Sophisticated Borrowing Constraints and Macroeconomic Dynamics[*]

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Abstract

In traditional macro-finance models, firms' debt contracts impose hard borrowing constraints, which require indiscriminate reductions of borrowing and investment when adverse shocks tighten these limits, giving rise to financial acceleration. We study the macroeconomic implications of "sophisticated borrowing constraints" akin to financial covenants among large U.S. nonfinancial firms, commonly specified based on firms' debt relative to operating earnings. We model these constraints as debt thresholds that trigger a transfer of control rights to creditors when they are violated, in which case creditors influence firms' decisions to maximize their value instead of cutting credit unconditionally to adhere to fixed ratios. At the micro level, our model is quantitatively consistent with empirical patterns of investment and earnings around covenant violations. At the macro level, sophisticated borrowing constraints do not generate financial acceleration, because constraint tightening and violations do not induce creditors to downscale firms indiscriminately.

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1 Introduction

In traditional macro-finance models, firms' debt contracts impose hard borrowing constraints (e.g., Kiyotaki and Moore, 1997). For example, firms can borrow up to a share of the value of their capital, or up to a multiple of their earnings. When firms hit the hard constraints, they need to cut borrowing indiscriminately to maintain these limits, resulting in sharp declines in investment, output, and earnings. At the macro level, this deleveraging generates "financial acceleration": firms' adjustments lead to drops in prices and quantities that further tighten borrowing constraints, which trigger further adjustment by firms and precipitate larger economic contractions in response to aggregate shocks.

In practice, large U.S. nonfinancial firms, which account for the majority of investment in the economy, face more sophisticated debt contracts. These contracts include legally binding financial covenants that commonly set borrowing thresholds based on operating earnings (Lian and Ma, 2021). When financial covenants are violated, creditors do not force firms to cut credit indiscriminately, but instead play an active role in firms' governance to improve their performance. Violations of financial covenants are common, and around 15% of all public nonfinancial firms can be in violation in a given year, as Nini, Smith, and Sufi (2012) document.

What are the macroeconomic implications of these debt contracts observed in practice? In this paper, we address this question by developing a new quantitative macro-finance framework with "sophisticated borrowing constraints" akin to financial covenants for large U.S. nonfinancial firms, commonly specified based on firms' debt relative to operating earnings. We conceptualize sophisticated borrowing constraints as debt thresholds whose violations trigger creditor intervention in firms' decisions. Following the corporate finance literature on covenant violations, we model these interventions as transferring control rights to creditors, who then shape firms' decisions to maximize creditor value. At the micro level, our model is quantitatively consistent with firm-level outcomes observed around covenant violations. At the macro level, we find that sophisticated borrowing constraints do not lead to financial acceleration because constraint tightening and the resulting violations do not induce creditors to downscale the firm unnecessarily.

Our model features heterogeneous firms financing their investment through defaultable long-term debt, subject to sophisticated borrowing constraints. In addition, firm managers are subject to agency problems and value some unproductive use of capital (e.g., empire building for their personal ambitions). Managerial agency problems are well recognized as a pervasive issue among firms, a key theoretical foundation for contingent creditor control, and an important element in explaining firms' outcomes around covenant violations (Jensen, 1986; Aghion and Bolton, 1992; Dewatripont and Tirole, 1994; Nini, Smith, and Sufi, 2012). We show that this model is quantitatively consistent with empirical patterns of investment and earnings around covenant violations. In the data, firms exhibit positive capital growth and declining earnings before covenant violation, but negative capital growth and rising earnings afterward. Our sophisticated borrowing constraint model replicates this empirical pattern because constraint violation is driven by a combination of increasing agency frictions (which explain the positive capital growth before violation) and decreasing productivity (which explains the negative earnings growth before violation). Creditor control after covenant violation curbs managerial agency frictions and curtails unproductive use of capital, which account for the reduction of capital combined with the increase of earnings. Our model also matches unconditional moments related to firms' borrowing, such as the fraction of firms newly violating a financial covenant each quarter, the total fraction of firms in the violation state, and the distribution of firm-level leverage.

An important feature of our model is that creditors' control rights are useful for alleviating agency frictions when firms underperform, but this does not imply that shareholders would be better off having creditors operate the firm permanently. This is consistent with the observation of Dewatripont and Tirole (1994) that creditor control is preferable on the downside, as they care particularly about downside risks but may forgo investment opportunities on the upside. Quantitatively, we find that shareholders prefer managers operating the firms away from borrowing constraints and only prefer creditors taking control in those states in which the firms are close to the constraints.

We embed our sophisticated borrowing constraints model into a canonical heterogeneous-firm New Keynesian framework (as in Ottonello and Winberry, 2020) to study its macroeconomic implications. As a benchmark, we contrast our results with those from a model that replaces sophisticated borrowing constraints with hard borrowing constraints, in which managers always retain control rights and must strictly adhere to borrowing limits based on the debt-to-earnings ratio.

We use our model to study the macroeconomic implications of sophisticated borrowing constraints, particularly the importance of financial acceleration in shaping macroeconomic outcomes. By financial acceleration, we refer to the general-equilibrium feedback mechanism with two components: (i) adverse economic outcomes tighten borrowing constraints, and (ii) the tightening of borrowing constraints further worsens economic outcomes.

We first analyze the second component of financial acceleration through the macroeconomic effects of a constraint-tightening shock. In hard-constraint models, the macroeconomic effects of this shock are strongly contractionary. This is because tightening hard constraints induces firms near the constraint to downscale indiscriminately to maintain these limits. In contrast, under sophisticated borrowing constraints, tightening these constraints generates a muted aggregate response, which is mildly expansionary. This muted response reflects the interplay of two opposing channels. The first is an extensive-margin channel, which captures responses arising when tightening induces some firms to violate the constraint and transition to creditor control. This channel improves firm outcomes, as creditors mitigate agency problems and do not engage in the contractionary precautionary behavior that characterizes managers' responses. The second is an intensive-margin channel, which captures responses to tightening holding the allocation of control rights fixed. This channel is contractionary, as managers respond precautionarily to avoid violating the constraint and relinquishing control. However, such precautionary behavior is much weaker than under hard constraints, because the decline in firms' capital after violating the sophisticated constraint is much milder than after hitting hard borrowing constraints.

We then study the importance of financial acceleration in determining the macroeconomic effects of monetary policy shocks. To do so, we compare full general-equilibrium responses, which incorporate financial acceleration, to counterfactual responses where borrowing constraints are governed by steady-state values and thus do not tighten in response to the shock. We find significant financial acceleration in the hard-constraint model but not in the sophisticated-constraint model. This difference reflects the constraint-tightening analysis above: in both models, the monetary policy shock depresses aggregate demand, lowering prices and earnings, which in turn tightens borrowing constraints. As shown above, this tightening generates further contractionary effects in the hard-constraint model but only muted effects in the sophisticated-constraint model.

Related literature. Our paper contributes to three strands of literature. First, we provide a new modeling framework for the literature on the macroeconomic impact of financial frictions. A core topic in this literature is financial acceleration: adverse economic outcomes exacerbate financial frictions, which in turn worsen economic outcomes (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999). Our model shows that with sophisticated borrowing constraints, which are central to debt contracts of large U.S. nonfinancial firms, financial acceleration can be muted. These findings align with observations about the macroeconomic dynamics in the Great Recession, during which nonfinancial firms experienced negative shocks, but financial acceleration primarily stemmed from households and financial institutions (Gertler and Gilchrist, 2018; Jordà et al., 2022).

Second, our analysis complements theoretical and empirical research on debt contracting. A large body of theoretical work studies creditor monitoring as a response to agency problems (Diamond, 1984; Jensen, 1986; Berlin and Loeys, 1988; Rajan and Winton, 1995; Hartman-Glaser, Mayer, and Milbradt, 2025), and models of contingent control are particularly related to our focus on covenants (Smith and Warner, 1979; Aghion and Bolton, 1992; Dewatripont and Tirole, 1994; Berlin and Mester, 1999; Garleanu and Zwiebel, 2009).[1] Moreover, empirical work on debt contracts has documented

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[1] Models of dynamic optimal contracts have also analyzed additional dimensions including limited commitment, the maturity structure of debt, the optimal number of creditors, and risk management (Hart and Moore, 1994, 1998; Bolton and Scharfstein, 1996; DeMarzo and Fishman, 2007; Rampini and Viswanathan, 2010, 2013; DeMarzo, 2019), but financial covenants are not their focus.

the importance of creditor influence following violations of debt covenants (Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012; Gong, 2021). We integrate this feature into macroeconomic analysis. We focus on debt limits as a function of firms' earnings given their prevalence in U.S. corporate debt contracts, as shown by a growing body of research (Greenwald, 2019; Lian and Ma, 2021; Drechsel, 2023; Caglio, Darst, and Kalemli-Ozcan, 2021; Adler, 2025; Zhao, 2025; Su, 2026; Su and Yan, 2026), and our sophisticated borrowing constraint approach complements the models that analyze these debt limits using hard constraints.

Third, our paper contributes to the literature connecting macroeconomic models with firm-level data (e.g., Khan and Thomas, 2013; Bloom et al., 2018; Ottonello and Winberry, 2020, 2024; Jeenas, 2023; Winberry et al., 2025). Our focus is on developing a macroeconomic framework that incorporates key features of debt contracts in practice. We do so by introducing sophisticated borrowing constraints that trigger transfers of control rights when they are violated. Bisetti, Li, and Yu (2024) examine how creditor control following covenant violations affects the risk profile of investment and the corresponding macroeconomic implications. We complement this work with a micro to macro approach, and provide a novel way to discipline heterogeneous-firm models with financial frictions, which uses firm-level outcomes around covenant violations. In matching these patterns, our model underscores that agency frictions are important at the micro level, and these frictions also influence macroeconomic dynamics (e.g., Arellano, Bai, and Kehoe, 2019; Terry, 2023). Our core mechanism can also be viewed as a financial channel that generates cleansing effects of downturns, which operates through reducing mismanagement instead of through firm exits (Caballero and Hammour, 1994).

