \tag{A.53}$$

Taking the difference in welfare and substituting using the iterated resource constraint

$$\begin{aligned}
\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = & -\sum_{t=0}^{\infty} \beta^t \omega \left[\frac{1}{2} c_{Tt}^2 - (\alpha+\gamma) l_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 l_{Tt}^2\right] \\
& + \sum_{t=0}^{\infty} \beta^t \omega \left[\frac{1}{2} \tilde{c}_{Tt}^2 - (\alpha+\gamma) \tilde{l}_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 \tilde{l}_{Tt}^2\right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\phi \overline{L}_T (l_{Tt} - \frac{1}{2} l_{Tt}^2)\right] + \sum_{t=0}^{\infty} \beta^t \left[\phi \overline{L}_T (\tilde{l}_{Tt} - \frac{1}{2} \tilde{l}_{Tt}^2)\right] \qquad \text{(A.54)} \\
= & -\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \omega (c_{Tt} - \tilde{c}_{Tt})^2 + \omega \tilde{c}_{Tt} (c_{Tt} - \tilde{c}_{Tt})\right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \omega (\alpha+\gamma)^2 (l_{Tt} - \tilde{l}_{Tt})^2 - \omega (\alpha+\gamma)^2 \tilde{l}_{Tt} (l_{Tt} - \tilde{l}_{Tt})\right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \phi \overline{L}_T (l_{Tt} - \tilde{l}_{Tt})^2 - \phi \overline{L}_T \tilde{l}_{Tt} (l_{Tt} - \tilde{l}_{Tt})\right] \qquad \text{(A.55)} \\
= & -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\omega z_t^2 + \left[\omega (\alpha+\gamma)^2 + \omega (\alpha+\gamma)\right] x_t^2\right], \qquad \text{(A.56)}
\end{aligned}$$

denoting deviations from the FB $z_t \equiv \log C_{Tt} - \log \tilde{C}_{Tt}$, $x_t \equiv \log L_{Tt} - \log \tilde{L}_{Tt}$, and using that $\phi \overline{L}_T = \omega(\alpha + \gamma)$, and as we now show the interaction terms are zero to second order.

Combining the second-order approximations of the resource constraint

$$c_{Tt} - \tilde{c}_{Tt} + \frac{1}{2} c_{Tt}^2 - \frac{1}{2} \tilde{c}_{Tt}^2 - (\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) - (\alpha+\gamma)^2 (l_{Tt}^2 - \tilde{l}_{Tt}^2) = R^* \check{f}_t^* - \check{f}_{t+1}^* \qquad \text{(A.57)}$$

$$\tilde{c}_{Tt}(c_{Tt} - \tilde{c}_{Tt}) - \tilde{c}_{Tt}(\alpha+\gamma)(l_{Tt} - \tilde{l}_{Tt}) + h.o.t. = \tilde{c}_{Tt}(R^* \check{f}_t^* - \check{f}_{t+1}^*), \qquad \text{(A.58)}$$

where $\check{f}_t^* \equiv f_t^* - \tilde{f}_t^*$.

Substituting for $\tilde{c}_{Tt}$ gives

$$\tilde{c}_{Tt}(c_{Tt}-\tilde{c}_{Tt}) - a_t(\alpha+\gamma)(l_{Tt}-\tilde{l}_{Tt}) - \tilde{l}_{Tt}(\alpha+\gamma)^2(l_{Tt}-\tilde{l}_{Tt}) = \tilde{c}_{Tt}(R^*\check{f}_t^* - \check{f}_{t+1}^*)$$
$$+(\gamma_{Tt}-\gamma)\log\overline{L}_T(\alpha+\gamma)(l_{Tt}-\tilde{l}_{Tt}) + (R^*\tilde{f}_t^* - \tilde{f}_{t+1}^*)(\alpha+\gamma)(l_{Tt}-\tilde{l}_{Tt}) + h.o.t. \tag{A.59}$$
$$\sum_{t=0}^{\infty}\beta^t\Big[\tilde{c}_{Tt}(c_{Tt}-\tilde{c}_{Tt}) - a_t(\alpha+\gamma)(l_{Tt}-\tilde{l}_{Tt}) - \tilde{l}_{Tt}(\alpha+\gamma)^2(l_{Tt}-\tilde{l}_{Tt}) = \sum_{t=0}^{\infty}\beta^t\Big[\tilde{c}_{Tt}(R^*\check{f}_t^* - \check{f}_{t+1}^*)\Big]$$
$$+(\gamma_{Tt}-\gamma)\log\overline{L}_T(\alpha+\gamma)(l_{Tt}-\tilde{l}_{Tt}) + (R^*\tilde{f}_t^* - \tilde{f}_{t+1}^*)(\alpha+\gamma)(l_{Tt}-\tilde{l}_{Tt})\Big] + h.o.t. \tag{A.60}$$

We set $\overline{A}$ such that $\sum_{t=0}^{\infty}\beta^t a_t x_t = \sum_{t=0}^{\infty}\beta^t\tilde{l}_{Tt}x_t$, and γ such that $\sum_{t=0}^{\infty}\beta^t x_t\Big[(\gamma_{Tt}-\gamma)\log\overline{L}_T + (R^*\tilde{f}_t^* - \tilde{f}_{t+1}^*)\Big] = 0$. The interaction terms simplify to

$$\sum_{t=0}^{\infty}\beta^t\Big[\omega\tilde{c}_{Tt}(c_{Tt}-\tilde{c}_{Tt}) - \omega(\alpha+\gamma)^2 l_{Tt}(l_{Tt}-\tilde{l}_{Tt}) - \omega(\alpha+\gamma)l_{Tt}(l_{Tt}-\tilde{l}_{Tt})\Big] = \sum_{t=0}^{\infty}\beta^t\Big[\tilde{c}_{Tt}(R^*\check{f}_t^* - \check{f}_{t+1}^*)\Big]$$
$$+h.o.t. \tag{A.61}$$
$$= \sum_{t=0}^{\infty}\beta^t\Big[\check{f}_{t+1}^*(\tilde{c}_{Tt+1}-\tilde{c}_{Tt})\Big] \tag{A.62}$$
$$= 0, \tag{A.63}$$

given from the FB optimality condition $\tilde{c}_{Tt} = \tilde{c}_{Tt+1}$ and $\check{f}_0^* = 0$.

Next solve the constraints in terms of z_t, x_t starting with the loglinear resource constraint

$$c_{Tt} - a_t - (\alpha+\gamma)l_{Tt} - (\gamma_{Tt}-\gamma)\log\overline{L}_T = R^* f_t^* - f_{t+1}^* \tag{A.64}$$
$$\tilde{c}_{Tt} - a_t - (\alpha+\gamma)\tilde{l}_{Tt} - (\gamma_{Tt}-\gamma)\log\overline{L}_T = R^*\tilde{f}_t^* - \tilde{f}_{t+1}^* \tag{A.65}$$
$$\Rightarrow z_t - (\alpha+\gamma)x_t = R^*\check{f}_t^* - \check{f}_{t+1}^*. \tag{A.66}$$

Next the MRS = MRT constraint for the XR-IP

$$\frac{\phi}{(\omega/C_{Tt})} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} \tag{A.67}$$

$$\log\phi - \log\omega + \log C_{Tt} = \log\alpha + \log A_t + (\alpha+\gamma-1)\log L_{Tt} + (\gamma_{Tt}-\gamma)\log L_{Tt}. \tag{A.68}$$

For the FB this condition is

$$\frac{\phi}{\left(\omega/\tilde{C}_{Tt}\right)} = (\alpha+\gamma_{Tt})A_t \tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1} \tag{A.69}$$

$$\log\phi - \log\omega + \log\tilde{C}_{Tt} = \log(\alpha+\gamma_{Tt}) + \log A_t + (\alpha+\gamma-1)\log\tilde{L}_{Tt} + (\gamma_{Tt}-\gamma)\log\tilde{L}_{Tt}. \tag{A.70}$$

Combining the XR-IP and FB gives

$$z_t = \psi_t + (\alpha+\gamma-1)x_t + (\gamma_{Tt}-\gamma)(\log L_{Tt} - \log\tilde{L}_{Tt}) \tag{A.71}$$

$$= \psi_t + (\alpha+\gamma-1)x_t, \tag{A.72}$$

where $\psi_t \equiv \log\alpha - \log(\alpha+\gamma_{Tt}) \leq 0$ and the second line uses a first-order approximation around the BT as shown in the third line below

$$(\gamma_{Tt}-\gamma)(\log L_{Tt} - \log\tilde{L}_{Tt}) = (\gamma_{Tt}-\gamma)(l_{Tt} - \tilde{l}_{Tt}) \tag{A.73}$$

$$= (\gamma_{Tt}-\gamma)l_{Tt} - (\gamma_{Tt}-\gamma)\tilde{l}_{Tt} \tag{A.74}$$

$$= 0. \tag{A.75}$$

Combining (A.56), (A.72) and (A.66), the approximate XR-IP problem is

$$\max_{\{z_t, x_t, \check{f}^*_{t+1}\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + \omega(\alpha+\gamma)\right]x_t^2\right]$$

$$s.t. \quad z_t = \psi_t + (\alpha+\gamma-1)x_t, \tag{A.76}$$

$$z_t - (\alpha+\gamma)x_t = R^*\check{f}^*_t - \check{f}^*_{t+1}, \tag{A.77}$$

$$\check{f}^*_0 = 0. \tag{A.78}$$

Substituting for $\beta R^* = 1$ and iterating (A.77) using the transversality condition $\lim_{s\to\infty}\beta^s f^*_{t+s} = 0$ gives

$$\sum_{t=0}^{\infty}\beta^t\left(z_t - (\alpha+\gamma)x_t\right) = 0, \tag{A.79}$$

which shows the Lemma.

A.5. Proof of Proposition 3

We first solve the optimal XR-IP problem in this case, then characterize the solution relative to the laissez-faire competitive equilibrium (LF-CE).

Combining the constraints, we can solve the XR-IP problem for x_t

$$\max_{\{x_t\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega(\psi_t + (\alpha+\gamma-1)x_t)^2 + \left[\omega(\alpha+\gamma)^2 + \omega(\alpha+\gamma)\right]x_t^2\right]$$

$$s.t. \quad \sum_{t=0}^{\infty}\beta^t\left[\psi_t - x_t\right] = 0. \tag{A.80}$$

Let λ be the multiplier on the lifetime resource constraint. The FOC for x_t is

$$-\beta^t\left[\omega(\psi_t + (\alpha+\gamma-1)x_t)(\alpha+\gamma-1) + \left[\omega(\alpha+\gamma)^2 + \omega(\alpha+\gamma)\right]x_t\right] = \beta^t\lambda. \tag{A.81}$$

We get a loglinear Euler equation to characterize the XR-IP solution

$$\psi_t(\alpha+\gamma-1)+\left[(\alpha+\gamma-1)^2+(\alpha+\gamma)^2+(\alpha+\gamma)\right]x_t^{IP}$$
$$=\psi_{t+1}(\alpha+\gamma-1)+\left[(\alpha+\gamma-1)^2+(\alpha+\gamma)^2+(\alpha+\gamma)\right]x_{t+1}^{IP} \tag{A.82}$$
$$\psi_t+\left[(\alpha+\gamma-1)+c\right]x_t^{IP}=\psi_{t+1}+\left[(\alpha+\gamma-1)+c\right]x_{t+1}^{IP}, \tag{A.83}$$

where $c=\frac{(\alpha+\gamma)^2+(\alpha+\gamma)}{(\alpha+\gamma-1)}<0$. Therefore

$$\psi_0+(\alpha+\gamma-1+c)x_0^{IP}=\psi_1+(\alpha+\gamma-1+c)x_1^{IP} \tag{A.84}$$
$$=\psi_t+(\alpha+\gamma-1+c)x_t^{IP} \tag{A.85}$$
$$\Rightarrow x_t^{IP}=\frac{(\psi_0-\psi_t)}{(\alpha+\gamma-1+c)}+x_0^{IP}. \tag{A.86}$$

To show that $x_t^{IP}<0$, i.e. $\tilde{L}_{Tt}>L_{Tt}^{IP}$ for all t, use the lifetime resource constraint

$$\sum_{t=0}^{\infty}\beta^t x_t=\sum_{t=0}^{\infty}\beta^t\psi_t \tag{A.87}$$
$$\frac{x_0^{IP}}{1-\beta}+\sum_{t=1}^{\infty}\beta^t\frac{(\psi_0-\psi_t)}{(\alpha+\gamma-1+c)}=\sum_{t=0}^{\infty}\beta^t\psi_t \tag{A.88}$$
$$x_0^{IP}=(1-\beta)\psi_0-\frac{\beta\psi_0}{(\alpha+\gamma-1+c)}+(1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t<0, \tag{A.89}$$

since $(\alpha+\gamma+c)<0$ and given $\gamma_{Tt}>\gamma_{Tt+1}>0$ for all t then $\psi_t<\psi_{t+1}<0$, $\sum_{t=1}^{\infty}\beta^t\psi_t<0$. Therefore

$$x_t^{IP}=\frac{(\psi_0-\psi_t)}{(\alpha+\gamma-1+c)}+(1-\beta)\psi_0-\frac{\beta\psi_0}{(\alpha+\gamma-1+c)}+(1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t \tag{A.90}$$
$$=\frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}-\frac{\psi_t}{(\alpha+\gamma-1+c)}+(1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t<0. \tag{A.91}$$

The LF-CE is characterized by

$$z_t = z_{t+1}, \tag{A.92}$$

$$z_t = \psi_t + (\alpha + \gamma - 1)x_t, \tag{A.93}$$

$$z_t - (\alpha + \gamma)x_t = R^* \check{f}_t^* - \check{f}_{t+1}^*, \tag{A.94}$$

$$\check{f}_0^* = 0. \tag{A.95}$$

Combining the first two equations

$$\psi_t + (\alpha + \gamma - 1)x_t^{CE} = \psi_{t+1} + (\alpha + \gamma - 1)x_{t+1}^{CE}, \tag{A.96}$$

so the CE allocation is given by the XR-IP with setting $c = 0$. Note that

$$x_t^{CE} = \frac{(\psi_0 - \psi_t)}{(\alpha + \gamma - 1)} + x_0^{CE}. \tag{A.97}$$

Therefore, from the lifetime resource constraint

$$x_0^{IP} = (1 - \beta)\psi_0 - \frac{\beta\psi_0}{(\alpha + \gamma - 1 + c)} + (1 - \beta)\frac{(\alpha + \gamma + c)}{(\alpha + \gamma - 1 + c)} \sum_{t=1}^{\infty} \beta^t \psi_t \tag{A.98}$$

$$x_0^{CE} = (1 - \beta)\psi_0 - \frac{\beta\psi_0}{(\alpha + \gamma - 1)} + (1 - \beta)\frac{(\alpha + \gamma)}{(\alpha + \gamma - 1)} \sum_{t=1}^{\infty} \beta^t \psi_t, . \tag{A.99}$$

Therefore

$$x_0^{IP} - x_0^{CE} = \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + \frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}\left[(\alpha+\gamma+c)(\alpha+\gamma-1)-(\alpha+\gamma)(\alpha+\gamma-1+c)\right] \tag{A.100}$$

$$= \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + \frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}\left[c(\alpha+\gamma-1)-(\alpha+\gamma)c\right] \tag{A.101}$$

$$= \underbrace{\frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c}_{>0} + \underbrace{\frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}[-c]}_{<0} \tag{A.102}$$

$$> \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + \frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}[-c] \tag{A.103}$$

$$= \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + \frac{\beta\psi_0}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}[-c] \tag{A.104}$$

$$= 0. \tag{A.105}$$

From the lifetime resource constraint

$$\sum_{t=0}^{\infty}\beta^t(x_t^{IP} - x_t^{CE}) = 0 \tag{A.106}$$

$$\underbrace{x_0^{IP} - x_0^{CE}}_{>0} = -\sum_{t=1}^{\infty}\beta^t(x_t^{IP} - x_t^{CE}), \tag{A.107}$$

so for at least one $t \geq 1$, $(x_t^{IP} - x_t^{CE}) < 0$.

Note, for any $t \geq 1$

$$x_t^{IP} - x_t^{CE} = (\psi_0 - \psi_t)\left[\frac{1}{(\alpha+\gamma-1+c)} - \frac{1}{(\alpha+\gamma-1)}\right] + x_0^{IP} - x_0^{CE} \tag{A.108}$$

$$= \frac{(\psi_0 - \psi_t)}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}\left[(\alpha+\gamma-1) - (\alpha+\gamma-1+c)\right] + x_0^{IP} - x_0^{CE} \tag{A.109}$$

$$= \frac{-(\psi_0 - \psi_t)}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c + x_0^{IP} - x_0^{CE}. \tag{A.110}$$

Therefore

$$x_{t+1}^{IP} - x_{t+1}^{CE} - (x_t^{IP} - x_t^{CE}) = \frac{-(\psi_0 - \psi_{t+1})}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c - \frac{-(\psi_0 - \psi_t)}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c \tag{A.111}$$

$$= \frac{\psi_{t+1} - \psi_t}{(\alpha+\gamma-1+c)(\alpha+\gamma-1)}c \tag{A.112}$$

$$< 0, \tag{A.113}$$

since $\psi_{t+1} - \psi_t > 0$ and $c < 0$, so $(x_t^{IP} - x_t^{CE})$ is strictly decreasing in t. Then, together with $(x_0^{IP} - x_0^{CE}) > 0$ and $(x_t^{IP} - x_t^{CE}) < 0$ for some t then it must be that $\exists\ \bar{t} > 0$ such that $(x_t^{IP} - x_t^{CE}) > 0$ (i.e. $L_{Tt}^{IP} > L_{Tt}^{CE}$) for $t < \bar{t}$ and $(x_t^{IP} - x_t^{CE}) < 0$ (i.e. $L_{Tt}^{IP} < L_{Tt}^{CE}$) for $t > \bar{t}$.

For both the XR-IP and LF-CE

$$C_{Tt} = \frac{\omega\alpha A_t}{\phi}\frac{1}{L_{Tt}^{1-\alpha-\gamma_{Tt}}}, \tag{A.114}$$

$$\mathcal{E}_t = \left(\frac{c_{Nt}}{A_t}\right)^{\frac{\alpha-1}{\alpha}} L_{Tt}^{1-\alpha-\gamma_{Tt}}, \tag{A.115}$$

where c_{Nt} coincides for the XR-IP and LF-CE.

Therefore, for $t < \bar{t}$ since $L_{Tt}^{IP} > L_{Tt}^{CE}$

$$C_{Tt}^{IP} < C_{Tt}^{CE}, \tag{A.116}$$

$$\mathcal{E}_t^{IP} > \mathcal{E}_t^{CE}. \tag{A.117}$$

By definition of the trade balance

$$TB_t = A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt}, \tag{A.118}$$

$$\Rightarrow TB_t^{IP} > TB_t^{CE}. \tag{A.119}$$

From the balance of payments

$$F_{t+1}^* = R^* F_t^* + A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt}, \tag{A.120}$$

$$\Rightarrow F_{t+1}^{*IP} > F_{t+1}^{*CE}, \tag{A.121}$$

for $t < \bar{t}$.

It is straightforward that similarly for $t > \bar{t}$ when $L_{Tt}^{IP} < L_{Tt}^{CE}$ that $C_{Tt}^{IP} > C_{Tt}^{CE}$, $\mathcal{E}_t^{IP} < \mathcal{E}_t^{CE}$, and $TB_t^{IP} < TB_t^{CE}$.

To examine the path of assets F_{t+1}^*, for both the XR-IP and CE

$$\psi_t - x_t = R^* \check{f}_t^* - \check{f}_{t+1}^*, \tag{A.122}$$

$$\check{f}_0^* = 0. \tag{A.123}$$

Therefore, at any t

$$\sum_{s=0}^{t} \beta^s [\psi_s - x_s] + \beta^t \check{f}_{t+1}^* = 0 \tag{A.124}$$

$$\beta^t \check{f}_{t+1}^* = \sum_{s=0}^{t} \beta^s [x_s - \psi_s]. \tag{A.125}$$

Substituting in for the solution for x_t gives

$$x_t - \psi_t = \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{\psi_t}{(\alpha+\gamma-1+c)} + (1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t - \psi_t \tag{A.126}$$

$$= \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{(\alpha+\gamma+c)\psi_t}{(\alpha+\gamma-1+c)} + (1-\beta)\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\sum_{t=1}^{\infty}\beta^t\psi_t \tag{A.127}$$

$$< \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{(\alpha+\gamma+c)\psi_t}{(\alpha+\gamma-1+c)} + \beta\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\psi_0 \tag{A.128}$$

$$< \frac{(1-\beta)\psi_0(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)} - \frac{(\alpha+\gamma+c)\psi_0}{(\alpha+\gamma-1+c)} + \beta\frac{(\alpha+\gamma+c)}{(\alpha+\gamma-1+c)}\psi_0 \tag{A.129}$$

$$= 0. \tag{A.130}$$

Therefore

$$\beta^t \check{f}^*_{t+1} = \sum_{s=0}^{t}\beta^s[x_s - \psi_s] < 0, \tag{A.131}$$

which shows that $\tilde{F}^*_{t+1} > F^{*IP}_{t+1}$ for all t.

Combining the expression above for $\check{f}^*_{t+1}$ for the XR-IP and CE

$$\beta^t[(\check{f}^*_{t+1})^{IP} - (\check{f}^*_{t+1})^{CE}] = \sum_{s=0}^{t}\beta^s[x_s^{IP} - x_s^{CE}]. \tag{A.132}$$

We know that

$$\sum_{s=0}^{\infty}\beta^s[x_s^{IP} - x_s^{CE}] = 0 \tag{A.133}$$

$$\sum_{s=0}^{t}\beta^s[x_s^{IP} - x_s^{CE}] + \sum_{s=t+1}^{\infty}\beta^s[x_s^{IP} - x_s^{CE}] = 0, \tag{A.134}$$

as well as that $(x_0^{IP} - x_0^{CE}) > 0$, $(x_s^{IP} - x_s^{CE})$ is strictly decreasing in s and $(x_t^{IP} - x_t^{CE}) < 0$

for all $t > \bar{t}$, therefore

$$\beta^t[(\check{f}^*_{t+1})^{IP} - (\check{f}^*_{t+1})^{CE}] = -\sum_{s=t+1}^{\infty} \beta^s[x_s^{IP} - x_s^{CE}] > 0 \tag{A.135}$$

$$\Rightarrow F^{*IP}_{t+1} > F^{*CE}_{t+1} \text{ for all } t. \tag{A.136}$$

A.6. Proof of Proposition 4

If the economy is not converging to the transition then $\gamma_{Tt} = \gamma_T$ for all $t \geq 0$.

If the economy is at the technological frontier then $\gamma_{Tt} = 0$ for all $t \geq 0$.

In either case the solution to the XR-IP problem shown in Propostion A.5 is

$$\psi + [(\alpha + \gamma - 1) + c]\, x_t^{IP} = \psi + [(\alpha + \gamma - 1) + c]\, x_{t+1}^{IP} \tag{A.137}$$

$$x_t^{IP} = x_{t+1}^{IP}, \tag{A.138}$$

and for the LF-CE is

$$\psi + (\alpha + \gamma - 1)x_t^{CE} = \psi + (\alpha + \gamma - 1)x_{t+1}^{IP} \tag{A.139}$$

$$x_t^{CE} = x_{t+1}^{CE}. \tag{A.140}$$

The other conditions are identical so the allocations for the optimal XR-IP and LF-CE must coincide.

A.7. Proof of Proposition 5

We first derive the XR-IP problem with international capital mobility. Given the financiers operate only in initial period by choosing Q_1, $Q_0 = 0$, and $Q_t = 0$ for all $t \geq 2$, and the economy starts below and converges to the frontier at $t = 1$ then $\gamma_{T0} > 0$ and $\gamma_{Tt} = 0$ for all

$t \geq 1$. The balance of payments condition is

$$C_{T0} - A_0 L_{T0}^{\alpha+\gamma_{T0}} = R^* F_0^* - F_1^* + \frac{Q_1}{\mathcal{E}_0} \tag{A.141}$$

$$C_{T1} - A_1 L_{T1}^{\alpha} = R^* F_1^* - F_2^* - R_t \frac{Q_1}{\mathcal{E}_1} \tag{A.142}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha} = R^* F_t^* - F_{t+1}^* \qquad \text{for } t \geq 2. \tag{A.143}$$

Combining and iterating these forward and using the transversality condition gives the intertemporal resource constraint

$$\sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left(A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt}\right) + Q_1 \left(\frac{1}{\mathcal{E}_0} - \frac{1}{\mathcal{E}_1}\right) \; = \; - R^* F_0^* \tag{A.144}$$

Working on this expression, substituting the optimality condition for the intermediaries $Q_1 = \frac{1}{\Gamma}\left[\mathcal{E}_0 - \frac{R^*}{R_1}\mathcal{E}_1\right]$ gives

$$\underbrace{\sum_{t=0}^{\infty} \frac{\left(A_t L_{Tt}^{\alpha+\gamma_{Tt}} - C_{Tt}\right)}{(R^*)^t}}_{\text{NPV net exports}} + \underbrace{\frac{1}{\Gamma}\left(1 - \frac{R^*}{R_1}\frac{\mathcal{E}_1}{\mathcal{E}_0}\right)\left(1 - \frac{R_1}{R^*}\frac{\mathcal{E}_0}{\mathcal{E}_1}\right)}_{\leq 0} \; = \; - R^* F_0^* \tag{A.145}$$

From the HH Euler equation

$$\frac{C_{Tt+1}}{C_{Tt}} = \beta R_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}. \tag{A.146}$$

Substituting the constraint for C_{Tt} gives

$$\frac{A_{t+1} L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}}{A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} = \beta R_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{A.147}$$

$$\frac{A_{t+1} L_{Tt+1}^{\alpha+\gamma_{Tt+1}-1}}{A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}} = \frac{R_{t+1}}{R^*} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}. \tag{A.148}$$

Substituting the constraint for C_{Tt} the intertemporal resource constraint from (A.145) then is

$$\sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} \right] = R^* F_0^*$$
$$-\frac{1}{\Gamma}\left(1 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}\right)\left(1 - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right) \tag{A.149}$$