Our results echo the view that institutions shaping corporate debt enforcement can influence macroeconomic fluctuations (e.g., Corbae and D'Erasmo, 2021; Bornstein and Castillo-Martinez, 2023; Guntin, 2023). Substantial financial amplification among nonfinancial firms can still exist when the contracting environment is less sophisticated (Kornejew et al., 2024; Ivashina et al., 2025). In addition, debt contracts among financial institutions can be very different (e.g., deposits, commercial paper, asset-backed securities, and repos). They generally do not enforce creditor control in the same manner (Ma and Scheinkman, 2019);[2] some impose hard borrowing constraints tied to the market value of financial securities, and the resulting financial acceleration has been well studied (Brunnermeier and Pedersen, 2009; Geanakoplos, 2010). Finally, household debt also cannot easily implement creditor control, and the resolution of financial distress is challenging. Prior research shows that it often contributes to persistent downturns (Mian, Rao, and Sufi, 2013), and modeling household borrowing constraints as soft versus hard has significant implications for the macroeconomic impact of a credit

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[2] One possibility is that payment default of financial institutions is costly and complex (given their short-term liabilities), so creditors cannot credibly use payment acceleration as a threat to implement creditor control. Financial regulation also restricts the scope of creditor control.

crunch (Alonso, 2018).

2 Corporate Debt Contracts and Financial Covenants

Financial covenants and enforcement. Corporate debt contracts in the US commonly impose legally binding requirements on firms using financial covenants. The most widely used financial covenants restrict total debt (or interest payments) relative to operating earnings in the form of EBITDA (earnings before interest, taxes, depreciation, and amortization), and those in loans are typically assessed for compliance on a quarterly basis (Lian and Ma, 2021).[3] Violations of financial covenants give creditors the ability to exert control and influence firms' operations. Specifically, a covenant violation triggers a "technical default," in which case creditors have legal power to accelerate payments and make the debt due immediately. Creditors rarely pursue acceleration, but use this threat to play an active role in firms' governance. They typically grant "waivers" and relax financial covenants for some period of time, on the condition that firms follow the actions that creditors demand to fix the business. The most common demands include restricting additional borrowing, reducing capital expenditures, and limiting acquisitions and operating expenses; they may also push for replacing the CEO (Nini, Smith, and Sufi, 2012). Some of these demands are formalized by amending the credit agreement with additional covenants restricting capital expenditures and other payments. In addition, "creditors work behind the scenes to affect changes in the way that the company is managed," and "creditors can offer advice to management and the board, quid pro quo, and suggest actions the company can take to maximize the chance of receiving a covenant waiver" (Nini, Smith, and Sufi, 2012). We provide a more detailed discussion and examples of how violations of financial covenants lead to creditor control in Appendix A.

These practices echo insights in the contract theory literature: the contingent transfer of control rights triggered by financial covenants helps approximate optimal contracts in models of incomplete contracts (e.g., Aghion and Bolton, 1992; Dewatripont and Tirole, 1994), in which it is beneficial for managers to have control rights in good times to preserve the upside and for creditors to have control rights to protect the downside. Moreover, loan contracts have relatively concentrated ownership,[4] while the shareholders of public companies are dispersed, so creditors can have stronger incentives and easier coordination compared to shareholders to impose control over company management. Accordingly, legal scholars have similarly observed: "When a business stumbles, creditors typically

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[3] Covenants in bonds are assessed whenever borrowers take significant actions (e.g., issuing new debt). Moreover, for large U.S. nonfinancial firms, monitoring-intensive syndicated loans typically become substantial when debt is high enough to be close to covenant violation.

[4] Even for syndicated loans, the average number of lenders is only around 8 (Blickle et al., 2022), which is very small compared with the number of shareholders.

enjoy powers that public shareholders never have, such as the ability to replace the managers and install those more to their liking... Loan covenants now are the principal mechanism for handling one of the most challenging problems in corporate governance" (Baird and Rasmussen, 2006).

In the data, financial covenants, as well as violations, are highly prevalent. For example, about half of large Compustat firms have financial covenants explicitly written in their loan contracts (Lian and Ma, 2021). As Nini, Smith, and Sufi (2012) document, each year around 15% of all public nonfinancial firms are in violation of financial covenants, and over 40% have violated a financial covenant at least once over their sample period (1997 to 2008).

Firm outcomes around covenant violations. To guide our model analysis, we examine firm outcomes around covenant violations, following a leading approach in the corporate finance literature (Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini, Smith, and Sufi, 2012). For this, we combine firm-level variables from quarterly U.S. Compustat with data on financial covenant violations disclosed in firms' financial statements, kindly shared by Greg Nini, which extend the data in Nini, Smith, and Sufi (2012) to 2016.

Using this approach, Figure 1 depicts the dynamics of capital and earnings around covenant violations. Panel (a) shows firms' capital in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by capital at the beginning of quarter t so that the differences over time come from the numerator rather than the denominator). Panel (b) shows firms' quarterly earnings in quarter $t+h$ relative to quarter t (normalized by capital at the beginning of quarter t). A striking pattern is that before covenant violation, firms have positive capital growth and declining earnings; after covenant violation, firms have negative capital growth and rising earnings. This pattern is consistent with the evidence from Nini, Smith, and Sufi (2012) Figure 4 (capital dynamics around covenant violation) and Figure 7 (earnings dynamics around covenant violation). In Figure B.1, we also verify that the baseline patterns of capital and earnings dynamics around covenant violation hold in macro recessions (2001 to 2002 and 2007 to 2009). On average, creditor intervention does improve earnings in macro recessions, even though some constrained lenders may cut credit supply suboptimally.[5] Our work focuses on the overall effect.

Firms' debt contracts in practice use a variety of covenant specifications (e.g., interest coverage ratio covenants that require interest payments to be less than a fraction of operating earnings, debt-to-EBITDA covenants that require total debt to be less than a multiple of operating earnings, as well as other formulas). Our baseline figures focus on firms with debt-to-EBITDA covenants (as this is the most common form), and we use this sample for our model calibration where the covenant is

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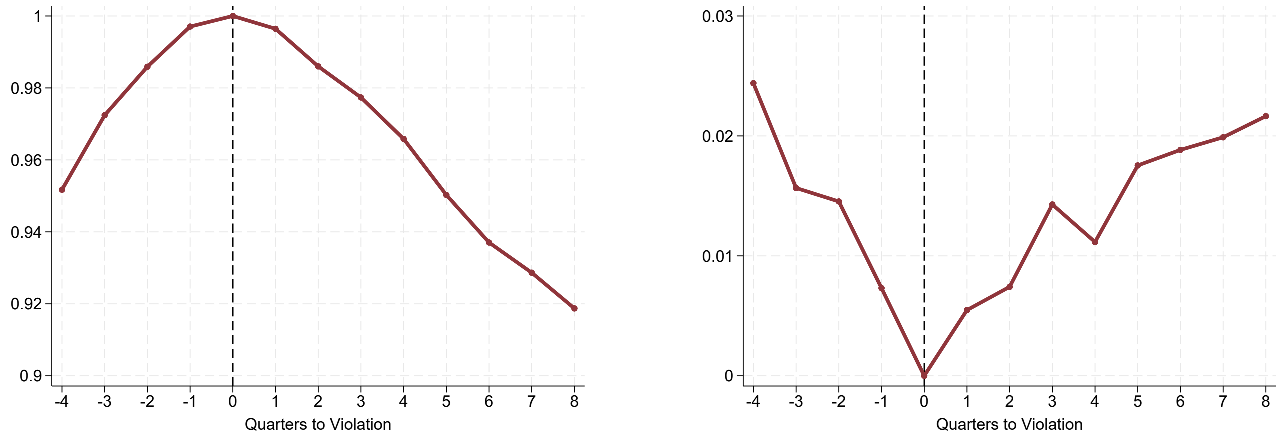
[5] In particular, Chodorow-Reich and Falato (2022) show that lenders with poor health cut credit supply following covenant violation during the Great Financial Crisis.

Figure 1: Dynamics of Firm Outcomes around Covenant Violations


(a) Capital (k_{t+h}/k_t)

(b) Earnings $((EBITDA_{t+h} - EBITDA_t)/k_t)$

Notes: Panel (a) shows firms' capital (property, plant, and equipment) in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by capital at the beginning of quarter t so that the differences over time come from the numerator rather than the denominator). Panel (b) shows firms' quarterly earnings in quarter $t+h$ relative to quarter t (normalized by capital at the beginning of quarter t). The line shows the median value for each quarter $t+h$. The sample includes US Compustat firms that have a debt-to-EBITDA covenant in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.

correspondingly written on debt-to-EBITDA. We verify that the core patterns in Figure 1 are robust when we consider all firms with financial covenants in Figure B.4, or all firms with earnings-based financial covenants (i.e., debt-to-EBITDA covenants plus interest coverage ratio covenants) in Figure B.5.

As additional analysis, Figure B.3 shows that firms' equity values decline before covenant violations and rise afterwards, consistent with Nini, Smith, and Sufi (2012) Figures 8 and 9. These patterns indicate that creditor interventions do not appear to destroy firm value.[6] Finally, Table B.1 shows the empirical results in regressions to check their robustness to controls. The outcome variables include the change in EBITDA in the next four quarters (normalized by beginning of quarter capital), and capital growth over the next four quarters. The regressions control for a number of firm characteristics (e.g., book leverage, EBITDA over capital, capital growth) in the previous 4 quarters as well as their squares and cubes following Nini, Smith, and Sufi (2012). The indicator variable "Violation" indicates the quarter of reporting an initial violation of financial covenants. The results show that future EBITDA is significantly higher and capital growth significantly lower following covenant violation, compared to the usual path of these variables.

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[6] We also do not see that creditors cut valuable long-term investment. For example, Figure B.2, Panel A shows that research and development (R&D) expenditures do not fall around covenant violations. This pattern contrasts with capital expenditures shown in Panel B. Creditors do not appear to cut R&D spending even as they reduce capital investment.

3 Model

We now develop a heterogeneous firm New Keynesian model featuring sophisticated borrowing constraints that capture key aspects of corporate borrowing constraint enforcement in Section 2, and study the macroeconomic implications. We describe the model in two steps. We begin by describing the core of our model, the "macro-finance block," and then proceed to describe the rest of the model.

3.1 The Macro-Finance Block

A unit mass of heterogeneous production firms produce and invest in capital, subject to sophisticated borrowing constraints. Time is discrete and infinite and there is no aggregate uncertainty. As further specified below, we study the perfect foresight transition path in response to unexpected aggregate shocks realized at the beginning of period 0. Each firm $j \in [0,1]$ produces an undifferentiated good with relative price p_t (in terms of the final good, described below) using the production function $y_{jt} = z_{jt}\tilde{k}_{jt}^{\alpha}\ell_{jt}^{\nu}$, where z_{jt} is the firm's productivity, $\tilde{k}_{jt}$ is the firm's capital input for production, ℓ_{jt} is the firm's labor input, and $\alpha + \nu < 1$. The productivity follows a log-AR(1) process, $\log z_{jt+1} = \rho_z \log z_{jt} + \epsilon_{jt+1}$, where $\epsilon_{jt+1} \sim \mathcal{N}\left(0, \sigma_z^2\right)$ is independent over time. The production firm generates operating earnings

$$\pi_t\left(z_{jt}, \tilde{k}_{jt}\right) \equiv \max_{\ell_{jt}} p_t z_{jt} \tilde{k}_{jt}^{\alpha} \ell_{jt}^{\nu} - w_t \ell_{jt} - f_c,$$

where w_t is the real wage and f_c is the average real fixed operating cost. Capital depreciates at rate δ, and firms can purchase new capital $k_{jt+1} - (1-\delta)k_{jt}$ in a competitive market with the relative price of capital $q_{k,t}$, subject to the adjustment costs

$$AC\left(k_{jt}, k_{jt+1}\right) = \psi_{ac}\left(\frac{k_{jt+1} - (1-\delta)\,k_{jt}}{k_{jt}}\right)^2 k_{jt}.$$

Motivated by the empirical evidence in Section 2, we introduce managerial agency frictions. Specifically, managers derive utility $H\left(a_{jt-1}, \tau_{jt}k_{jt}\right) = a_{jt-1}\left(\tau_{jt}k_{jt}\right)^{\alpha_H} - C_H \mathbb{1}\left(\tau_{jt}k_{jt} > 0\right)$ by using a fraction $\tau_{jt} \in [0,1]$ of capital for unproductive use, with $\alpha_H \in (0,1)$, $C_H > 0$, and the remaining capital used as input for production, $\tilde{k}_{jt} = \left(1 - \tau_{jt}\right)k_{jt}$. a_{jt} captures the idiosyncratic shock to agency frictions, which we refer to as the "agency shock." It follows a log-AR(1) process $\log a_{jt} = \left(1-\rho_a\right)\log\left(\mu_a\right) + \rho_a \log a_{jt-1} + \epsilon_{jt}^{a}$, where $\epsilon_{jt}^{a} \sim \mathcal{N}\left(0, \sigma_a^2\right)$ is independent over time and independent of $\{\epsilon_{js}\}_{s=0}^{\infty}$. C_H captures the fixed cost of unproductive use of capital, which can be microfounded as the cost managers incur to avoid being caught diverting capital. The choice of $\tau_{jt} \in [0,1]$ is made at $t-1$, and the utility $H\left(a_{jt-1}, \tau_{jt}k_{jt}\right)$ is also obtained at $t-1$, reflecting the time lag for managers to implement projects that yield personal benefits.[7] This explains why a_{jt-1} is relevant for the utility associated with

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[7] If those decisions are instead made in the same period as production, then managers can easily lower τ_{jt+1} to boost

unproductive use of period t capital. Creditors do not value the unproductive use of capital.

The production firm can finance its expenditures by both debt and equity. In terms of debt financing, the firm can borrow externally through a long-term, defaultable, real debt where the payment declines at a factor $\gamma \in [0,1]$. We use $b_{jt} \geq 0$ to denote the payment of long-term debt due at period t. The firm can issue new debt $b_{jt+1} - \gamma b_{jt}$ given the price schedule $q_{b,t}(\cdots)$, which captures the post-coupon-payment period-t price of debt that promises one unit of payment at $t+1$, γ at $t+2$, γ^2 at $t+3$, and so on. The price schedule is determined by creditors (financial intermediaries owned by the representative household) who competitively purchase and trade firms' debt, as further specified below.

The issuance of new debt is subject to the "sophisticated borrowing constraint," which governs the transition between two different states representing the allocation of control rights. In the normal state ($\omega_{jt} = n$), managers make decisions to maximize their value. In the violation state ($\omega_{jt} = v$), creditors make decisions to maximize their value. The transition from the normal state to the violation state is given by the sophisticated borrowing constraint, summarized by the mapping $\Gamma_b\left(b_{jt+1}, \overline{b}_{jt+1}\right) \mapsto \omega_{jt+1}$, where $\overline{b}_{jt+1}$ is the threshold debt level that triggers covenant violation and the transition to the violation state. If the firm is currently in the normal state $\omega_{jt} = n$, it stays in the normal state ($\omega_{jt+1} = n$) if the next period debt is below the threshold, $b_{jt+1} \leq \overline{b}_{jt+1}$, and transitions to the violation state ($\omega_{jt+1} = v$) if the next period debt is above the threshold, $b_{jt+1} > \overline{b}_{jt+1}$:

$$\omega_{jt+1} = \Gamma_b\left(b_{jt+1}, \overline{b}_{jt+1}\right) = \begin{cases} n, & \text{if } b_{jt+1} \leq \overline{b}_{jt+1}, \\ v, & \text{if } b_{jt+1} > \overline{b}_{jt+1}. \end{cases} \tag{1}$$

Following the contractual evidence of Lian and Ma (2021) about the popularity of earnings-based covenants, we further specify the threshold debt level $\overline{b}_{jt+1}$ based on the ratio of the face value of debt and earnings:

$$\tilde{q}_b^{\text{face}} \overline{b}_{jt+1} = \phi \pi_{t+1}\left(z_{jt+1}, \tilde{k}_{jt+1}\right), \tag{2}$$

where $\tilde{q}_b^{\text{face}} = \sum_{k=0}^{\infty} \beta^k \gamma^k = \frac{1}{1-\beta\gamma}$ is the face value of one unit of debt, β is the representative household's discount factor, and ϕ is the debt-to-earnings limit.[8'9] The contingent transfer of control rights

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earnings so that the sophisticated constraint $b_{jt+1} \leq \overline{b}_{jt+1}$ is satisfied based on (2), and they can always avoid violation.

[8] Following the practice of financial covenants, we assume that the threshold in (2) is specified based on the book value of debt, $\tilde{q}_b^{\text{face}}$, computed using a risk-free valuation of debt. Specifically, $\tilde{q}_b^{\text{face}}$ is the pre-coupon-payment risk-free face value of debt that promises one unit of payment in the current period, γ units of payment in the next period, and so on. Additionally, we let the face value $\tilde{q}_b^{\text{face}}$ be constant at its steady-state level. This is because financial covenants use book value of existing debt, which does not vary with changes in interest rates. Moreover, if $\pi_{t+1}\left(z_{jt+1}, \tilde{k}_{jt+1}\right) < 0$, we simply set $\overline{b}_{jt+1} = 0$.

[9] Also, in practice, the threshold in (2) is typically specified in terms of operating earnings over the past twelve months. For tractability, we instead use current-period (quarterly) earnings to avoid an additional state variable for rolling earnings, which would make the model computationally intractable.

engineered by sophisticated borrowing constraints helps approximate optimal contracts in models of incomplete contracts, such as Aghion and Bolton (1992) and Dewatripont and Tirole (1994). They show why it is beneficial for managers to have control rights in good times to preserve the upside, but for creditors to have control rights to protect the downside. One may also wonder why we do not model post-violations as a bargaining outcome between creditors and borrowers. This is because creditors have the legal power to accelerate payments. They rarely do so, as it is inefficient, but they use this power to impose creditor control and influence firms' decisions (Nini, Smith, and Sufi, 2012).

The firm can also issue equity. The literature has identified various frictions associated with equity issuance (e.g., Gomes, 2001; Hennessy and Whited, 2007; Guo et al., 2025), as well as with adjusting dividend payments (e.g., Jermann and Quadrini, 2012, and references therein). We capture these frictions with the cost function $AD\left(d_{jt}\right) = \left(\psi_d + \psi_e \mathbb{I}\left(d_{jt} < 0\right)\right) d_{jt}^2$, where d_{jt} denotes dividend payments to shareholders, and negative d_{jt} indicates equity issuance; $\psi_d > 0$ captures costs associated with adjusting dividends (we center this cost around zero given that around 90% of firm-quarters in Compustat have zero dividend payments); and $\psi_e > 0$ represents the cost of equity issuance. The firm pays corporate tax $T_{c,jt} = \tau_c \left[\pi_t\left(z_{jt}, \tilde{k}_{jt}\right) - \delta q_{k,t} k_{jt} - (1-\beta)\tilde{q}_b^{\text{face}} b_{jt}\right]$, where τ_c is the corporate tax rate.[10]

From both normal and violation states $\omega_{jt} \in \{n, v\}$, firms can be liquidated for two possible reasons. First, firms liquidate with an exogenous exit probability $\chi_d \in (0,1)$. Second, when the total value of continuation (joint between shareholders and creditors, further specified below) is lower than that of liquidation, determined by $\xi q_{k,t} k_{jt}$, where $\xi \in [0,1]$ captures the liquidation value per dollar of capital.