The XR-IP problem is then to solve for the tradables block C_{Tt}, L_{Tt}, and F_{t+1}^*

$$\max_{\{C_{Tt}, L_{Tt}, F_{t+1}^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\omega(\alpha + \gamma_{Tt} - 1) \log L_{Tt} - \phi L_{Tt}\right] + constant$$
$$s.t. \quad \sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} \right] = R^* F_0^*,$$
$$-\frac{1}{\Gamma}\left(1 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}\right)\left(1 - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right) \tag{A.150}$$
$$F_0^* \text{ given.} \tag{A.151}$$

The Lagrangian for the model with intermediaries is

$$\begin{aligned}\mathcal{L} = &\sum_{t=0}^{\infty} \beta^t \left[\omega(\alpha + \gamma_{Tt} - 1) \log L_{Tt} - \phi L_{Tt}\right] \\ &+ \lambda \Bigg[R^* F_0^* - \sum_{t=0}^{\infty} \frac{1}{(R^*)^t} \left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} \right] \\ &+ \frac{1}{\Gamma}\left(2 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}} - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right) \Bigg].\end{aligned} \tag{A.152}$$

The FOCs are

$$\frac{\partial \mathcal{L}}{\partial L_{T0}} = \omega(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - \phi - \lambda\left[\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)A_0 L_{T0}^{\alpha+\gamma_{T0}-2} - (\alpha+\gamma_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1}\right]$$
$$-\lambda\left[\frac{1}{\Gamma}\left[(\alpha+\gamma_{T0}-1)\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-2}}{A_1 L_{T1}^{\alpha-1}} - (\alpha+\gamma_{T0}-1)\frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}}}\right]\right] = 0 \quad (A.153)$$

$$\frac{\partial \mathcal{L}}{\partial L_{T1}} = \beta\left[\omega(\alpha-1)\frac{1}{L_{T1}} - \phi\right] - \lambda\frac{1}{R^*}\left[\frac{\omega\alpha}{\phi}(\alpha-1)A_1 L_{T1}^{\alpha-2} - \alpha A_1 L_{T1}^{\alpha-1}\right]$$
$$-\lambda\left[\frac{1}{\Gamma}\left[(1-\alpha)\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha}} - (1-\alpha)\frac{A_1 L_{T1}^{\alpha-2}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right]\right] = 0 \quad (A.154)$$

$$\frac{\partial \mathcal{L}}{\partial L_{Tt}} = \beta^t\left[\omega(\alpha-1)\frac{1}{L_{Tt}} - \phi\right] - \lambda\frac{1}{(R^*)^t}\left[\frac{\omega\alpha}{\phi}(\alpha-1)A_t L_{Tt}^{\alpha-2} - \alpha A_t L_{Tt}^{\alpha-1}\right] = 0 \text{ for } t \geq 2. \quad (A.155)$$

Given that A_t is constant for $t \geq 2$, the final equation gives the usual XR-IP intertemporal optimality condition for $t \geq 2$

$$L_{T2} = L_{Tt+1}. \quad (A.156)$$

Combining the FOCs gives

$$\frac{1}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\theta_0(L_{T0}, L_{T1}, \Gamma) = \frac{1}{A_1 L_{T1}^{\alpha-1}}\theta_1(L_{T0}, L_{T1}, \Gamma) \quad (A.157)$$
$$= \frac{1}{A_1 L_{Tt}^{\alpha-1}} \text{ for } t \geq 2 \quad (A.158)$$

where

$$\theta_0(L_{T0}, L_{T1}, \Gamma) \equiv \frac{\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - \alpha}{\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)\frac{1}{L_{T0}} - (\alpha+\gamma_{T0}) + \frac{1}{\Gamma}(\alpha+\gamma_{T0}-1)\left[\frac{A_0 L_{T0}^{-1}}{A_1 L_{T1}^{\alpha-1}} - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{2(\alpha+\gamma_{T0})-1}}\right]} \quad (A.159)$$

$$\theta_1(L_{T0}, L_{T1}, \Gamma) \equiv \frac{\frac{\omega\alpha}{\phi}(\alpha-1)\frac{1}{L_{T1}} - \alpha}{\frac{\omega\alpha}{\phi}(\alpha-1)\frac{1}{L_{T1}} - \alpha + R^*\frac{1}{\Gamma}(1-\alpha)\left[\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{2\alpha-1}} - \frac{A_1 L_{T1}^{-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right]}. \quad (A.160)$$

With no financial intermediaries $\Gamma \to \infty$ then back to baseline XR-IP model and $\theta_0(L_{T0})$ and $\theta_1 = 1$.

The solution to the model for L_{T0} and L_{T1} is characterized equation (A.157) above and from the balance of payments after substituting the optimality conditions

$$G\left(L_{T0}, \theta_0, \theta_1\right) - \underbrace{H(L_{T0}, L_{T1})}_{\leq 0} - \frac{1}{\beta} F_0^* = 0 \tag{A.161}$$

where

$$\begin{aligned} G\left(L_{T0}, \theta_0, \theta_1\right) \equiv & \left(1 + \beta \frac{\theta_1}{\theta_0} + \frac{\beta^2}{1-\beta} \frac{1}{\theta_0}\right) \frac{\omega \alpha}{\phi} L_{T0}^{\alpha + \gamma_{T0} - 1} \\ & - L_{T0}^{\alpha+\gamma_{T0}} - \left(\beta \left[\frac{\theta_1}{\theta_0}\right]^{\frac{\alpha}{\alpha-1}} + \frac{\beta^2}{1-\beta} \left[\frac{1}{\theta_0}\right]^{\frac{\alpha}{\alpha-1}}\right) \left(L_{T0}^{\alpha+\gamma_{T0}-1}\right)^{\frac{\alpha}{\alpha-1}}, \end{aligned} \tag{A.162}$$

$$H(L_{T0}, L_{T1}) \equiv \frac{1}{\Gamma} \left(1 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}\right) \left(1 - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right) \leq 0. \tag{A.163}$$

Summarize the XR-IP solution with intermediaries

$$G\left(L_{T0}^{IP-B}, \theta_0(L_{T0}, L_{T1}, \Gamma), \theta_1(L_{T0}, L_{T1}, \Gamma)\right) - \underbrace{H(L_{T0}^{IP-B}, L_{T1}^{IP-B})}_{\leq 0} - \frac{1}{\beta} F_0^* = 0, \tag{A.164}$$

where below we prove that $\theta_1(L_{T0}, L_{T1}, \Gamma) \in (0,1)$ and $\theta_0(L_{T0}, L_{T1}, \Gamma) \in (0,1)$.

The solution for the baseline XR-IP model with no intermediaries

$$G\left(L_{T0}^{IP}, \theta_0(L_{T0}, L_{T1}, \infty), 1\right) - 0 - \frac{1}{\beta} F_0^* = 0, \tag{A.165}$$

where $\theta_0(L_{T0}, L_{T1}, \infty) \in (0,1)$.

The solution for the LF-CE is

$$G\left(L_{T0}^{CE}, 1, 1\right) - 0 - \frac{1}{\beta} F_0^* = 0. \tag{A.166}$$

We now compare the LF-CE vs. XR-IP with intermediaries ("IP-B") allocations.

The LF-CE allocation without intermediaries is

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} = A_1 L_{T1}^{\alpha-1}, \tag{A.167}$$

where L_{T0} satisfies the intertemporal resource constraint. In the economy with intermediaries this leads to $H(L_{T0}^{CE}, L_{T1}^{CE}) = 0$, so it is feasible.

Now consider a small increase in L_{T0}, $dL_{T0} > 0$, from the LF-CE allocation in the economy with intermediaries. We show that this, combined with a decrease in L_{T1}, $dL_{T1} < 0$ increases HH utility and is feasible. The change in welfare from each is

$$\frac{\partial W}{\partial L_{T0}} dL_{T0} = \left[\omega(\alpha+\gamma_{T0}-1)\frac{1}{A_0 L_{T0}} - \phi\right] dL_{T0} \tag{A.168}$$

$$\frac{\partial W}{\partial L_{T1}} dL_{T1} = \beta\left[\omega(\alpha-1)\frac{1}{A_1 L_{T1}} - \phi\right] dL_{T1} \tag{A.169}$$

Consider the welfare neutral change around the LF-CE allocation

$$dW = \frac{\partial W}{\partial L_{T0}} dL_{T0} + \frac{\partial W}{\partial L_{T1}} dL_{T1} = 0 \tag{A.170}$$

$$\left[\omega(\alpha+\gamma_{T0}-1)\frac{1}{A_0 L_{T0}} - \phi\right] dL_{T0} + \beta\left[\omega(\alpha-1)\frac{1}{A_1 L_{T1}} - \phi\right] dL_{T1} = 0 \tag{A.171}$$

The resource constraint is:

$$RC = R^* F_0^* - \sum_{t=0}^{\infty} \frac{1}{(R^*)^t}\left[\frac{\omega\alpha}{\phi} A_t L_{Tt}^{\alpha+\gamma_{Tt}-1} - A_t L_{Tt}^{\alpha+\gamma_{Tt}}\right] + \frac{1}{\Gamma}\left(2 - \frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}} - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right). \tag{A.172}$$

The change in the resource constraint from the change in L_{T0} and L_{T1} is:

$$\begin{aligned}
\Delta RC = & -\left[\frac{\omega\alpha}{\phi}(\alpha+\gamma_{T0}-1)A_0 L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1}\right]dL_{T0}\\
& -\beta\left[\frac{\omega\alpha}{\phi}(\alpha-1)A_1 L_{T1}^{\alpha-2}-\alpha A_1 L_{T1}^{\alpha-1}\right]dL_{T1}\\
& \underbrace{-\frac{1}{\Gamma}\frac{1}{L_{T0}}\left[(\alpha+\gamma_{T0}-1)\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}-(\alpha+\gamma_{T0}-1)\frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right]dL_{T0}}_{=0}\\
& \underbrace{-\frac{1}{\Gamma}\frac{1}{L_{T1}}\left[(1-\alpha)\frac{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}{A_1 L_{T1}^{\alpha-1}}-(1-\alpha)\frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}}\right]dL_{T1}}_{=0} \quad \text{(A.173)}\\
\Delta RC = & \ \gamma_{T0}A_0 L_{T0}^{\alpha+\gamma_{T0}-1}dL_{T0} > 0 \quad \text{(A.174)}
\end{aligned}$$

So this utility neutral change leaves resources left over to be able to increase tradable consumption and raise overall welfare. Therefore, we can raise welfare in the economy with intermediaries relative to the LF-CE by increasing L_{T0} and decreasing L_{T1}. We can do so similarly for decreasing any L_{Tt} for $t \geq 2$. If we do the opposite change and decrease L_{T0} and increase L_{T1}, the signs are reversed and this will leave utility constant but reduce resources. A perturbation of L_{T1} and L_{Tt} for any $t \geq 2$ leads to no change in welfare or resources.

Therefore, this shows that locally around the LF-CE allocation

$$L_{T0}^{IP-B} > L_{T0}^{CE}, \quad \text{(A.175)}$$

$$C_{T0}^{IP-B} < C_{T0}^{CE}, \quad \text{(A.176)}$$

$$\mathcal{E}_0^{IP-B} > \mathcal{E}_0^{CE}. \quad \text{(A.177)}$$

We now compare the baseline XR-IP vs. XR-IP with intermediaries allocations. The XR-IP allocation in the economy with no intermediaries is

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} = \frac{1}{\theta_0}A_1 L_{T1}^{\alpha-1} \quad \text{(A.178)}$$

where

$$\theta_0(L_{T0}, L_{T1}, \Gamma = \infty) \equiv \frac{\frac{\omega\alpha}{\phi}(\alpha + \gamma_{T0} - 1)\frac{1}{L_{T0}} - \alpha}{\frac{\omega\alpha}{\phi}(\alpha + \gamma_{T0} - 1)\frac{1}{L_{T0}} - (\alpha + \gamma_{T0}) + \frac{1}{\Gamma}(\alpha + \gamma_{T0} - 1)\left[\frac{A_0 L_{T0}^{-1}}{A_1 L_{T1}^{\alpha-1}} - \frac{A_1 L_{T1}^{\alpha-1}}{A_0 L_{T0}^{2(\alpha+\gamma_{T0})-1}}\right]}. \tag{A.179}$$

and $\Gamma = \infty$ implies $\theta_0(L_{T0}) \in (0, 1)$, and L_{T0} satisfies the intertemporal resource constraint.

In the economy with intermediaries $H(L_{T0}^{IP}, L_{T1}^{IP}) < 0$ enters the resource constraint, so the baseline XR-IP allocation without intermediaries is not feasible here. We therefore consider a small change from the XR-IP allocation with intermediaries in the baseline economy without intermediaries. The term $H(L_{T0}, L_{T1}) = 0$ so there are additional resources left over, and the allocation with intermediaries cannot be optimal in this case.

Again consider the welfare neutral change of increasing L_{T0}, $dL_{T0} > 0$ and decreasing L_{T1}, $dL_{T1} < 0$.

$$dW = \frac{\partial W}{\partial L_{T0}} dL_{T0} + \frac{\partial W}{\partial L_{T1}} dL_{T1} = 0 \tag{A.180}$$

$$\left[\omega(\alpha + \gamma_{T0} - 1)\frac{1}{A_0 L_{T0}} - \phi\right] dL_{T0} + \beta\left[\omega(\alpha - 1)\frac{1}{A_1 L_{T1}} - \phi\right] dL_{T1} = 0 \tag{A.181}$$

The change in the resource constraint in the economy without intermediaries is

$$\begin{aligned} \Delta RC = & -\left[\frac{\omega\alpha}{\phi}(\alpha + \gamma_{T0} - 1)A_0 L_{T0}^{\alpha+\gamma_{T0}-2} - (\alpha + \gamma_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1}\right] dL_{T0} \\ & - \beta\left[\frac{\omega\alpha}{\phi}(\alpha - 1)A_1 L_{T1}^{\alpha-2} - \alpha A_1 L_{T1}^{\alpha-1}\right] dL_{T1} \end{aligned} \tag{A.182}$$

$$\Delta RC = \gamma_{T0} A_0 L_{T0}^{\alpha+\gamma_{T0}-1} dL_{T0} > 0 \tag{A.183}$$

after following the same steps as above, and $\Delta RC > 0$ since $dL_{T0} > 0$.