The timing of events within each period t is as follows. At the beginning of each period, idiosyncratic shocks to existing firms' productivity and agency frictions occur, the allocation of control rights for existing firms is determined, and liquidation occurs for the two aforementioned reasons. New firms enter the economy, and the mass of entrants equals the mass of firms being liquidated. Each entrant starts in the normal state, and its initial borrowing, capital, fraction of capital in unproductive use, and technology and agency shocks are drawn from the distribution of $x = (b, k, \tau, z, a)$ of firms in the normal state in the previous period. Then, each existing and new firm (excluding liquidated ones) makes decisions as described below.

We now write the optimization problem in each state recursively. Once the allocation of control rights has been determined, the relevant state variables for each firm are (ω, x). We omit subscripts for current variables (e.g., b for b_{jt}) and use the superscript ' for future variables (e.g., b' for b_{jt+1}) for notational simplicity.

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[10] $(1-\beta)\tilde{q}_b^{\text{face}} b_{jt}$ captures steady-state interest (but not principal) payments on debt with promised total payments of b in the current period, reflecting that only interest payments, not principal payments, are tax-deductible.

The manager's value. In the normal state ($\omega = n$) where the sophisticated borrowing constraint is not violated, the manager of each production firm holds control rights and makes decisions to maximize the manager's value. Specifically, they optimally choose next period's debt b', capital k', the fraction of capital for unproductive use τ', and the dividend payment d. The manager's value $V_{m,t}(n,x)$ can be written as the solution to their optimization problem:

$$V_{m,t}(n,x) = \max_{k',b'\geq 0,\tau'\in[0,1],d} d - AD(d) + H\left(a,\tau' k'\right) + \mathbb{E}\left[\Lambda_{t+1} V^0_{m,t+1}\left(\omega',x'\right)|z,a\right], \tag{3}$$

where Λ_{t+1} is the representative household's stochastic discount factor (SDF), further specified below, and is subject to the transition of firm states, $\omega' = \Gamma_b\left(b', \overline{b}'\right)$, and the flow budget constraint:

$$\underbrace{b}_{\text{debt payment}} + \underbrace{d}_{\text{dividends}} = \underbrace{\pi_t\left(z,(1-\tau)k\right)}_{\text{earnings}} + \underbrace{q_{b,t}\left(n,b',k',\tau',z,a\right)\left(b'-\gamma b\right)}_{\text{debt issuance}} - \underbrace{T_{c,t}(x)}_{\text{tax}} - \underbrace{\left[q_{k,t}\left(k'-(1-\delta)k\right) + AC\left(k,k'\right)\right]}_{\text{investment}}, \tag{4}$$

where $V^0_{m,t+1}\left(\omega',x'\right)$ captures the manager's value in the next period incorporating the possibility of liquidation, further specified below. We use $\left(k'_t(n,x),\ b'_t(n,x),\ \tau'_t(n,x),\ d_t(n,x)\right)$ to denote the optimally chosen managerial decisions in (3) at period t, and $\omega'_t(n,x)$ and $x'_t(n,x)$ to denote the corresponding (stochastic) laws of motion of the state variables.

The manager's value in the violation state is determined passively, where $d_t(v,x)$, $\tau'_t(v,x)$, $k'_t(v,x)$, $\omega'_t(v,x)$, and $x'_t(v,x)$ are determined by the optimally chosen creditor decisions in (7) below:

$$V_{m,t}(v,x) = d_t(v,x) - AD\left(d_t(v,x)\right) + H\left(a,\tau'_t(v,x)\,k'_t(v,x)\right) + \mathbb{E}\left[\Lambda_{t+1} V^0_{m,t+1}\left(\omega'_t(v,x), x'_t(v,x)\right)|z,a\right]. \tag{5}$$

Production firms are owned by the representative household, which receives the dividend payments. Their value, termed shareholder value, is given by the manager's value without the agency-friction $H(a,\tau k)$ term. Specifically, for $\omega \in \{n,v\}$,

$$V_{s,t}(\omega,x) = d_t(\omega,x) - AD\left(d_t(\omega,x)\right) + \mathbb{E}\left[\Lambda_{t+1} V^0_{s,t+1}\left(\omega'_t(\omega,x), x'_t(\omega,x)\right)|z,a\right], \tag{6}$$

where $d_t(\omega,x)$, $\omega'_t(\omega,x)$, and $x'_t(\omega,x)$ are determined by the optimally chosen manager and creditor decisions in (3) and (7).

The creditor's value. In the violation state ($\omega = v$) where the sophisticated borrowing constraint is violated, the creditor obtains control rights and makes decisions to maximize the creditor's value. They do not derive value from unproductive use of capital and optimally choose next period capital k', the fraction of capital for unproductive use τ', the dividend payment d, and the fraction of (current) debt coupon payments to forgive ζ. The total value of debt to creditors who hold it at the beginning of t, $V_{c,t}(v,x)$, can then be written as the solution to their optimization problem:

$$V_{c,t}(v,x) = \max_{k',\tau'\in[0,1],d\geq 0,\zeta\in[0,1]} (1-\zeta)\,b + \mathbb{E}\left[\Lambda_{t+1} V^0_{c,t+1}\left(\omega',x'\right)|z,a\right] \tag{7}$$

and is subject to the transition of firm states (9) and the flow budget constraint:

$$\text{s.t.} \qquad \underbrace{(1-\zeta)\,b}_{\text{debt payment (allowing forgiveness)}} + \underbrace{d}_{\text{dividend}} = \underbrace{\pi_t\left(z,(1-\tau)\,k\right)}_{\text{earnings}} - \underbrace{T_{c,t}\left(x\right)}_{\text{tax}} - \underbrace{\left[q_{k,t}\left(k'-(1-\delta)\,k\right)+AC\left(k,k'\right)\right]}_{\text{investment}}, \tag{8}$$

where $V^0_{c,t+1}\left(\omega',x'\right)$ captures the creditor's value in the next period incorporating the possibility of liquidation, further specified below. We use $\left(k'_t(v,x),\ \tau'_t(v,x),\ d_t(v,x),\ \zeta_t(v,x)\right)$ to denote the optimally chosen creditor decisions in (7) at period t, and $\omega'_t(v,x)$ and $x'_t(v,x)$ to denote the corresponding (stochastic) laws of motion of the state variables.

The creditor's problem in (7) embeds three assumptions regarding creditor control in the violation state. First, we let debt naturally mature, $b' = \gamma b$. This is consistent with median debt issuance being effectively zero during violations in the data.[11] Firms that are in violation cannot issue new debt given the covenant restrictions, and they typically do not have excess cash flows to repurchase additional debt either (if they had, they could have repurchased debt prior to violation to avoid it). Second, we restrict $d \geq 0$, since equity issuance in violation is rare too with a median of zero.[12] For example, classic debt overhang problems would make equity issuance difficult in this case (Myers, 1977).

Third, we assume that the transition from the violation state back to the normal state is exogenous, governed by the transition probability λ. That is,

$$\omega_{jt+1} = \begin{cases} n, & \text{with probability } \lambda, \\ v, & \text{with probability } 1-\lambda. \end{cases} \tag{9}$$

The transition back to the normal state is not governed by the original threshold because creditors may suspend or modify the original covenants for some period of time (see the discussion in the previous section and Appendix A). In addition, how long it takes to resolve problems caused by managerial agency frictions (e.g., inefficient capital expenditures, pet projects, bureaucracies) may be difficult to fully anticipate (if the duration of the initial covenant modification is not sufficient, creditors typically extend it while making additional requests for operational adjustments). For simplicity, we model the transition to the normal state as an exogenous stochastic event.[13]

The creditor's value in the normal state is determined passively by the optimally chosen managerial decisions in (3). Specifically, the total value of debt to creditors who hold it at the beginning of t is

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[11] For firms in violation, the median quarterly net debt issuance relative to capital is -0.0025 and the mean is 0.0129.

[12] For firms in violation, the median quarterly net equity issuance relative to capital is 0 and the mean is 0.0268.

[13] If transitioning back to the normal state is instead determined by whether the constraint $b_{jt+1} \leq \overline{b}_{jt+1}$ is satisfied, almost all violating firms will transition back immediately. This is because, as creditors do not value the unproductive use of capital, they significantly improve earnings by avoiding such use when they have control rights (set $\tau' = 0$) and the constraint $b_{jt+1} \leq \overline{b}_{jt+1}$ is satisfied immediately.

given by:

$$V_{c,t}(n,x) = b + \frac{\gamma b}{b'_t(n,x)} \mathbb{E}\left[\Lambda_{t+1} V^0_{c,t+1}\left(\omega'_t(n,x), x'_t(n,x)\right) | z, a\right]. \tag{10}$$

Compared to (7), it captures that, in the normal state, there is no debt forgiveness but there is the possibility of debt dilution ($b' \neq \gamma b$). For the latter, because creditors can competitively trade firms' debt, all holders of debt at the beginning of $t+1$ have the same valuation; therefore, creditors who hold it at the beginning of t own a fraction $\frac{\gamma b}{b'_t(n,x)}$ of value at $t+1$.

Liquidation. In the case of liquidation, the recovery value is $\xi q_{k,t} k_{jt}$, where $\xi \in [0,1]$ captures the liquidation value per dollar of capital. The value of the creditor, manager, and shareholder in liquidation is then given by

$$V^l_{c,t}(x) = \min\left\{\xi q_{k,t} k, \tilde{q}^{\text{face}}_b b\right\} \quad \text{and} \quad V^l_{m,t}(x) = V^l_{s,t}(x) = \max\left\{\xi q_{k,t} k - \tilde{q}^{\text{face}}_b b, 0\right\}. \tag{11}$$

For $i \in \{m, c, s\}$ and $\omega \in \{n, v\}$, the value functions incorporating liquidation are then given by

$$V^0_{i,t}(\omega, x) = \chi^l_t(\omega, x) V^l_{i,t}(x) + \left(1 - \chi^l_t(\omega, x)\right) V_{i,t}(\omega, x), \tag{12}$$

where the probability of liquidation $\chi^l_t(\omega, x)$ is given by

$$\chi^l_t(\omega, x) = \begin{cases} \chi_d & V_{c,t}(\omega,x) + V_{s,t}(\omega,x) \geq V^l_{c,t}(x) + V^l_{s,t}(x) \\ 1 & V_{c,t}(\omega,x) + V_{s,t}(\omega,x) < V^l_{c,t}(x) + V^l_{s,t}(x) \end{cases}, \tag{13}$$

which incorporates two reasons for the transition to liquidation: exogenous exit with probability $\chi_d \in (0,1)$ and cases where the total continuation value, jointly for shareholders and creditors, is lower than the liquidation value. The latter case mirrors the practice of the bankruptcy system in the US, where the supervising court decides on continuation or liquidation based on the total continuation value versus the liquidation value.