Therefore, this utility-neutral reallocation further increases available resources for tradable consumption to raise utility. This must be preferred to the welfare neutral change of decreasing L_{T0} and increasing L_{T1} which strictly reduces the resources available for consumption. A similar change in both L_{T1} and L_{Tt} for any $t \geq 2$ yields no change in utility or the resource constraint. This must also be strictly preferred to simply increasing utility from just changing one of L_{T0} and L_{T1} as changing both also allows for additional resources.

Therefore, this shows that locally around the IP-B allocation

$$L_{T0}^{IP} > L_{T0}^{IP-B} \tag{A.184}$$

$$C_{T0}^{IP} < C_{T0}^{IP-B} \tag{A.185}$$

$$\mathcal{E}_0^{IP} > \mathcal{E}_0^{IP-B}. \tag{A.186}$$

The results for CA_0 directly follow, which shows the Proposition.

A.8. Proof of Proposition 6

We first describe the model variant in which households can save or borrow in foreign currency and the government can impose a capital control tax.

Households. Households can save or borrow in foreign currency at R^* and the government imposes a time-varying capital control tax τ_t. The household budget constraint expressed in domestic currency is given by

$$P_{Tt}C_{Tt} + P_{Nt}C_{Nt} + \frac{1}{(1+\tau_t)}\mathcal{E}_t B_{t+1}^* = W_t L_t + \Pi_t + T_t + \mathcal{E}_t R^* B_t^*, \tag{A.187}$$

where B_{t+1}^* are the foreign currency bonds purchased in t that mature in $t+1$ and R^* is the foreign currency interest rate. The other elements of the household problem are as in the baseline model.

The household's problem is to choose allocations $\{C_t, C_{Tt}, C_{Nt}, L_t, B_{t+1}^*\}_{t=0}^{\infty}$ that maximize utility (1), subject to the aggregation technology (2), the sequence of budget constraints (A.187), given a sequence of prices, profits and transfers, and an initial level of bonds B_0^*.

The first-order conditions that characterize the solution to the household's problem are

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} = p_t \left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}, \tag{A.188}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_t}{P_{Tt}} = \phi L_t^{\nu}, \tag{A.189}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R^*(1+\tau_t)\frac{P_{Tt}}{P_{Tt+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}. \tag{A.190}$$

Firms. As in Section 2.

Government. The government budget balances each period with revenue from the capital control tax τ_t distributed lump-sum to the household

$$-\frac{\tau_t}{(1+\tau_t)}\mathcal{E}_t B_{t+1}^* = T_t. \tag{A.191}$$

Rest of the world. The domestic economy consumes c_{Tt} and produces $A_t L_{Tt}^{\alpha+\gamma_t}$ of the tradable good, and saves B_{t+1}^* abroad at the real interest rate R^*. The value in domestic currency must be equal, giving the balance of payments

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_t} = R^* B_t^* - B_{t+1}^*. \tag{A.192}$$

As in the baseline model, we assume the law of one price holds for tradable goods and normalize the foreign currency price of tradables, so that $P_{Tt} = \mathcal{E}_t$.

We now show the proposition for the model.

Competitive Equilibrium. The competitive equilibrium allocation $\{C_{Tt}, C_{Nt}, L_{Tt}, L_{Nt}, B_{t+1}^*\}_{t=0}^{\infty}$ is characterized by combining households' and firms' optimality conditions and market clearing to give

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}}, \tag{A.193}$$

$$\frac{\phi\left(L_{Tt}+L_{Nt}\right)^{\nu}}{\left(\omega/C_{Tt}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.194}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R^*(1+\tau_t)\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}, \tag{A.195}$$

$$C_{Nt} = A_t L_{Nt}^{\alpha}, \tag{A.196}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_t} = R^* B_t^* - B_{t+1}^*. \tag{A.197}$$

By setting the sequence of capital control taxes

$$\tau_t = \frac{\theta(\mathbf{x}_{t+1}, \gamma_{Tt+1})}{\theta(\mathbf{x}_t, \gamma_{Tt})} - 1, \tag{A.198}$$

where $\mathbf{x}_t \equiv \{C_t^{IP}, C_{Tt}^{IP}, C_{Nt}^{IP}, L_t^{IP}, L_{Tt}^{IP}, L_{Nt}^{IP}, A_t\}$ is the optimal exchange rate industrial policy allocation, the competitive equilibrium conditions (A.193)–(A.197) are equivalent to the XR-IP and, therefore, attain the some allocation.

A.9. Proof of Proposition 7

The first-order conditions that characterize the solution to the household's problem are

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} = p_t\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}, \tag{A.199}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\frac{W_{Tt}}{P_{Tt}} = \phi_T L_{Tt}^{\nu}, \tag{A.200}$$

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\frac{W_{Nt}}{P_{Nt}} = \phi_N L_{Nt}^{\nu}, \tag{A.201}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} = \beta R_{t+1}\frac{P_{Tt}}{P_{Tt+1}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}. \tag{A.202}$$

Note we can also equate the marginal utility of C_{Nt} with the marginal disutility of L_{Tt}

which gives

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma} \frac{W_{Tt}}{P_{Nt}} = \phi_T L_{Tt}^{\nu}, \tag{A.203}$$

$$\Rightarrow \frac{W_{Tt}}{W_{Nt}} = \frac{\phi_T L_{Tt}^{\nu}}{\phi_N L_{Nt}^{\nu}}. \tag{A.204}$$

Firms profits are given by $\Pi_{it} = P_{it} A_t l_{it}^{\alpha} L_{it}^{\gamma_{it}} - W_{it} l_{it}$, which gives rise to the following aggregate labor demand

$$\alpha A_t L_{it}^{\alpha+\gamma_{it}-1} = W_{it}/P_{it}. \tag{A.205}$$

For the competitive equilibrium then

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = p_t, \tag{A.206}$$

$$\frac{\phi_T L_{Tt}^{\nu}}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1}, \tag{A.207}$$

$$\frac{\phi_N L_{Nt}^{\nu}}{((1-\omega)/C_{Nt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Nt}^{\alpha-1}. \tag{A.208}$$

From combining the latter two equations using the nontradable market clearing condition $C_{Nt} = A_t L_{Nt}^{\alpha}$

$$\frac{\phi_T}{(\omega/C_{Tt})^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}, \tag{A.209}$$

$$\frac{\phi_N}{(1-\omega)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t^{1-\frac{1}{\eta}} L_{Nt}^{\alpha-1-\frac{\alpha}{\eta}-\nu}. \tag{A.210}$$

These will serve as implementability conditions for the XR-IP problem.

For Cole and Obstfeld (1991) preferences ($\sigma = \eta = 1$) the second equation is given by

$$\frac{\phi_N}{(1-\omega)^{\frac{1}{\eta}}} = \alpha L_{Nt}^{-1-\nu} \tag{A.211}$$

$$L_{Nt} = \left[\frac{\alpha\,(1-\omega)^{\frac{1}{\eta}}}{\phi_N}\right]^{\frac{1}{1+\nu}}, \tag{A.212}$$

so the nontradable block $\{C_{Nt}, L_{Nt}\}$ is exogenous.

The tradables constraint is given by

$$\frac{\phi_T}{(\omega/C_{Tt})} = \alpha A_t L_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}. \tag{A.213}$$

The FB allocation is characterized by

$$\frac{\phi_T}{\left(\omega/\tilde{C}_{Tt}\right)} = (\alpha+\gamma_{Tt}) A_t \tilde{L}_{Tt}^{\alpha+\gamma_{Tt}-1-\nu}. \tag{A.214}$$

We now derive the approximate problem as in Lemma 1, where in the BT allocation

$$\overline{C}_T = \overline{A}\ \overline{L}_T^{\alpha+\gamma}, \tag{A.215}$$

$$\frac{\phi_T}{\left(\omega/\overline{C}_T\right)} = (\alpha+\gamma)\overline{A}\ \overline{L}_T^{\alpha+\gamma-1-\nu}. \tag{A.216}$$

Therefore in the BT allocation

$$\overline{L}_T = \left[\frac{\omega(\alpha+\gamma)}{\phi_T}\right]^{\frac{1}{1+\nu}} \tag{A.217}$$

The first-order loglinear approximation of the MRS = MRT constraint is

$$z_t = \psi_t + (\alpha+\gamma-1-\nu)x_t. \tag{A.218}$$

The welfare function is

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega \log C_{Tt} - \phi_T \frac{L_{Tt}^{1+\nu}}{1+\nu} \right], \tag{A.219}$$

where the other terms are exogenous. A second-order Taylor expansion for the tradable labor term around the BT

$$-\phi_T \frac{L_{Tt}^{1+\nu}}{1+\nu} = -\phi_T \frac{\overline{L}_T^{1+\nu}}{1+\nu} e^{(1+\nu)l_{Tt}} = -\phi_T \frac{\overline{L}_T^{1+\nu}}{1+\nu} - \phi_T \overline{L}_T^{1+\nu} l_{Tt} - \frac{1}{2}\phi_T(1+\nu)\overline{L}_T^{1+\nu} l_{Tt}^2 \tag{A.220}$$

Therefore, welfare in terms of deviations and ignoring terms independent of c_{Tt} and l_{Tt}

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\omega c_{Tt} - \phi_T \overline{L}_T^{1+\nu} (l_{Tt} - \frac{1}{2}(1+\nu) l_{Tt}^2) \right], \tag{A.221}$$

similarly for the FB $\tilde{\mathbb{W}}_0$.

Taking a second-order approximation of the resource constraint gives

$$\begin{aligned}
\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = & -\sum_{t=0}^{\infty} \beta^t \omega \left[\frac{1}{2} c_{Tt}^2 - (\alpha+\gamma) l_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 l_{Tt}^2 \right] \\
& + \sum_{t=0}^{\infty} \beta^t \omega \left[\frac{1}{2} \tilde{c}_{Tt}^2 - (\alpha+\gamma) \tilde{l}_{Tt} - \frac{1}{2}(\alpha+\gamma)^2 \tilde{l}_{Tt}^2 \right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\phi_T \overline{L}_T^{1+\nu} (l_{Tt} - \frac{1}{2}(1+\nu) l_{Tt}^2) \right] + \sum_{t=0}^{\infty} \beta^t \left[\phi_T \overline{L}_T^{1+\nu} (\tilde{l}_{Tt} - \frac{1}{2}(1+\nu) \tilde{l}_{Tt}^2) \right]
\end{aligned} \tag{A.222}$$

$$\begin{aligned}
= & -\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \omega (c_{Tt} - \tilde{c}_{Tt})^2 + \omega \tilde{c}_{Tt} (c_{Tt} - \tilde{c}_{Tt}) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \omega (\alpha+\gamma)^2 (l_{Tt} - \tilde{l}_{Tt})^2 - \omega (\alpha+\gamma)^2 \tilde{l}_{Tt} (l_{Tt} - \tilde{l}_{Tt}) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \phi_T (1+\nu) \overline{L}_T^{1+\nu} (l_{Tt} - \tilde{l}_{Tt})^2 - \phi_T (1+\nu) \overline{L}_T^{1+\nu} \tilde{l}_{Tt} (l_{Tt} - \tilde{l}_{Tt}) \right]
\end{aligned} \tag{A.223}$$

$$= -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\omega z_t^2 + \left[\omega (\alpha+\gamma)^2 + (1+\nu) \omega (\alpha+\gamma) \right] x_t^2 \right], \tag{A.224}$$

following the same steps as Lemma 1 and using that $\phi_T \overline{L}_T^{1+\nu} = \omega(\alpha+\gamma)$.

Therefore, the approximate XR-IP problem relative to the FB is

$$\max_{\{z_t, x_t, \check{f}^*_{t+1}\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega z_t^2 + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right]x_t^2\right]$$

$$s.t. \quad z_t = \psi_t + (\alpha+\gamma-1-\nu)x_t, \tag{A.225}$$

$$z_t - (\alpha+\gamma)x_t = R^*\check{f}^*_t - \check{f}^*_{t+1}, \tag{A.226}$$

$$\check{f}^*_0 = 0. \tag{A.227}$$

Combining the constraints and iterating gives

$$\sum_{t=0}^{\infty}\beta^t\left[\psi_t - (1+\nu)x_t\right] = 0, \tag{A.228}$$

imposing the transversality condition for net foreign assets $\lim_{s\to\infty}\beta^s f^*_{t+s} = 0$.

We can solve the XR-IP problem for x_t

$$\max_{\{x_t\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\omega(\psi_t + (\alpha+\gamma-1-\nu)x_t)^2 + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right]x_t^2\right]$$

$$s.t. \quad \sum_{t=0}^{\infty}\beta^t\left[\psi_t - (1+\nu)x_t\right] = 0. \tag{A.229}$$

Let λ be the multiplier on the lifetime resource constraint. The FOC for x_t is

$$-\beta^t\left[\omega(\psi_t + (\alpha+\gamma-1-\nu)x_t)(\alpha+\gamma-1-\nu) + \left[\omega(\alpha+\gamma)^2 + (1+\nu)\omega(\alpha+\gamma)\right]x_t\right] = \beta^t(1+\nu)\lambda. \tag{A.230}$$

We get a loglinear Euler equation to characterize the XR-IP solution

$$\psi_t(\alpha+\gamma-1-\nu) + \left[(\alpha+\gamma-1-\nu)^2 + (\alpha+\gamma)^2 + (1+\nu)(\alpha+\gamma)\right]x_t^{IP}$$

$$= \psi_{t+1}(\alpha+\gamma-1-\nu) + \left[(\alpha+\gamma-1-\nu)^2 + (\alpha+\gamma)^2 + (1+\nu)(\alpha+\gamma)\right]x_{t+1}^{IP} \tag{A.231}$$

$$\psi_t + \left[(\alpha+\gamma-1-\nu) + c\right]x_t^{IP} = \psi_{t+1} + \left[(\alpha+\gamma-1-\nu) + c\right]x_{t+1}^{IP}, \tag{A.232}$$

where $c = \frac{(\alpha+\gamma)^2+(1+\nu)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)} < 0$. Therefore,

$$x_t^{IP} = \frac{(\psi_0 - \psi_t)}{(\alpha + \gamma - 1 - \nu + c)} + x_0^{IP}. \tag{A.233}$$

The LF-CE is characterized by

$$\psi_t + (\alpha + \gamma - 1 - \nu)x_t^{CE} = \psi_{t+1} + (\alpha + \gamma - 1 - \nu)x_{t+1}^{CE}, \tag{A.234}$$

and

$$x_t^{CE} = \frac{(\psi_0 - \psi_t)}{(\alpha + \gamma - 1 - \nu)} + x_0^{CE}. \tag{A.235}$$

Substituting into the lifetime resource constraint

$$\sum_{t=0}^{\infty} \beta^t x_t = \frac{1}{1+\nu} \sum_{t=0}^{\infty} \beta^t \psi_t \tag{A.236}$$

$$\frac{x_0^{IP}}{1-\beta} + \sum_{t=1}^{\infty} \beta^t \frac{(\psi_0 - \psi_t)}{(\alpha + \gamma - 1 - \nu + c)} = \frac{1}{1+\nu} \sum_{t=0}^{\infty} \beta^t \psi_t \tag{A.237}$$

$$x_0^{IP} = \frac{(1-\beta)}{(1+\nu)}\psi_0 - \frac{\beta\psi_0}{(\alpha + \gamma - 1 - \nu + c)} + \frac{(1-\beta)}{(1+\nu)} \frac{(\alpha + \gamma + c)}{(\alpha + \gamma - 1 - \nu + c)} \sum_{t=1}^{\infty} \beta^t \psi_t. \tag{A.238}$$

Similarly for the LF-CE gives

$$x_0^{CE} = \frac{(1-\beta)}{(1+\nu)}\psi_0 - \frac{\beta\psi_0}{(\alpha + \gamma - 1 - \nu)} + \frac{(1-\beta)}{(1+\nu)} \frac{(\alpha + \gamma)}{(\alpha + \gamma - 1 - \nu)} \sum_{t=1}^{\infty} \beta^t \psi_t. \tag{A.239}$$

Therefore

$$x_0^{IP} - x_0^{CE} = \frac{\beta\psi_0}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c + \frac{(1-\beta)\sum_{t=1}^{\infty}\beta^t\psi_t}{(1+\nu)(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}\left[c(\alpha+\gamma-1-\nu)-(\alpha+\gamma)c\right] \tag{A.240}$$

$$= \underbrace{\frac{\beta}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c}_{<0} \; \underbrace{\left[\psi_0 - \frac{(1-\beta)}{\beta}\sum_{t=1}^{\infty}\beta^t\psi_t\right]}_{<0} \tag{A.241}$$

$$> \frac{\beta}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c\left[\psi_0 - \frac{(1-\beta)}{\beta}\frac{\beta}{1-\beta}\psi_0\right] \tag{A.242}$$

$$= 0. \tag{A.243}$$

To determine the sign of $\frac{\partial(x_0^{IP}-x_0^{CE})}{\partial\nu}$ observe that

$$\frac{\partial(x_0^{IP}-x_0^{CE})}{\partial\nu} = \underbrace{\frac{\partial Q}{\partial\nu}}_{>0}\underbrace{\left[\psi_0 - \frac{(1-\beta)}{\beta}\sum_{t=1}^{\infty}\beta^t\psi_t\right]}_{<0} \tag{A.244}$$

$$< 0 \tag{A.245}$$

$$\text{where } Q \equiv \frac{\beta}{(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)}c. \tag{A.246}$$

To see that $\frac{\partial Q}{\partial\nu} > 0$ note that

$$\frac{\partial c}{\partial\nu} = \frac{(\alpha+\gamma)(\alpha+\gamma-1-\nu)+[(\alpha+\gamma)^2+(1+\nu)(\alpha+\gamma)]}{(\alpha+\gamma-1-\nu)^2} \tag{A.247}$$

$$= \frac{2(\alpha+\gamma)^2}{(\alpha+\gamma-1-\nu)^2}. \tag{A.248}$$

Then

$$\frac{\partial Q}{\partial \nu} = \frac{\beta}{[(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)]^2}\times \left[\frac{\partial c}{\partial \nu}(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu) - c\left[(\alpha+\gamma-1-\nu)(-1+\frac{\partial c}{\partial \nu}) - (\alpha+\gamma-1-\nu+c)\right]\right] \tag{A.249}$$

$$= \underbrace{\frac{\beta}{[(\alpha+\gamma-1-\nu+c)(\alpha+\gamma-1-\nu)]^2}}_{>0}\times \underbrace{\left[2(\alpha+\gamma)^2 + c(\alpha+\gamma-1-\nu) + c(\alpha+\gamma-1-\nu+c)\right]}_{>0} \tag{A.250}$$

$$> 0 \tag{A.251}$$

This implies $(x_0^{IP} - x_0^{CE}) > 0$, i.e. the approximation of $\frac{L_{T0}^{IP}}{L_{T0}^{CE}} > 1$, is decreasing in ν. The Frisch elasticity of labor supply is ν^{-1}, so if the labor supply becomes more elastic $\downarrow \nu$, then $\frac{L_{T0}^{IP}}{L_{T0}^{CE}} \uparrow$.

For both the XR-IP and LF-CE

$$C_{Tt} = \frac{\omega\alpha A_t}{\phi}\frac{1}{L_{Tt}^{1-\alpha-\gamma_{Tt}}}, \tag{A.252}$$

$$\mathcal{E}_t = \frac{\omega}{1-\omega}\frac{C_{Nt}}{C_{Tt}}, \tag{A.253}$$

where C_{Nt} is coincides for the XR-IP and LF-CE. The results for L_{T0} then imply that

$$\frac{\mathcal{E}_0^{IP}}{\mathcal{E}_0^{CE}} > 1, \tag{A.254}$$

$$\frac{C_{T0}^{CE}}{C_{T0}^{IP}} > 1, \tag{A.255}$$

$$\frac{CA_0^{IP}}{CA_0^{CE}} > 1, \tag{A.256}$$

$$\frac{F_1^{*IP}}{F_1^{*CE}} > 1, \tag{A.257}$$

are decreasing in ν which shows the Proposition.

A.10. Proof of Proposition 8

In the economy with 2 traded goods sectors, the household utility function is:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \phi_{T1} \frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi_{T2} \frac{L_{T2t}^{1+\nu_2}}{1+\nu_2} - \phi_N \frac{L_{Nt}^{1+\nu_N}}{1+\nu_N} \right], \tag{A.258}$$

where the budget constraint in domestic currency is given by

$$P_{T1t}C_{T1t} + P_{T2t}C_{T2t} + P_{Nt}C_{Nt} + B_{t+1} = W_{T1t}L_{T1t} + W_{T2t}L_{T2t} + W_{Nt}L_{Nt} + \Pi_t + T_t + R_t B_t, \tag{A.259}$$

with aggregate consumption

$$C_t = \left[\omega^{\frac{1}{\eta}} (C_{Tt})^{1-\frac{1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} (C_{Nt})^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{A.260}$$

where the tradable good is a CES over varieties $1, 2$

$$C_{Tt} = \left[(1/2)^{\frac{1}{\rho}} (C_{T1t})^{1-\frac{1}{\rho}} + (1/2)^{\frac{1}{\rho}} (C_{T2t})^{1-\frac{1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{A.261}$$

with elasticity of substitution $\rho > 0$ and for $\rho = 1$

$$C_{Tt} = C_{T1t}^{1/2} C_{T2t}^{1/2}. \tag{A.262}$$

Define the aggregate price of tradables P_{Tt}

$$P_{Tt} = \left((1/2) P_{T1t}^{1-\rho} + (1/2) P_{T2t}^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{A.263}$$

and for $\rho = 1$, $P_{Tt} = P_{T1t}^{1/2} P_{T2t}^{1/2}$, and similarly for the foreign currency price P_{Tt}^*.

Assume the law of one price holds for each variety of tradable good $P_{T1t} = \mathcal{E}_t P_{T1t}^*$ and $P_{T2t} = \mathcal{E}_t P_{T2t}^*$.

The first-order conditions that characterize the solution to the household's problem are

$$\left(\frac{\omega}{2C_{T1t}}\right)^{\frac{1}{\rho}} = \frac{P_{T1t}}{P_{T2t}}\left(\frac{\omega}{2C_{T2t}}\right)^{\frac{1}{\rho}} \tag{A.264}$$

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{P_{Nt}}{P_{T1t}}\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}}\left(\frac{C_{Tt}}{2C_{T1t}}\right)^{\frac{1}{\rho}}, \tag{A.265}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt}}{2C_{T1t}}\right)^{\frac{1}{\rho}}\frac{W_{T1t}}{P_{T1t}} = \phi_{T1}L_{T1t}^{\nu_1}, \tag{A.266}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt}}{2C_{T2t}}\right)^{\frac{1}{\rho}}\frac{W_{T2t}}{P_{T2t}} = \phi_{T2}L_{T2t}^{\nu_2}, \tag{A.267}$$

$$\left(\frac{1-\omega}{C_{Nt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\frac{W_{Nt}}{P_{Nt}} = \phi_N L_{Nt}^{\nu_N}, \tag{A.268}$$

$$\left(\frac{\omega}{C_{Tt}}\right)^{\frac{1}{\eta}} C_t^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt}}{2C_{T1t}}\right)^{\frac{1}{\rho}} = \beta R_{t+1}\frac{P_{T1t}}{P_{T1t+1}}\left(\frac{\omega}{C_{Tt+1}}\right)^{\frac{1}{\eta}} C_{t+1}^{\frac{1}{\eta}-\sigma}\left(\frac{C_{Tt+1}}{2C_{T1t+1}}\right)^{\frac{1}{\rho}}. \tag{A.269}$$

Tradable sector $1,2$ and nontradable firms choose labor to maximize their profits, which gives rise to the following aggregate labor demand

$$\alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1} = W_{T1t}/P_{T1t} \tag{A.270}$$

$$\alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1} = W_{T2t}/P_{T2t} \tag{A.271}$$

$$\alpha A_t L_{Nt}^{\alpha-1} = W_{Nt}/P_{Nt}. \tag{A.272}$$

Normalize $P^*_{T1t} = 1$, which gives $P_{T1t} = \mathcal{E}_t$. Let $p^*_t \equiv \frac{P^*_{T2t}}{P^*_{T1t}} = \frac{P_{T2t}}{P_{T1t}}$ and $p_{1t} \equiv \frac{P_{Nt}}{P_{T1t}}$. Normalize $P_{Nt} \equiv 1$ then $p_{1t} = \mathcal{E}_t^{-1}$.