The debt price schedule. Because creditors competitively purchase and trade firms' debt at the firm-specific price schedule $q_{b,t}\left(\omega, b', k', \tau', z, a\right)$, the creditors' value and the price schedule are linked by, for $\omega \in \{n, v\}$,[14]

$$V_{c,t}(\omega, x) = (1 - \zeta_t(\omega, x)) b + \gamma b q_{b,t}\left(\omega, b'_t(\omega, x), k'_t(\omega, x), \tau'_t(\omega, x), z, a\right). \tag{14}$$

The first term in (14) captures the debt payment at t. The second term captures the period-t value of debt that promises γb units of payment at $t+1$, where γ reflects debt maturing. Together with the creditor's value in (7) and (10), we obtain the usual recursive expression for the competitive price

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[14] We define $\zeta_t(n, x) = 0$ (no debt forgiveness in the normal state) and $b'_t(v, x) = \gamma b$ (debt naturally matures in the violation state).

schedule:[15]

$$q_{b,t}\left(\omega, b', k', \tau', z, a\right) = \mathbb{E}\left[\Lambda_{t+1}\left(1-\chi^{l}_{t+1}\left(\omega', x'\right)\right)\left(\left(1-\zeta_{t+1}\left(\omega', x'\right)\right)+\gamma q_{b,t+1}\left(\cdots\right)\right)|z,a\right] \tag{15}$$
$$+\mathbb{E}\left[\Lambda_{t+1}\chi^{l}_{t+1}\left(\omega', x'\right)\min\left\{\xi q_{k,t+1}\frac{k'}{b'}, \tilde{q}_{b}^{\text{face}}\right\}|z,a\right],$$

where $x' = \left(b', k', \tau', z', a'\right)$ and ω' are determined by (1) and (9). The first term captures the debt's value if the firm is not liquidated at $t+1$, and the second term captures the debt's value if the firm is liquidated at $t+1$.

3.2 The Macro Block and Equilibrium

We follow Ottonello and Winberry (2020) to parsimoniously generate a New Keynesian Phillips curve that relates nominal variables to the real economy by introducing sticky-price retailers. There is a unit mass of retailers $i \in [0,1]$. Each retailer i produces a differentiated variety $\tilde{y}_{it}$ using the production firms' undifferentiated good as its input: $\tilde{y}_{it} = y_{it}$, where y_{it} is the amount of the undifferentiated good demanded by retailer i. Retailers set a nominal price for their variety $\tilde{p}_{it}$ but must pay a quadratic price adjustment cost $\frac{\varphi}{2}\left(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}}-1\right)^{2} Y_t$.

A representative, competitive final good producer purchases differentiated varieties from retailers and uses a CES production technology that incorporates retailers' varieties as inputs, with an elasticity of substitution γ_{CES}: $Y_t = \left(\int \tilde{y}_{it}^{\frac{\gamma_{CES}-1}{\gamma_{CES}}} di\right)^{\frac{\gamma_{CES}}{\gamma_{CES}-1}}$. Together, this gives rise to a New Keynesian Phillips curve (linearized for simplicity):

$$\log \Pi_t = \frac{\gamma_{CES}-1}{\varphi} \times \log \frac{p_t}{p^*} + \beta \log \Pi_{t+1}, \tag{16}$$

where p_t is the relative price of output from financially constrained production firms, and $p^* = \frac{\gamma_{CES}-1}{\gamma_{CES}}$ is its steady-state value. The Phillips Curve links the macro block to the macro-finance block through the relative price p_t. When aggregate demand increases, sticky-price retailers expand production of their differentiated goods and raise demand for the heterogeneous firms' produced goods, y_{it}, which increases their relative price, p_t, and generates inflation through (16).

A representative, competitive capital goods producer produces new capital goods using the technology $\Phi\left(\frac{I_t}{K_t}\right)K_t$, where I_t is units of the final good used to produce capital, K_t is the aggregate start-of-period capital, and $\Phi(x) = \frac{\delta^{\frac{1}{\eta}}}{1-1/\eta}(x)^{1-1/\eta} - \frac{\delta}{\eta-1}$. Its optimality pins down the relative price of capital as $q_{k,t} = \left(\frac{I_t/K_t}{\delta}\right)^{1/\eta}$. The evolution of aggregate capital is then given by

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + K_t\left(1-\delta\right). \tag{17}$$

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[15] In (15), the full argument list of $q_{b,t+1}$ is $\left(\omega', b'_{t+1}\left(\omega', x'\right), k'_{t+1}\left(\omega', x'\right), \tau'_{t+1}\left(\omega', x'\right), z', a'\right)$.

The monetary authority sets the nominal risk-free interest rate R_t^{nom} according to the Taylor rule

$$\log R_t^{\text{nom}} = \log \frac{1}{\beta} + \varphi_\pi \log \Pi_t + \epsilon_t^m, \tag{18}$$

where ϵ_t^m is the monetary policy shock and $\varphi_\pi > 1$ is the Taylor coefficient.

There is a representative household with preferences over consumption C_t and labor supply L_t with life-time expected utility

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \Psi L_t\right)\right],$$

where β is the discount factor and Ψ parametrizes the disutility of labor. The household owns all firms in the economy, serving as a shareholder of both production firms and financial intermediaries that lend to them. The stochastic discount factor and nominal interest rate are linked through the Euler equation for bonds, $\Lambda_{t+1} = \beta \frac{C_t}{C_{t+1}} = \frac{\Pi_{t+1}}{R_t^{\text{nom}}}$, where R_t^{nom} is the nominal risk-free interest rate, and Π_{t+1} is the inflation between period t and $t+1$.

The economy starts from a steady state with all ϵ_t^m equal to zero. An unexpected shock to monetary policy, $\{\epsilon_t^m\}_{t=0}^{\infty}$, is realized at the beginning of period 0, and we study the perfect-foresight transition back to the steady state.

An equilibrium is a set of value functions $\left\{V_{i,t}(\omega,x), V_{i,t}^0(\omega,x)\right\}_{\omega=n,v,i=m,c,s}$, decision rules $\left\{k_t'(\omega,x), \tau_t'(\omega,x), d_t(\omega,x), b_t'(n,x), \zeta_t(v,x)\right\}_{\omega=n,v}$, measure of firms $\mu_t(\omega,x)$, debt price schedule $\left\{q_{b,t}(\omega,b',k',\tau',z,a)\right\}_{\omega=n,v}$, probability of liquidation $\left\{\chi_t^l(\omega,x)\right\}_{\omega=n,v}$, and aggregate variables $\left\{C_t, Y_t, L_t, K_t, I_t, w_t, p_t, q_{k,t}, \Pi_t, R_t^{\text{nom}}, \Lambda_{t+1}\right\}$ such that (i) managers optimize in the normal state, creditors optimize in the violation state, and the probability of liquidation is given by (13), (ii) creditors price competitively, (iii) the household optimizes, (iv) the measure of firms is consistent with decision rules, (v) goods, labor, and capital markets clear, (vi) monetary policy follows (18).

3.3 The Hard Borrowing Constraint Model

We will contrast the above model with sophisticated borrowing constraints with a model with hard borrowing constraints. The key difference from the sophisticated borrowing constraint model specified above is that, consistent with the literature on hard borrowing constraint models (e.g., Khan and Thomas, 2013; Bianchi and Mendoza, 2018; Drechsel, 2023), agents always respect the hard borrowing constraint, so there is no violation state. In the baseline hard borrowing-constraint problem, we also remove managerial agency frictions, as these are not part of standard hard borrowing-constraint models. Besides these two differences, the rest of the production firm's problem is exactly the same as above. The relevant firm state is $x = (b,k,z)$ and the firm's value V_t^{hard} can be written as the solution to the following optimization problem:

$$V_t^{\text{hard}}(x) = \max_{b' \geq 0, k', d} d - AD(d) + \mathbb{E}\left[\Lambda_{t+1} V_{t+1}^{\text{hard},0}(x') | z\right] \tag{19}$$

subject to the flow budget constraint

$$d = \pi_t(z,k) - b + q_{b,t}^{\text{hard}}(b', k', z) \cdot (b' - \gamma b) - T_{c,t}(x) - \left[q_{k,t}(k' - (1-\delta)k) + AC(k,k')\right], \tag{20}$$

and the hard borrowing constraint

$$\tilde{q}_b^{\text{face}} b' \leq \phi^{\text{hard}} \mathbb{E}\left[\pi_{t+1}(z', k') | z\right], \tag{21}$$

where $V_{t+1}^{\text{hard},0}(x')$ incorporates the possibility of the two types of liquidation similar to (13), and $q_{b,t}^{\text{hard}}(\cdot)$ is priced competitively similar to (15).

The enforcement of hard borrowing constraints (21) differs from that of sophisticated borrowing constraints. When a firm hits hard borrowing constraints, it must cut credit indiscriminately to remain within borrowing limits.