For the competitive equilibrium (CE) then

$$\left(\frac{C_{T1t}}{C_{T2t}}\right)^{\frac{1}{\rho}} = p_t^*, \tag{A.273}$$

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}}\left(\frac{2C_{T1t}}{C_{Tt}}\right)^{\frac{1}{\rho}} = p_{1t}, \tag{A.274}$$

$$\frac{\phi_{T1}L_{T1t}^{\nu_1}}{(\omega/C_{Tt})^{\frac{1}{\eta}}\,C_t^{\frac{1}{\eta}-\sigma}\,(C_{Tt}/(2C_{T1t}))^{\frac{1}{\rho}}} = \alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1}, \tag{A.275}$$

$$\frac{\phi_{T2}L_{T2t}^{\nu_2}}{(\omega/C_{Tt})^{\frac{1}{\eta}}\,C_t^{\frac{1}{\eta}-\sigma}\,(C_{Tt}/(2C_{T2t}))^{\frac{1}{\rho}}} = \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1}, \tag{A.276}$$

$$\frac{\phi_N L_{Nt}^{\nu_N}}{((1-\omega)/C_{Nt})^{\frac{1}{\eta}}\,C_t^{\frac{1}{\eta}-\sigma}} = \alpha A_t L_{Nt}^{\alpha-1}. \tag{A.277}$$

For Cole and Obstfeld (1991) preferences ($\sigma = \eta = 1$) the nontradable block is exogenous, and $\rho = 1$. The CE conditions simplify to

$$\left(\frac{C_{T1t}}{C_{T2t}}\right) = p_t^*, \tag{A.278}$$

$$\left(\frac{2(1-\omega)}{\omega}\frac{C_{T1t}}{C_{Nt}}\right) = p_{1t}, \tag{A.279}$$

$$\frac{\phi_{T1}}{(\omega/(2C_{T1t}))} = \alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1-\nu_1}, \tag{A.280}$$

$$\frac{\phi_{T2}}{(\omega/(2C_{T2t}))} = \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1-\nu_2}, \tag{A.281}$$

$$\left(\frac{\omega}{2C_{T1t}}\right) = \beta R_{t+1}\frac{P_{T1t}}{P_{T1t+1}}\left(\frac{\omega}{2C_{T1t+1}}\right). \tag{A.282}$$

The balance of payments is given by substituting firm profits, nontradable market clearing and the government BC into the household BC

$$P_{T1t}C_{T1t} + P_{T2t}C_{T2t} = P_{T1t}A_t L_{T1t}^{\alpha+\gamma_{T1t}} + P_{T2t}A_t L_{T2t}^{\alpha+\gamma_{T2t}} + \mathcal{E}_t R^* F_t^* - \mathcal{E}_t F_{t+1}^* \tag{A.283}$$

$$C_{T1t} + p_t^* C_{T2t} = A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p_t^* A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F_t^* - F_{t+1}^*, \tag{A.284}$$

using that $P_{T1t} = \mathcal{E}_t$.

For the laissez-faire (LF) CE, combining

$$\left(\frac{2(1-\omega)}{\omega}\frac{C_{T1t}}{C_{Nt}}\right) = P_{T1t}^{-1}, \tag{A.285}$$

$$\left(\frac{\omega}{2C_{T1t}}\right) = \beta R_{t+1}\frac{P_{T1t}}{P_{T1t+1}}\left(\frac{\omega}{2C_{T1t+1}}\right), \tag{A.286}$$

gives

$$\beta R_{t+1} = \frac{C_{Nt+1}}{C_{Nt}} = \frac{A_{t+1}}{A_t}. \tag{A.287}$$

Therefore, from UIP and $\beta R^* = 1$

$$R_{t+1} = R^*\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \tag{A.288}$$

$$\beta R_{t+1} = \beta R^*\frac{P_{T1t+1}}{P_{T1t}} \tag{A.289}$$

$$\beta R_{t+1}\frac{P_{T1t}}{P_{T1t+1}} = 1. \tag{A.290}$$

For the LF CE then can solve for the 5 variables $\{C_{T1t}, C_{T2t}, L_{T1t}, L_{T2t}, F^*_{t+1}\}$ with the following 5 equations

$$\left(\frac{C_{T1t}}{C_{T2t}}\right) = p_t^*, \tag{A.291}$$

$$\frac{\phi_{T1}}{(\omega/(2C_{T1t}))} = \alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1-\nu_1}, \tag{A.292}$$

$$\frac{\phi_{T2}}{(\omega/(2C_{T2t}))} = \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1-\nu_2}, \tag{A.293}$$

$$C_{T1t} + p_t^* C_{T2t} = A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p_t^* A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F_t^* - F_{t+1}^*, \tag{A.294}$$

$$\left(\frac{\omega}{2C_{T1t}}\right) = \left(\frac{\omega}{2C_{T1t+1}}\right). \tag{A.295}$$

Equations (A.291), (A.292), (A.293) will serve as implementability conditions for the

XR-IP problem, given by

$$\max_{\{C_{T1t},C_{T2t},L_{T1t},L_{T2t},F^*_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2}\log C_{T1t} + \frac{\omega}{2}\log C_{T2t} - \phi_{T1}\frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi_{T2}\frac{L_{T2t}^{1+\nu_2}}{1+\nu_2}\right] \quad \text{subject to} \tag{A.296}$$

$$\begin{aligned}
\left(\frac{C_{T1t}}{C_{T2t}}\right) &= p_t^*,\\
\frac{\phi_{T1}}{(\omega/(2C_{T1t}))} &= \alpha A_t L_{T1t}^{\alpha+\gamma_{T1t}-1-\nu_1},\\
\frac{\phi_{T2}}{(\omega/(2C_{T2t}))} &= \alpha A_t L_{T2t}^{\alpha+\gamma_{T2t}-1-\nu_2},\\
C_{T1t} + p_t^* C_{T2t} &= A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p_t^* A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F_t^* - F_{t+1}^*.
\end{aligned}$$

Note that the first best (FB) problem

$$\max_{\{C_{T1t},C_{T2t},L_{T1t},L_{T2t},F^*_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2}\log C_{T1t} + \frac{\omega}{2}\log C_{T2t} - \phi_{T1}\frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi_{T2}\frac{L_{T2t}^{1+\nu_2}}{1+\nu_2}\right] \quad \text{subject to} \tag{A.297}$$

$$C_{T1t} + p_t^* C_{T2t} = A_t L_{T1t}^{\alpha+\gamma_{T1t}} + p_t^* A_t L_{T2t}^{\alpha+\gamma_{T2t}} + R^* F_t^* - F_{t+1}^*,$$

gives the optimality condition in the FB

$$\left(\frac{\tilde{C}_{T1t}}{\tilde{C}_{T2t}}\right) = p_t^*. \tag{A.298}$$

We now derive the approximate problem as in Lemma 1, where in the BT allocation

$$\overline{C}_{T1} = \overline{A}\ \overline{L}_{T1}^{\alpha+\gamma_1} \tag{A.299}$$

$$\frac{\phi_{T1}}{\left(\omega/(2\overline{C}_{T1})\right)} = (\alpha+\gamma_1)\overline{A}\ \overline{L}_{T1}^{\alpha+\gamma_1-1-\nu_1} \tag{A.300}$$

$$\overline{C}_{T2} = \overline{A}\ \overline{L}_{T2}^{\alpha+\gamma_2} \tag{A.301}$$

$$\frac{\phi_{T2}}{\left(\omega/(2\overline{C}_{T2})\right)} = (\alpha+\gamma_2)\overline{A}\ \overline{L}_{T2}^{\alpha+\gamma_2-1-\nu_2}. \tag{A.302}$$

Therefore in the BT allocation

$$\overline{L}_{T1}^{1+\nu_1} = \frac{\omega(\alpha+\gamma_1)}{2\phi_{T1}} \tag{A.303}$$

$$\overline{L}_{T2}^{1+\nu_2} = \frac{\omega(\alpha+\gamma_2)}{2\phi_{T2}}. \tag{A.304}$$

For the constraint on consumption across tradable sectors for the XR-IP and FB

$$\log C_{T1t} - \log C_{T2t} = \log \tilde{C}_{T1t} - \log \tilde{C}_{T2t} \tag{A.305}$$

$$z_{1t} = z_{2t}, \tag{A.306}$$

where $z_{jt} \equiv \log C_{Tjt} - \log \tilde{C}_{Tjt}$.

The first-order loglinear approximation of the MRS = MRT constraint in each sector is

$$z_{1t} = \psi_{1t} + (\alpha + \gamma_1 - 1 - \nu_1)x_{1t}, \tag{A.307}$$

$$z_{2t} = \psi_{2t} + (\alpha + \gamma_2 - 1 - \nu_2)x_{2t}, \tag{A.308}$$

where $\psi_{jt} \equiv \log \alpha - \log(\alpha + \gamma_{Tjt}) \leq 0$.

The welfare function is

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2} \log C_{T1t} + \frac{\omega}{2} \log C_{T2t} - \phi_{T1} \frac{L_{T1t}^{1+\nu_1}}{1+\nu_1} - \phi_{T2} \frac{L_{T2t}^{1+\nu_2}}{1+\nu_2} \right], \tag{A.309}$$

where other terms are exogenous. The tradable consumption terms relative to the BT are

$$\frac{\omega}{2} \log C_{T1t} + \frac{\omega}{2} \log C_{T2t} = \frac{\omega}{2} c_{T1t} + \frac{\omega}{2} c_{T2t} + \frac{\omega}{2} \log \overline{C}_{T1} + \frac{\omega}{2} \log \overline{C}_{T2}, \tag{A.310}$$

where $c_{Tjt} \equiv \log C_{Tjt} - \log \overline{C}_{Tj}$. A second-order Taylor expansion for the tradable labor term around the BT

$$-\phi_{T1}\frac{L_{T1t}^{1+\nu_1}}{1+\nu_1}-\phi_{T2}\frac{L_{T2t}^{1+\nu_2}}{1+\nu_2}= -\phi_{T1}\frac{\overline{L}_{T1}^{1+\nu_1}}{1+\nu_1}-\phi_{T1}\overline{L}_{T1}^{1+\nu_1}l_{T1t}-\frac{1}{2}\phi_{T1}(1+\nu_1)\overline{L}_{T1}^{1+\nu_1}l_{T1t}^2$$
$$-\phi_{T2}\frac{\overline{L}_{T2}^{1+\nu_2}}{1+\nu_2}-\phi_{T2}\overline{L}_{T2}^{1+\nu_2}l_{T2t}-\frac{1}{2}\phi_{T2}(1+\nu_2)\overline{L}_{T2}^{1+\nu_2}l_{T2t}^2 \quad \text{(A.311)}$$

Therefore, substituting $\phi_{Tj}\overline{L}_{Tj}^{1+\nu_j} = \omega(\alpha+\gamma_j)/2$, welfare in terms of deviations and ignoring terms independent of c_{Tjt} and l_{Tjt}

$$\mathbb{W}_0=\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega}{2}c_{T1t}+\frac{\omega}{2}c_{T2t}-\frac{\omega}{2}(\alpha+\gamma_1)(l_{T1t}-\frac{1}{2}(1+\nu_1)l_{T1t}^2)-\frac{\omega}{2}(\alpha+\gamma_2)(l_{T2t}-\frac{1}{2}(1+\nu_2)l_{T2t}^2)\right], \quad \text{(A.312)}$$

and similarly for the FB $\tilde{\mathbb{W}}_0$.

To derive the loglinear balance of payments constraint relative to the BT

$$C_{T1t}+p_t^*C_{T2t}=A_tL_{T1t}^{\alpha+\gamma_{T1t}}+p_t^*A_tL_{T2t}^{\alpha+\gamma_{T2t}}+R^*F_t^*-F_{t+1}^* \quad \text{(A.313)}$$

$$2\frac{C_{T1t}}{\overline{C}_{T1}}-\frac{A_t}{\overline{A}}\frac{L_{T1t}^{\alpha+\gamma_{T1t}}}{\overline{L}_{T1}^{\alpha+\gamma_1}}-\frac{C_{T1t}/\overline{C}_{T1}}{C_{T2t}/\overline{C}_{T2}}\frac{A_t}{\overline{A}}\frac{L_{T2t}^{\alpha+\gamma_{T2t}}}{\overline{L}_{T2}^{\alpha+\gamma_2}}=R^*\frac{F_t^*}{\overline{Y}_{T1}}-\frac{F_{t+1}^*}{\overline{Y}_{T1}} \quad \text{(A.314)}$$

$$2e^{c_{T1t}}-e^{a_t+(\alpha+\gamma_{T1t})l_{T1t}+(\gamma_{T1t}-\gamma_1)\log\overline{L}_{T1}}-e^{a_t+c_{T1t}-c_{T2t}+(\alpha+\gamma_{T2t})l_{T2t}+(\gamma_{T2t}-\gamma_2)\log\overline{L}_{T2}}=R^*f_t^*+f_{t+1}^*, \quad \text{(A.315)}$$

where $f_t^*\equiv\frac{F_t^*}{\overline{Y}_{T1}}$. A first-order approximation of the LHS around the BT gives

$$c_{T1t}+c_{T2t}-2a_t-(\alpha+\gamma_1)l_{T1t}-(\gamma_{T1t}-\gamma_1)\log\overline{L}_{T1}-(\alpha+\gamma_2)l_{T2t}-(\gamma_{T2t}-\gamma_2)\log\overline{L}_{T2}=R^*f_t^*-f_{t+1}^*, \quad \text{(A.316)}$$

and similarly for the FB. Therefore

$$z_{1t}+z_{2t}-(\alpha+\gamma_1)x_{1t}-(\alpha+\gamma_2)x_{2t}=R^*\check{f}_t^*-\check{f}_{t+1}^*. \quad \text{(A.317)}$$

For the welfare function, taking a second-order approximation of the resource constraint gives

$$\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2} c_{T1t} + \frac{\omega}{2} c_{T2t} - \frac{\omega}{2}(\alpha+\gamma_1)(l_{T1t} - \frac{1}{2}(1+\nu_1) l_{T1t}^2) - \frac{\omega}{2}(\alpha+\gamma_2)(l_{T2t} - \frac{1}{2}(1+\nu_2) l_{T2t}^2) \right] \tag{A.318}$$

$$- \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega}{2} \tilde{c}_{T1t} + \frac{\omega}{2} \tilde{c}_{T2t} - \frac{\omega}{2}(\alpha+\gamma_1)(\tilde{l}_{T1t} - \frac{1}{2}(1+\nu_1) \tilde{l}_{T1t}^2) - \frac{\omega}{2}(\alpha+\gamma_2)(\tilde{l}_{T2t} - \frac{1}{2}(1+\nu_2) \tilde{l}_{T2t}^2) \right] \tag{A.319}$$

$$\begin{aligned}
= & -\sum_{t=0}^{\infty} \beta^t \frac{\omega}{2} \left[\frac{1}{2}(c_{T1t} - \tilde{c}_{T1t})^2 + \frac{1}{2}(c_{T2t} - \tilde{c}_{T2t})^2 + \tilde{c}_{T1t}(c_{T1t} - \tilde{c}_{T1t}) + \tilde{c}_{T2t}(c_{T2t} - \tilde{c}_{T2t}) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \frac{\omega}{2} \left[\frac{1}{2}(\alpha+\gamma_1)^2 (l_{T1t} - \tilde{l}_{T1t})^2 - (\alpha+\gamma_1)^2 \tilde{l}_{T1t}(l_{T1t} - \tilde{l}_{T1t}) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \frac{\omega}{2} \left[\frac{1}{2}(\alpha+\gamma_2)^2 (l_{T2t} - \tilde{l}_{T2t})^2 - (\alpha+\gamma_2)^2 \tilde{l}_{T2t}(l_{T2t} - \tilde{l}_{T2t}) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \frac{\omega}{2} \left[\frac{1}{2}(\alpha+\gamma_1)(1+\nu_1)(l_{T1t} - \tilde{l}_{T1t})^2 - (\alpha+\gamma_1)(1+\nu_1) \tilde{l}_{T1t}(l_{T1t} - \tilde{l}_{T1t}) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \frac{\omega}{2} \left[\frac{1}{2}(\alpha+\gamma_2)(1+\nu_2)(l_{T2t} - \tilde{l}_{T2t})^2 - (\alpha+\gamma_2)(1+\nu_2) \tilde{l}_{T2t}(l_{T2t} - \tilde{l}_{T2t}) \right] \\
= & -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \omega \Bigg\{ \frac{1}{2} z_{1t}^2 + \frac{1}{2} z_{2t}^2 + \frac{1}{2} \left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1) \right] x_{1t}^2 \\
& + \frac{1}{2} \left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2) \right] x_{2t}^2 \Bigg\},
\end{aligned} \tag{A.320}$$

following the same steps as Lemma 1 and using that $\phi_{Tj} \overline{L}_{Tj}^{1+\nu_j} = \omega(\alpha+\gamma_j)/2$.

Therefore, the approximate XR-IP problem relative to the FB is

$$\max_{\{z_{1t},z_{2t},x_{1t},x_{2t},\check{f}^*_{t+1}\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\omega\Bigg\{\frac{1}{2}z_{1t}^2+\frac{1}{2}z_{2t}^2+\frac{1}{2}\left[(\alpha+\gamma_1)^2+(1+\nu_1)(\alpha+\gamma_1)\right]x_{1t}^2 + \frac{1}{2}\left[(\alpha+\gamma_2)^2+(1+\nu_2)(\alpha+\gamma_2)\right]x_{2t}^2\Bigg\}$$

$$s.t. \quad z_{1t} = z_{2t}, \tag{A.321}$$

$$z_{1t} = \psi_{1t} + (\alpha+\gamma_1-1-\nu_1)x_{1t}, \tag{A.322}$$

$$z_{2t} = \psi_{2t} + (\alpha+\gamma_2-1-\nu_2)x_{2t}, \tag{A.323}$$

$$z_{1t}+z_{2t}-(\alpha+\gamma_1)x_{1t}-(\alpha+\gamma_2)x_{2t} = R^*\check{f}^*_t - \check{f}^*_{t+1}, \tag{A.324}$$

$$\check{f}^*_0 = 0. \tag{A.325}$$

Combining the constraints we get the condition for x_{1t} and x_{2t}

$$\psi_{1t} + (\alpha+\gamma_1-1-\nu_1)x_{1t} = \psi_{2t} + (\alpha+\gamma_2-1-\nu_2)x_{2t} \tag{A.326}$$

$$x_{2t} = U_t + Vx_{1t},\ U_t \equiv \frac{\psi_{1t}-\psi_{2t}}{(\alpha+\gamma_2-1-\nu_2)},\ V \equiv \frac{(\alpha+\gamma_1-1-\nu_1)}{(\alpha+\gamma_2-1-\nu_2)} > 0. \tag{A.327}$$

Combining the constraints and iterating the resource constraint gives

$$\sum_{t=0}^{\infty}\beta^t\left[\psi_{1t}-(1+\nu_1)x_{1t}+\psi_{2t}-(1+\nu_2)x_{2t}\right] = 0. \tag{A.328}$$

We can solve the XR-IP problem for x_{1t}, x_{2t}

$$\max_{\{x_{1t},x_{2t}\}_{t=0}^{\infty}} \quad -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\omega\Bigg\{(\psi_{1t}+(\alpha+\gamma_1-1-\nu_1)x_{1t})^2+\frac{1}{2}\left[(\alpha+\gamma_1)^2+(1+\nu_1)(\alpha+\gamma_1)\right]x_{1t}^2 + \frac{1}{2}\left[(\alpha+\gamma_2)^2+(1+\nu_2)(\alpha+\gamma_2)\right]x_{2t}^2\Bigg\}$$

$$s.t. \quad x_{2t} = U_t + Vx_{1t}$$

$$\sum_{t=0}^{\infty}\beta^t\left[\psi_{1t}-(1+\nu_1)x_{1t}+\psi_{2t}-(1+\nu_2)x_{2t}\right] = 0. \tag{A.329}$$

Substitute for x_{2t}, and let λ be the multiplier on the lifetime resource constraint. The FOC for x_{1t} is

$$-\beta^t \frac{1}{2}\omega\Big[2(\psi_{1t} + (\alpha+\gamma_1-1-\nu_1)x_{1t})(\alpha+\gamma_1-1-\nu_1) + \left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right] x_{1t} \\ + \left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right](U_t + Vx_{1t})V\Big] = \beta^t[1+\nu_1+(1+\nu_2)V]\lambda. \tag{A.330}$$

We get a loglinear Euler equation for x_{1t} to characterize the XR-IP solution

$$\psi_{1t}(\alpha+\gamma_1-1-\nu_1) + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]U_tV \\ + \left\{(\alpha+\gamma_1-1-\nu_1)^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right] + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]V^2\right\}x_{1t} \\ = \psi_{1t+1}(\alpha+\gamma_1-1-\nu_1) + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]U_{t+1}V \\ + \left\{(\alpha+\gamma_1-1-\nu_1)^2 + \frac{1}{2}\left[(\alpha+\gamma_1)^2 + (1+\nu_1)(\alpha+\gamma_1)\right] + \frac{1}{2}\left[(\alpha+\gamma_2)^2 + (1+\nu_2)(\alpha+\gamma_2)\right]V^2\right\}x_{1t+1}. \tag{A.331}$$

Substituting $z_{1t} = \psi_{1t} + (\alpha+\gamma_1-1-\nu_1)x_{1t} = z_{2t}$, the Euler equation simplifies to

$$z_{1t} + \left[\frac{1}{2}D_1 + \frac{1}{2}D_2\right]z_{1t} - \left[\frac{1}{2}D_1\psi_{1t} + \frac{1}{2}D_2\psi_{2t}\right] \\ = z_{1t+1} + \left[\frac{1}{2}D_1 + \frac{1}{2}D_2\right]z_{1t+1} - \left[\frac{1}{2}D_1\psi_{1t+1} + \frac{1}{2}D_2\psi_{2t+1}\right], \tag{A.332}$$

where $D_j = \frac{(\alpha+\gamma_j)^2+(1+\nu_j)(\alpha+\gamma_j)}{(\alpha+\gamma_j-1-\nu_j)^2} > 0$.

The single sector model Euler equation for $z_t = \psi_t + (\alpha+\gamma-1-\nu)x_t$ is

$$z_t + Dz_t - D\psi_t = z_{t+1} + Dz_{t+1} - D\psi_{t+1}, \tag{A.333}$$

where $D = \frac{(\alpha+\gamma)^2+(1+\nu)(\alpha+\gamma)}{(\alpha+\gamma-1-\nu)^2} > 0$.

We can map the multiple sectors model to the form

$$z_t + D^M z_t - D^M \psi_t^M = z_{t+1} + D^M z_{t+1} - D^M \psi_{t+1}^M, \tag{A.334}$$

by setting $D^M = \frac{1}{2}[D_1 + D_2]$. Then to map

$$D^M \psi_t^M = \frac{1}{2} D_1 \psi_{1t} + \frac{1}{2} D_2 \psi_{2t} \tag{A.335}$$

$$\psi_t^M = \frac{1}{D^M}\left[\frac{1}{2} D_1 \psi_{1t} + \frac{1}{2} D_2 \psi_{2t}\right] \tag{A.336}$$

$$= \frac{D_1}{D_1 + D_2}\psi_{1t} + \frac{D_2}{D_1 + D_2}\psi_{2t}. \tag{A.337}$$

Next, to show the connection between z_{1t} and the log deviation of the first-best exchange rate $\epsilon_t \equiv \log(\mathcal{E}_t) - \log(\tilde{\mathcal{E}}_t)$ observe the optimality condition

$$\left(\frac{2(1-\omega)}{\omega}\frac{C_{T1t}}{C_{Nt}}\right) = p_{1t} = \mathcal{E}_t^{-1}. \tag{A.338}$$

Taking logs and combining with the same expression for the FB exchange rate $\tilde{\mathcal{E}}_t$ gives

$$z_{1t} = -\epsilon_t, \tag{A.339}$$

and similarly for the single-tradable-sector model $z_t = -\epsilon_t$. Substituting these into (A.332) and (A.333) shows the proposition.

Finally, we can also show that D_j is increasing in the sector j labor elasticity ν_j^{-1} and externality γ_j observe

$$\frac{\partial D_j}{\partial \nu_j} = \frac{(\alpha+\gamma_j)(\alpha+\gamma_j-1-\nu_j)^2 - [(\alpha+\gamma_j)^2 + (1+\nu_j)(\alpha+\gamma_j)]2(\alpha+\gamma_j-1-\nu_j)(-1)}{(\alpha+\gamma_j-1-\nu_j)^4} \tag{A.340}$$

$$= \frac{(\alpha+\gamma_j)(\alpha+\gamma_j-1-\nu_j)[3(\alpha+\gamma_j)+1+\nu_j]}{(\alpha+\gamma_j-1-\nu_j)^4} \tag{A.341}$$

$$< 0, \tag{A.342}$$

and

$$\frac{\partial D_j}{\partial \gamma_j} = \frac{[2(\alpha+\gamma_j)+(1+\nu_j)](\alpha+\gamma_j-1-\nu_j)^2 - [(\alpha+\gamma_j)^2+(1+\nu_j)(\alpha+\gamma_j)]2(\alpha+\gamma_j-1-\nu_j)}{(\alpha+\gamma_j-1-\nu_j)^4} \tag{A.343}$$

$$= \frac{(\alpha+\gamma_j-1-\nu_j)\left[[2(\alpha+\gamma_j)+(1+\nu_j)](\alpha+\gamma_j-1-\nu_j) - 2[(\alpha+\gamma_j)^2+(1+\nu_j)(\alpha+\gamma_j)]\right]}{(\alpha+\gamma_j-1-\nu_j)^4} \tag{A.344}$$

$$> 0. \tag{A.345}$$

B. Model Extensions

B.1. Nontradable externalities

Consider a generalization of the baseline model in which $\gamma_{Nt} > 0$ for $t \geq 0$. In principle, the presence of time- and sector-specific production externalities can imply different paths of exchange rate policies depending on the relative strength of externalities. However, the following characterizes the optimal policy under Assumption 2 for an arbitrary path of externalities in the nontradable sector, $\gamma_{Nt} > 0$.

Appendix A.5 showed that under Assumption 2 the optimal exchange rate industrial policy depends only on the tradable block of the model for any arbitrary path of γ_{Nt}.

Nontradable production and consumption are independent of the optimal policy, which can be seen from equation (A.35) and nontradable goods market clearing.

In this case, the modified Euler equation for the optimal exchange rate industrial policy takes the form

$$\left(\frac{\omega}{C_{Tt}}\right) = \beta R^* \frac{\theta(L_{Tt+1}, \gamma_{Tt+1})}{\theta(L_{Tt}, \gamma_{Tt})} \left(\frac{\omega}{C_{Tt+1}}\right), \tag{B1}$$

which does not depend on γ_{Nt}.

Therefore, the optimal policy directly follows Proposition 3.

B.2. Fixed Labor Supply

We first characterize the competitive equilibrium and solve the optimal exchange rate industrial policy problem with fixed labor supply, then characterize the solution relative to the laissez-faire competitive equilibrium.

With fixed labor supply of 1 unit by the households, the labor market-clearing condition is $L_{Tt} + L_{Nt} = 1$.

Competitive equilibrium. From the households' first-order conditions for C_{Tt} and C_{Nt}, combined with the firms' optimal labor demand, and nontradable goods market-clearing

$$\left(\frac{(1-\omega)}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}} \tag{B2}$$

$$\left(\frac{(1-\omega)}{\omega}C_{Tt}\right)^{\frac{1}{\eta}} = \frac{A_t^{\frac{1}{\eta}} L_{Tt}^{\alpha+\gamma_{Tt}-1}}{(1-L_{Tt})^{\alpha-1-\frac{\alpha}{\eta}}}, \tag{B3}$$

which characterizes the competitive equilibrium allocation and is the implementability constraint for the optimal exchange rate industrial policy problem. The remaining conditions for the laissez-faire competitive equilibrium are as in the baseline model.