We consider two additional extensions featuring different versions of hard borrowing-constraint models. First, we introduce a hard borrowing-constraint model with managerial agency frictions. Second, we consider a hard borrowing-constraint model where the borrowing constraint takes the form of an asset-based borrowing constraint.

$$\tilde{q}_b^{\text{face}} b' \leq \phi^{\text{hard}} q_{k,t+1} k'. \tag{22}$$

4 Firm-Level Implications of Sophisticated Borrowing Constraints

4.1 Model Parameterization and Validation

We calibrate the model in two steps. First, we exogenously assign a subset of parameters. Second, we choose the remaining parameters to match firm-level moments in the data. We describe each step next.

Fixed parameters. Table 1 lists the parameters that are exogenously assigned. The model period is one quarter, so we set the discount factor to $\beta = 0.990$. We choose total returns to scale $\alpha + \nu = 0.850$, a commonly used value in the heterogeneous-firm literature, and set $\nu = 0.610$, close to the value documented in Karabarbounis and Neiman (2014). We set the quarterly depreciation rate to $\delta = 2.5\%$, also a commonly used value in the literature. We set ϕ, the capacity of earnings-based borrowing, to 14, following the contractual evidence in Lian and Ma (2021).[16] The liquidation value per dollar of

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[16] Lian and Ma (2021) find that the capacity of earnings-based borrowing, measured as the debt-to-annual-earnings ratio, is approximately 3.5, which corresponds to a debt-to-quarterly-earnings of 14.

capital is set to $\xi = 0.350$, based on Kermani and Ma (2023). We set the quarterly exogenous exit rate to $\chi_d = 0.5\%$ based on the exit rate of large firms with more than 500 employees in the Census Business Dynamics Statistics (BDS) database, and the corporate tax rate to $\tau_c = 0.3$ based on estimates of the effective corporate tax rate in recent decades in Dyreng et al. (2017). The elasticity of substitution across differentiated varieties is $\gamma_{CES} = 10$, and the slope of the New Keynesian Phillips curve, which determines φ, is $\kappa \equiv (\gamma_{CES} - 1)/\varphi = 0.1$, both taken from Ottonello and Winberry (2020). The elasticity of aggregate investment with respect to the relative price of capital goods is $\eta = 4$, also from Ottonello and Winberry (2020). We set the Taylor rule coefficient to $\varphi_\pi = 1.5$. Finally, the disutility of labor $\Psi = 0.618$ is chosen so that the steady-state real wage is normalized to one.

Table 1: Fixed Parameters

Parameter	Value	Description
Panel 1: Micro		
β	0.990	Discount Factor
α	0.240	Capital Share
ν	0.610	Labor Share
δ	0.025	Depreciation Rate
ξ	0.350	Recovery in Liq.
ϕ	14.000	Debt to Earnings Limit
χ_d	0.005	Exogenous Exit Prob.
τ_c	0.300	Corporate Tax
Panel 2: Macro		
γ_{CES}	10.000	Elasticity of Substitution
κ	0.100	Philips Curve Slope
η	4.000	Elasticity of Capital
φ_π	1.500	Taylor Rule Coefficient
Ψ	0.618	Disutility of Labor

Notes: This table displays parameters exogenously fixed in the calibration. A model period is one quarter.

Calibrated parameters. In the second step of the calibration, we choose the parameters listed in Table 2 to target key moments describing the dynamics of investment, earnings, and financing, both unconditionally and around covenant violations, reported in Table 3 and Figure 1. We compute these moments using the same sample as in Figure 1 (Compustat firms with debt/EBITDA covenants). In terms of unconditional moments, we target the standard deviations of the earnings-to-capital ratio and investment rates ($\frac{k' - (1-\delta)k}{k}$) (0.028 and 0.056); the median leverage (0.320), the median dividend-to-capital ratio (0.000), and the median equity issuance-to-capital ratio (0.001); the fraction of firms newly violating a financial covenant each quarter (2.5%); and the total fraction of firms in the violation state (15%). For firm outcomes around covenant violations, we match the dynamics of median capital and earnings changes shown in Figure 1, normalized by capital in the first quarter of covenant violation t, corresponding to k_{jt+h}/k_{jt} and $\left(\pi_{jt+h}-\pi_{jt}\right)/k_{jt}$ in the model, for $h \in \{-4,-3,\cdots,4\}$.

While all parameters are jointly chosen to match the full set of moments, we describe identification in terms of three groups of moments. First, the unconditional standard deviations of the earnings-to-capital ratio and investment rates are primarily linked to the parameters governing the stochastic process for idiosyncratic productivity shocks, (ρ_z, σ_z), and capital adjustment costs, ψ_{ac} (see Appendix Figure D.1).

Second, the dynamics of capital and earnings around covenant violations and the violation rates are mainly linked to the parameters governing agency frictions and creditor control (see Appendix Figures D.2-D.4). As we further discuss below, more volatile and persistent agency shocks (σ_a, ρ_a) induce a negative correlation between capital and earnings, which helps explain their dynamics before covenant violations. The parameters governing the attractiveness of allocating capital to unproductive use, (μ_a, α_H, C_H), play a central role in explaining the rate of violation and the post-violation dynamics of capital and earnings: when managers derive higher utility from diversion, they avoid frequent violations and allocate higher fractions of capital to unproductive use, which leads to larger increases in earnings when creditors obtain control and curtail such allocation. Additionally, the maturity of debt, γ, governs how quickly debt matures when creditors are in control, also affecting post-violation dynamics, and the probability of transitioning from the violation state to the normal state, λ, primarily affects the violation rate in the steady state.

Finally, median leverage, dividend-to-capital ratio, and equity issuance-to-capital ratio are mainly linked to the parameters governing dividend adjustment and equity issuance costs, ψ_d and ψ_e, and fixed operating costs, f_c (see Appendix Figure D.5).

Table 2: Fitted Parameters

Parameter	Value	Description
Panel 1: Technology		
ρ_z	0.876	Persistence of TFP Shocks
σ_z	0.099	SD of TFP Shocks
ψ_{ac}	0.115	Capital Adj Cost
Panel 2: Agency Frictions and Creditor's Control		
ρ_a	0.704	Persistence of Agency Shocks
σ_a	0.225	SD of Agency Shocks
μ_a	0.066	Mean of Agency Shocks
α_H	0.426	Agency Curvature
C_H	0.055	Agency Fixed Cost
γ	0.977	Coupon Decay Rate
λ	0.107	Prob. of Transition to Normal from Violation
Panel 3: Financial Frictions		
ψ_d	0.458	Dividend Adj Cost
ψ_e	0.619	Equity Issuance Cost
f_c	0.046	Fixed Operating Cost

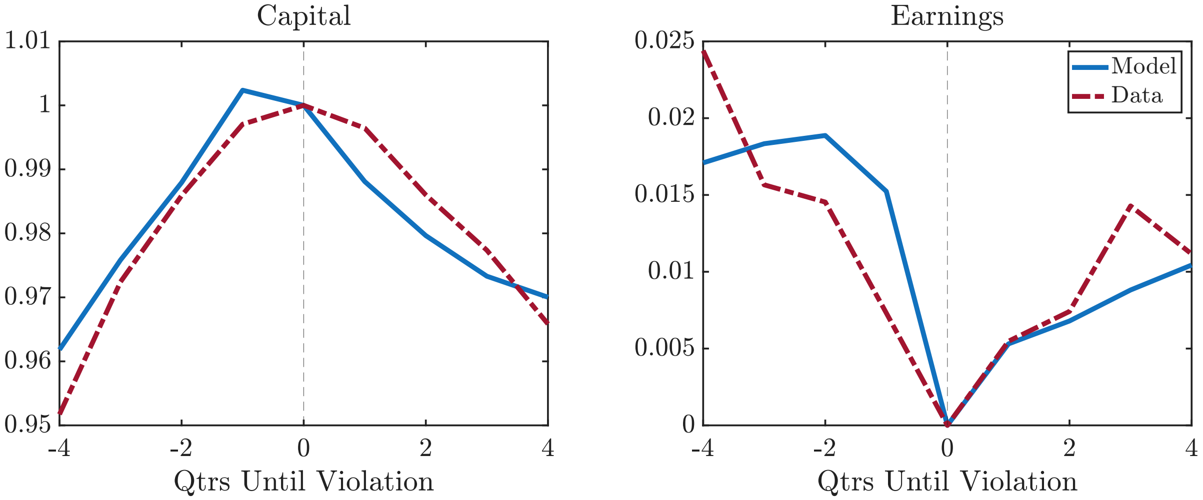
Notes: This table displays parameters that are chosen to match moments in Table 3 and Figure 1.

Table 3: Unconditional Targeted Moments and Model Fit

	Model	Data
Median Leverage	0.293	0.320
New Violation Rate	0.019	0.025
Frac. in Violation	0.146	0.150
Median Dividend/Capital	0.024	0.000
Median Equity/Capital	0.000	0.001
SD of Earnings/Capital	0.028	0.028
SD of Investment Rate	0.081	0.056

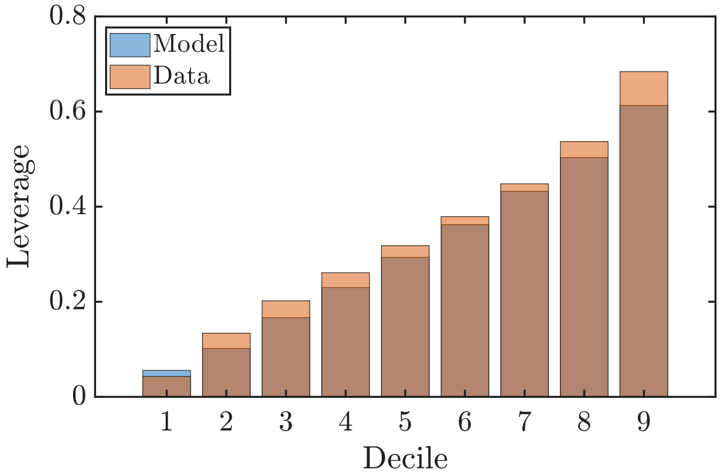
Notes: This table displays the empirical moments targeted in our calibration and the corresponding values generated by the steady state of the model. Leverage, dividend-to-capital ratio, equity-issuance-to-capital ratio, earnings-to-capital ratio, and investment rate correspond to $\tilde{q}_b^{\text{face}} b/k$, $\max\{d,0\}/k$, $\max\{-d,0\}/k$, $\frac{\pi}{k}$, and $\frac{k'-(1-\delta)k}{k}$ in the model, respectively. We use assets as a proxy for capital when calculating unconditional moments in this table.

Figure 2: Targeted Dynamics Around Covenant Violations and Model Fit



Notes: The blue solid line in the left panel shows the median firm's capital in period $t+h$ normalized by its capital at t when it initially violates the sophisticated borrowing constraint, i.e., k_{t+h}/k_t in the model. The blue solid line in the right panel shows the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The lines are based on all firms violating the sophisticated borrowing constraint in the steady state of the model. The red dashed lines in both panels correspond to their empirical counterparts, i.e., the median lines in Figure 1.

Figure 3: Distribution of Leverage: Model vs. Data



Notes: The red bar plots the deciles of firm leverage in the data, using a sample of US Compustat firms that have a debt-to-EBITDA covenant according to DealScan. The blue bar plots the deciles of firm leverage in the steady state of the model, defined as $\tilde{q}_b^{\text{face}} b/k$.

Despite being overidentified, in the sense that the model features 13 fitted parameters and 23 targeted moments (7 unconditional moments and 16 moments describing the dynamics of earnings and capital around covenant violations), the model matches the targeted moments reasonably well. In particular, Figure 2 shows that the model closely tracks the dynamics of capital and earnings around covenant violations, although it implies a slightly less gradual decline in earnings before the violation. The calibrated parameters in Table 2 are broadly comparable to existing estimates in the literature. In addition, the average duration of covenant violations, $\frac{1}{\lambda} \approx 9$ quarters, is close to that observed in our Compustat sample of firms combined with the data on covenant violations in Nini, Smith, and Sufi (2012).

Validation. Figure 3 shows that our model generates a distribution of leverage, which is not targeted in the calibration, that is closely aligned with that observed in the data. In addition, Appendix Table D.1 shows that the firm size distribution implied by the model, illustrated by the lower and upper interquartile distances relative to the median of the capital distribution, is also aligned with its empirical counterpart.

Appendix Table D.1 further shows that the model is broadly consistent with several unconditional moments of investment and earnings that are not targeted in the calibration. First, it produces a positive comovement between investment rates and earnings-to-capital ratios, albeit stronger than in the data, as well as a negative comovement between covenant violation rates and earnings-to-capital ratios. Second, the model predicts positive autocorrelation in both investment rates and earnings-to-capital ratios, although the autocorrelation of earnings-to-capital ratios is larger than in the data.

4.2 Unpacking Firm Outcomes Around Covenant Violations

Policies under manager and creditor control. To better understand how our model matches the dynamics of capital and earnings before covenant violations (under manager control) and after violations (under creditor control), we now zoom in on policies under manager and creditor control.

We start with the normal state under manager control. Panel (a) of Figure 4 plots the manager's chosen next period capital $k'(n,x)$ in the steady state and the corresponding expected earnings $\mathbb{E}\left[\pi'|z,a\right]$ (determined by $k'(n,x)$ and the manager's chosen $\tau'(n,x)$) as functions of current productivity z and agency shock a.[17]

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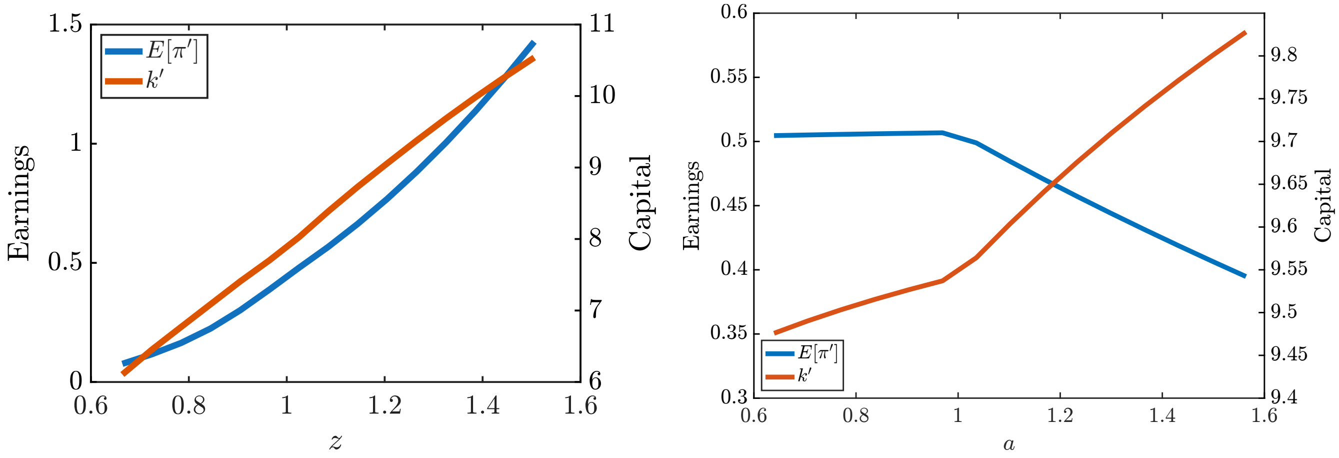
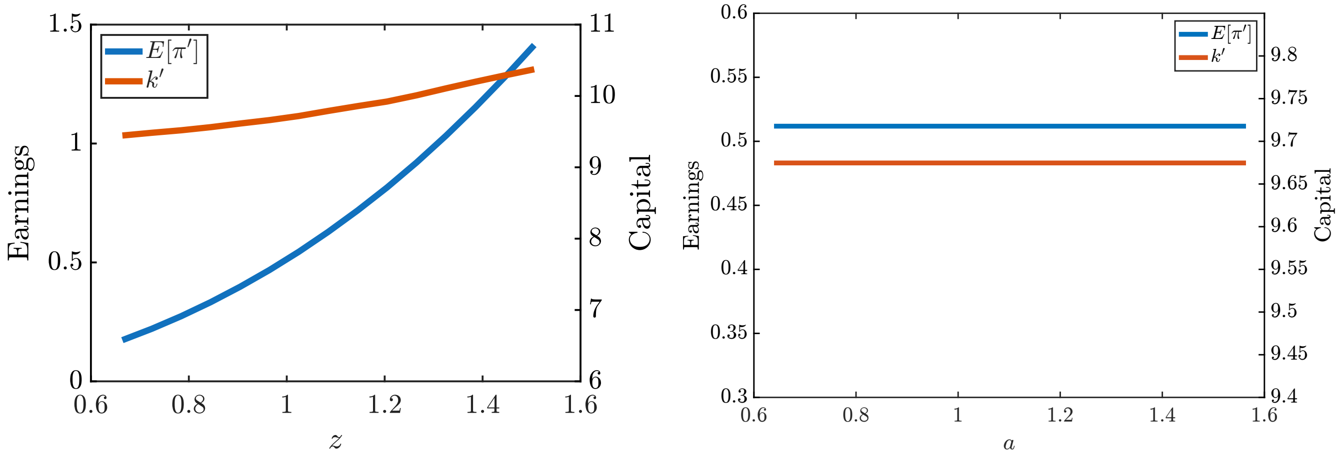
[17] We omit the subscript t when describing steady-state value functions and decision rules. For example, $k'(n,x)$ captures the manager's optimally chosen capital in the normal state in the steady state.

Figure 4: Capital and Earnings under Manager and Creditor Control

(a) Manager control



(b) Creditor control



Notes: The red lines plot next period capital, k', as a function of current productivity z (left subfigures) and the agency shock a (right subfigures). The blue lines plot next period expected earnings, $\mathbb{E}\left[\pi' \mid z, a\right]$, as a function of current productivity z (left subfigures) and the agency shock a (right subfigures). Every non-moving state variable is fixed at the median of the ergodic distribution. The plots in panel (a) are based on the manager's problem in (3), evaluated in the steady state when the firm is in the normal state. The plots in panel (b) are based on the creditor's problem in (7), evaluated in the steady state when the firm is in the violation state.

The left subfigure shows that with higher productivity, z, the manager chooses higher capital, as the marginal product of capital will be persistently higher. Earnings will also be higher due to both the direct effect of productivity and the higher chosen capital. The right subfigure shows that with higher agency shocks, a, the manager also chooses higher capital, as the unproductive use of capital becomes more attractive. However, earnings decrease with a because the manager allocates a higher fraction of capital, $\tau'(n,x)$, to unproductive use.

We now turn to the violation state under creditor control. We first comment on a few features of the creditor's optimal policy in (7). First, the creditor optimally chooses not to allocate any capital to unproductive use, $\tau'(v,x) = 0$, because they do not derive any value from it. As further explained below, such an alleviation of managerial agency frictions is key for the model to match the recovery of earnings after covenant violations in Figure 1. Second, the creditor optimally chooses not to pay dividends, $d(v,x) = 0$.[18] This is because, from the flow budget in (8), creditors can use any funds that would have been paid as dividends (which do not increase creditor value) to increase next period capital, k', thereby increasing creditor value. Third, when the debt burden, b, is high relative to capital k and productivity z, creditors may choose debt forgiveness ($\zeta(v,x) > 0$), although this is rare in the ergodic distribution of our model.

Panel (b) of Figure 4 plots the creditor's chosen next period capital $k'(v,x)$ in the steady state and the corresponding expected earnings $\mathbb{E}\left[\pi'|z,a\right]$ (determined by $k'(v,x)$ and $\tau'(v,x)$) as functions of current productivity z and agency shock a. Compared to Panel (a), capital $k'(v,x)$ increases much less with z. This can be seen from the flow constraint (8) under creditor control when dividend payments are set to zero:

$$(1-\zeta(v,x))\,b = \pi\left(z,(1-\tau)\,k\right) - T_c\left(x\right) - q_k\left(k'(v,x)-(1-\delta)\,k\right) - AC\left(k,k'(v,x)\right). \tag{23}$$

Under creditor control, z only affects $k'(v,x)$ through the direct effect on current earnings $\pi\left(z,(1-\tau)\,k\right)$. This contrasts with the case under manager control, where $k'(n,x)$ also increases with z because it raises the marginal product of capital in the future. Earnings also increase somewhat less with z compared with manager control, but are more sensitive to z than capital.