Exchange rate industrial policy. The optimal exchange rate industrial policy problem with fixed labor supply is

$$\max_{\{C_{it},L_{it},F_{t+1}^*\}_{t\geq 0}^{i=T,N}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \quad \text{subject to} \tag{B4}$$

$$\left(\frac{1-\omega}{\omega}\frac{C_{Tt}}{C_{Nt}}\right)^{\frac{1}{\eta}} = \frac{L_{Tt}^{\alpha+\gamma_{Tt}-1}}{L_{Nt}^{\alpha-1}},$$

$$L_{Tt} + L_{Nt} = 1,$$

$$C_{Tt} - A_t L_{Tt}^{\alpha+\gamma_{Tt}} = R^* F_t^* - F_{t+1}^*,$$

the consumption aggregator definition (2), and the market-clearing conditions for nontradable goods (12).

Analytical case. Suppose that the economy starts below the steady-state level of productivity and converges to it in the next period.

After substituting for nontradable consumption and labor, the XR-IP problem is given by

$$\max_{\{C_{Tt},L_{Tt},F^*_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty}\beta^t[\omega\log C_{Tt}+(1-\omega)\alpha\log(1-L_{Tt})]$$

$$s.t. \quad \frac{\omega}{C_{Tt}}=\frac{(1-L_{Tt})^{-1}}{A_tL_{Tt}^{\alpha+\gamma_{Tt}-1}}\left(1-\omega\right), \tag{B5}$$

$$C_{Tt}-A_tL_{Tt}^{\alpha+\gamma_{Tt}}=R^*F^*_t-F^*_{t+1}, \tag{B6}$$

$$F^*_0 \text{ given.} \tag{B7}$$

Substituting out C_{Tt} gives

$$\max_{\{L_{Tt},F^*_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty}\beta^t[\omega(\alpha+\gamma_{Tt}-1)\log L_{Tt}+[\omega+(1-\omega)\alpha]\log(1-L_{Tt})]+constant$$

$$s.t. \quad \frac{\omega}{(1-\omega)}A_tL_{Tt}^{\alpha+\gamma_{Tt}-1}-\frac{1}{(1-\omega)}A_tL_{Tt}^{\alpha+\gamma_{Tt}}=R^*F^*_t-F^*_{t+1}, \tag{B8}$$

$$F^*_0 \text{ given.} \tag{B9}$$

The first-order conditions are

$$\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-[\omega+(1-\omega)\alpha]\frac{1}{(1-L_{Tt})}$$
$$=\lambda_t\left[\frac{\omega}{(1-\omega)}(\alpha+\gamma_{Tt}-1)A_tL_{Tt}^{\alpha+\gamma_{Tt}-2}-\frac{1}{(1-\omega)}(\alpha+\gamma_{Tt})A_tL_{Tt}^{\alpha+\gamma_{Tt}-1}\right], \tag{B10}$$

$$\lambda_t=\beta R^*\lambda_{t+1}. \tag{B11}$$

From the first first-order condition

$$\frac{(1-L_{Tt})^{-1}}{A_tL_{Tt}^{\alpha+\gamma_{Tt}-1}}(1-\omega)\left[\frac{\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-(\alpha+\omega\gamma_t)}{\omega(\alpha+\gamma_{Tt}-1)\frac{1}{L_{Tt}}-(\alpha+\gamma_{Tt})}\right]=\lambda_t.$$

Substituting λ_t from the second first-order condition gives the modified XR-IP Euler equation in this case

$$\left(\frac{\omega}{C_{Tt}}\right) = \beta R^* \frac{\theta(L_{Tt+1}, \gamma_{Tt+1})}{\theta(L_{Tt}, \gamma_{Tt})} \left(\frac{\omega}{C_{Tt+1}}\right), \tag{B12}$$

$$\theta(L_{Tt}, \gamma_{Tt}) \equiv \frac{\omega(\alpha + \gamma_{Tt} - 1)\frac{1}{L_{Tt}} - (\alpha + \omega\gamma_{Tt})}{\omega(\alpha + \gamma_{Tt} - 1)\frac{1}{L_{Tt}} - (\alpha + \gamma_{Tt})} \in (0, 1]. \tag{B13}$$

We now characterize the optimal XR-IP solution.

For $t \geq 1$ with $\gamma_{Tt} = 0$, from (B13) $\theta_t = \theta_{t+1} = 1$, therefore

$$L_{T1} = \left[\frac{A_1}{A_{t+1}}\right]^{\frac{1}{1-\alpha}} L_{Tt+1}, \tag{B14}$$

$$C_{T1} = C_{Tt+1}. \tag{B15}$$

For $t = 0$

$$\frac{(1 - L_{T0})^{-1}}{A_0 L_{T0}^{\alpha+\gamma_{T0}-1}} \theta_0 = \frac{(1 - L_{T1})^{-1}}{A_1 L_{T1}^{\alpha-1}}, \tag{B16}$$

$$\Rightarrow (1 - L_{T0}) A_0 L_{T0}^{\alpha+\gamma_{T0}-1} = (1 - L_{T1}) A_1 L_{T1}^{\alpha-1} \theta_0. \tag{B17}$$

The sequence of foreign currency bonds $\{F^*_{t+1}\}_{t=0}^{\infty}$ given F^*_0 is determined by the balance of payments

$$C_{T0} - A_0 L_{T0}^{\alpha+\gamma_{T0}} = R^* F^*_0 - F^*_1, \tag{B18}$$

$$C_{Tt} - A_t L_{Tt}^{\alpha} = R^* F^*_t - F^*_{t+1} \text{ for } t \geq 1. \tag{B19}$$

Substituting (B14) and C_{T0}, C_{T1} from (B5), and iterating the second balance of payments equation forward

$$\frac{\beta}{(1-\beta)} \frac{\omega}{(1-\omega)} A_1 (1 - L_{T1}) L_{T1}^{\alpha-1} - A_1 \widetilde{A}_1 L_{T1}^{\alpha} = F^*_1, \tag{B20}$$

and substituting for F_1^* into the first balance of payments equation

$$\frac{\omega}{(1-\omega)}A_0(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} + \frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}A_1(1-L_{T1})L_{T1}^{\alpha-1} - A_1\widetilde{A}_1 L_{T1}^{\alpha} - \frac{1}{\beta}F_0^* = 0. \tag{B21}$$

Substituting the optimality condition (B17)

$$\frac{\omega}{(1-\omega)}A_0(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} + \frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}\frac{1}{\theta_0}A_0(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1}$$
$$- A_1\widetilde{A}_1 L_{T1}^{\alpha} - \frac{1}{\beta}F_0^* = 0 \tag{B22}$$

$$H(L_{T0}, \theta_0(L_{T0})) = L_{T1}, \tag{B23}$$

$$H(L_{T0}, \theta_0(L_{T0})) \equiv \frac{1}{(A_1\widetilde{A}_1)^{\frac{1}{\alpha}}}\Bigg\{\frac{\omega}{(1-\omega)}A_0(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}}$$
$$+\frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}\frac{1}{\theta_0}A_0(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1} - \frac{1}{\beta}F_0^*\Bigg\}^{\frac{1}{\alpha}}. \tag{B24}$$

We solve for L_{T0} in the XR-IP by plugging L_{T1} from (B23) into (B17)

$$(1-L_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1} - (1-H(L_{T0},\theta_0))\,A_1\,(H(L_{T0},\theta_0))^{\alpha-1}\,\theta_0 = 0 \tag{B25}$$

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} - (1-H(L_{T0},\theta_0))\,A_1\,(H(L_{T0},\theta_0))^{\alpha-1}\,\theta_0 = 0. \tag{B26}$$

Following the same steps to solve for L_{T0} in the laissez-faire competitive equilibrium

$$A_0 L_{T0}^{\alpha+\gamma_{T0}-1} - A_0 L_{T0}^{\alpha+\gamma_{T0}} - (1-H(L_{T0},1))\,A_1\,(H(L_{T0},1))^{\alpha-1} = 0. \tag{B27}$$

We can sign the following

$$\frac{\partial H}{\partial L_{T0}} = \frac{1}{A_1\widetilde{A}_1}\frac{1}{\alpha}H(L_{T0},\theta_0)^{1-\alpha}\Bigg\{\frac{\omega}{1-\omega}\left(-(1-\alpha-\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}\right)$$
$$-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}+\frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}\frac{1}{\theta_0}A_0\left[-(1-\alpha-\gamma_{T0})L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})L_{T0}^{\alpha+\gamma_{T0}-1}\right]\Bigg\} \tag{B28}$$

$$< 0, \tag{B29}$$

$$\frac{\partial H}{\partial \theta_0} = -\frac{1}{A_1\widetilde{A}_1}\frac{1}{\alpha}H(L_{T0},\theta_0)^{1-\alpha}\frac{\beta}{(1-\beta)}\frac{\omega}{(1-\omega)}\frac{1}{\theta_0^2}(1-L_{T0})L_{T0}^{\alpha+\gamma_{T0}-1} < 0. \tag{B30}$$

Treating L_{T0} as a function of θ_0 and differentiating (B26) with respect to θ_0 gives

$$\begin{aligned}
&-[(1-\alpha-\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-2}-(\alpha+\gamma_{T0})A_0L_{T0}^{\alpha+\gamma_{T0}-1}]\frac{\partial L_{T0}}{\partial \theta_0}\\
&+A_1(H(L_{T0},\theta_0))^{\alpha-1}\theta_0\left[\frac{\partial H}{\partial L_{T0}}\frac{\partial L_{T0}}{\partial \theta_0}+\frac{\partial H}{\partial \theta_0}\right]\\
&+(1-\alpha)(1-H(L_{T0},\theta_0))A_1\theta_0(H(L_{T0},\theta_0))^{\alpha-1}\left[\frac{\partial H}{\partial L_{T0}}\frac{\partial L_{T0}}{\partial \theta_0}+\frac{\partial H}{\partial \theta_0}\right]\\
&-(1-H(L_{T0},\theta_0))A_1(H(L_{T0},\theta_0))^{\alpha-1}=0.
\end{aligned} \tag{B31}$$

Since $\frac{\partial H}{\partial \theta_0} < 0$

$$\begin{aligned}
&A_1(H(L_{T0},\theta_0))^{\alpha-1}\theta_0\left[\frac{\partial H}{\partial \theta_0}\right]\\
&+(1-\alpha)(1-H(L_{T0},\theta_0))A_1\theta_0(H(L_{T0},\theta_0))^{\alpha-1}\left[\frac{\partial H}{\partial \theta_0}\right]\\
&-(1-H(L_{T0},\theta_0))A_1(H(L_{T0},\theta_0))^{\alpha-1} < 0.
\end{aligned} \tag{B32}$$

Therefore, the remaining terms must satisfy

$$\frac{\partial L_{T0}}{\partial \theta_0}\Bigg\{ - [(1-\alpha-\gamma_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-2} - (\alpha+\gamma_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1}] + A_1(H(L_{T0},\theta_0))^{\alpha-1}\theta_0\left[\frac{\partial H}{\partial L_{T0}}\right] + (1-\alpha)(1-H(L_{T0},\theta_0))A_1\theta_0(H(L_{T0},\theta_0))^{\alpha-1}\left[\frac{\partial H}{\partial L_{T0}}\right]\Bigg\} > 0. \quad \text{(B33)}$$

The terms in braces are negative, since $\frac{\partial H}{\partial L_{T0}} < 0$, which means that it must be that

$$\frac{\partial L_{T0}}{\partial \theta_0} < 0. \quad \text{(B34)}$$

This shows that for the XR-IP solution when $\theta_0 < 1$, compared with the CE solution to (B27),

$$L_{T0}^{IP} > L_{T0}^{CE}. \quad \text{(B35)}$$

For both the XR-IP and CE

$$C_{T0} = \frac{\omega}{1-\omega}(1-L_{T0})A_0 L_{T0}^{\alpha+\gamma_{T0}-1} \quad \text{(B36)}$$

$$\Rightarrow \frac{\partial C_{T0}}{\partial L_{T0}} < 0, \quad \text{(B37)}$$

$$\mathcal{E}_0 = \frac{(1-L_{T0})^{\alpha-1}}{L_{T0}^{\alpha+\gamma_{T0}-1}} \quad \text{(B38)}$$

$$\Rightarrow \frac{\partial \mathcal{E}_0}{\partial L_{T0}} > 0. \quad \text{(B39)}$$

Therefore, since $L_{T0}^{IP} > L_{T0}^{CE}$

$$C_{T0}^{IP} < C_{T0}^{CE}, \quad \text{(B40)}$$

$$\mathcal{E}_0^{IP} > \mathcal{E}_0^{CE}. \quad \text{(B41)}$$

By definition of the current account balance

$$CA_0 = L_{T0}^{\alpha+\gamma_{T0}} - C_{T0} + (R^* - 1)F_0^*, \tag{B42}$$

$$\Rightarrow CA_0^{IP} > CA_0^{CE}. \tag{B43}$$

As shown above, given F_0^*, for both the IP and CE

$$F_1^* = R^* F_0^* + A_0 L_{T0}^{\alpha+\gamma_{T0}} - C_{T0}, \tag{B44}$$

$$\Rightarrow F_1^{*IP} > F_1^{*CE}. \tag{B45}$$

This shows the same result as Proposition 3 in the initial period for the model with fixed labor supply.