Moreover, Panel (b) of Figure 4 shows that, under creditor control, next period capital and expected earnings are no longer sensitive to the agency shock, a. This is because creditors choose $\tau'(v,x) = 0$, making their expected earnings insensitive to the agency shock a, and from the flow constraint (23), $k'(v,x)$ does not depend on a either.

Dynamics around the violation of sophisticated borrowing constraints. We now examine what drives the dynamics of capital and earnings before and after covenant violations. Figure 5 shows that firms violating sophisticated borrowing constraints experience negative productivity shocks (decreases in z) and positive agency shocks (increases in a) before violations. After violations, z increases and a decreases. Moreover, unproductive capital use τ (see Appendix Figure D.6) increases before violations due to positive agency shocks and decreases after violations as creditor control alleviates managerial agency frictions.

The decline in earnings before violations in Figure 2 is explained by both negative productivity shocks and positive agency shocks before violations. From Panel (a) of Figure 4, both shocks lower

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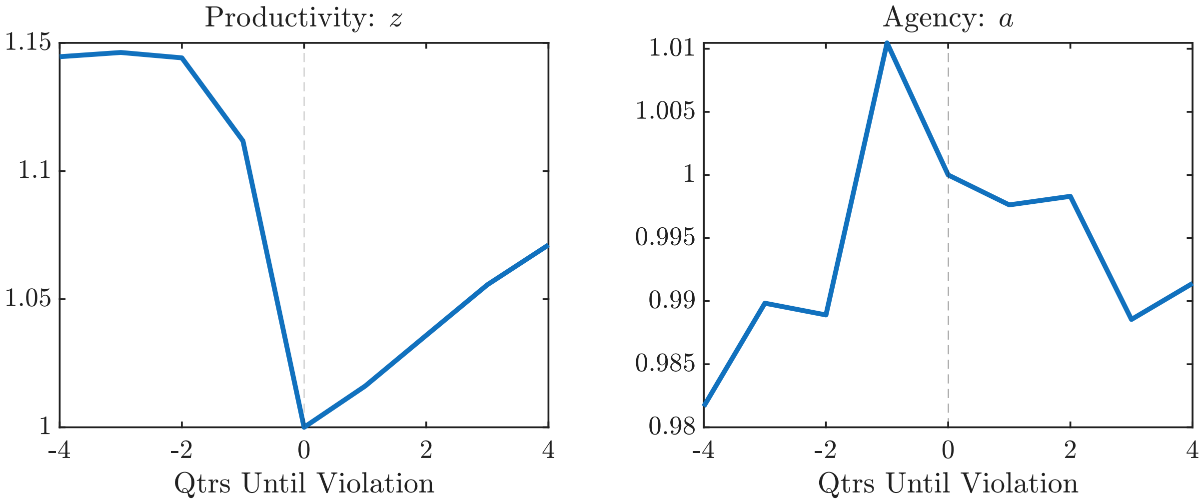
[18] We have verified that, in the violation state, creditors choose $\tau'(v,x) = 0$ and $d(v,x) = 0$ across the entire ergodic distribution of state variables in our model.

Figure 5: Firms' Idiosyncratic Shocks around Covenant Violations



Notes: The left panel plots the median firm's idiosyncratic productivity shock z in period $t+h$ normalized by its productivity shock at t, i.e., z_{t+h}/z_t in the model. The right panel plots the median firm's idiosyncratic agency shock a in period $t+h$ normalized by its agency shock at t when it initially violates the sophisticated borrowing constraint, i.e., a_{t+h}/a_t in the model. The lines are based on all firms violating the sophisticated borrowing constraint in the steady state of the model.

firms' earnings and contribute to violations of sophisticated borrowing constraints in (2). The increase in earnings after violations in Figure 2 is explained by i) creditor control after violations, which eliminates unproductive capital use ($\tau' = 0$), alleviates managerial agency frictions, and raises earnings, and ii) increases in z following violations.

The increase in capital before covenant violations in Figure 2 is explained by positive agency shocks. Without agency shocks, capital would otherwise have decreased due to negative productivity shocks (according to Panel (a) of Figure 4). The decrease in capital after violations in Figure 2 is mostly explained by creditors divesting capital after violations because their demand for capital declines when unproductive capital use is eliminated.

Without agency frictions, productivity shocks alone cannot jointly explain the dynamics of capital and earnings before and after covenant violations. The productivity process in Figure 5 would counterfactually lead to a decrease in capital before violations and an increase in capital after violations.

To further illustrate the importance of managerial agency frictions in explaining the dynamics of capital and earnings around covenant violations, we consider a variant of our model in which managerial agency frictions are shut down ($H(\cdots) = 0$).[19] We recalibrate this model using the same procedure as in our baseline model, matching both the unconditional empirical moments in Table 3 and the firm outcomes around covenant violations in Figure 1. Appendix Figure D.7 shows that such a model has difficulty explaining the dynamics of capital and earnings before and after covenant viola-

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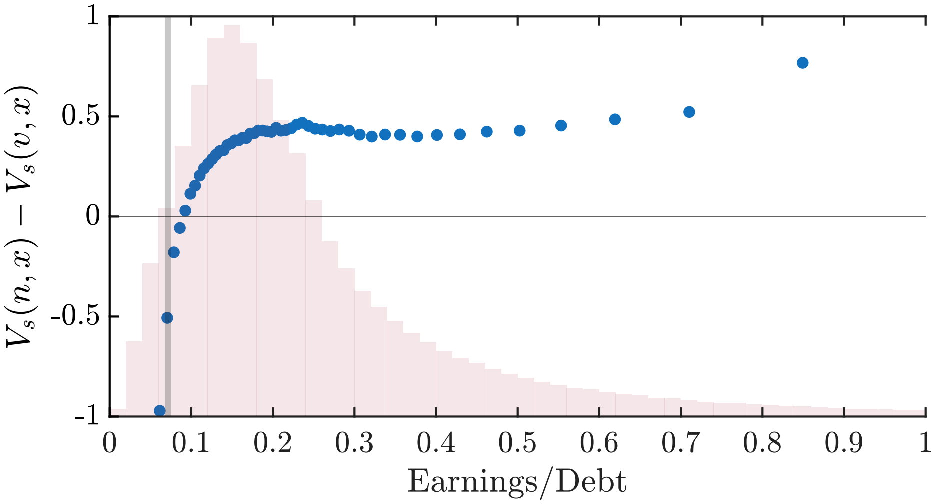
[19] Appendices Tables D.2 and D.3 report the calibrated parameters in this model and its fairly well fit to the unconditional moments.

Figure 6: Differences in Shareholder Value between Manager and Creditor Control



Notes: The figure plots the difference in the shareholder value function (defined in (6)) under manager control and creditor control, $V_s(n,\cdot) - V_s(v,\cdot)$, as a function of firms' earnings-to-debt ratios ($\pi/\left(\tilde{q}_b^{\text{face}} b\right)$ in the model). Each dot represents a percentile of the earnings-to-debt ratio in the ergodic distribution of the steady state of our model. The pink bar plot shows the probability density of the earnings-to-debt ratio in the ergodic distribution of our model. The grey line indicates the earnings-to-debt ratio corresponding to $1/\phi$.

tions despite directly targeting it.

Finally, despite not being targeted, our baseline model with agency frictions can also explain the empirical pattern in which firms' equity value declines before covenant violations (due to a combination of negative productivity shocks and positive agency shocks) and rises afterward (as creditor control alleviates managerial agency frictions and productivity recovers). Appendix Figure D.8 plots firms' cumulative equity returns based on the shareholder value, $V_s(\cdot)$ (defined in (6)), and shows that it is broadly consistent with its empirical counterpart in Figure B.3.

4.3 Do Shareholders Prefer Manager or Creditor Control?

After violations of sophisticated borrowing constraints, creditor control eliminates unproductive capital use, alleviates managerial agency frictions, and improves firms' earnings. One may wonder whether creditor control always improves the firm's performance. That is not the case. The creditor's capital policy in Panel (b) of Figure 4 helps explain why. Under creditor control, capital increases little with productivity: creditors do not raise capital in response to increases in the future marginal product of capital.

To analyze this more systematically, Figure 6 plots the difference in shareholder value between manager control and creditor control, $V_s(n,\cdot) - V_s(v,\cdot)$, as a function of firms' earnings-to-debt ratios. Away from sophisticated borrowing constraints, when the earnings-to-debt ratio is high, shareholder value is higher under manager control. There, the benefits of managers' closer-to-optimal capital policy outweigh the costs of managerial agency frictions arising from the unproductive use of capital. Close to or after violations of sophisticated borrowing constraints, when the earnings-to-debt ratio is low, shareholder value is higher under creditor control. In those states, shareholders prefer creditor control, as manager control leads to both agency frictions in terms of unproductive capital use and strong precautionary behavior, because managers want to avoid violating sophisticated borrowing constraints, since they can no longer derive utility from unproductive capital use. Such precautionary behavior can be seen in Figure 4, where capital is lower under manager control than under creditor control for low levels of productivity (when violations of sophisticated borrowing constraints become a concern). For most of the ergodic distribution of our model, shareholders would prefer managerial control to creditor control.

This finding is consistent with Dewatripont and Tirole (1994), who argue that a contingent transfer of control rights from managers to creditors upon poor firm performance is an effective way for shareholders to mitigate managerial agency frictions when managers cannot be directly contracted on. Because managerial contracts are inherently incomplete and shareholders of public companies are too dispersed to coordinate direct oversight, the contingent transfer of control through debt covenants serves as a practical governance mechanism.

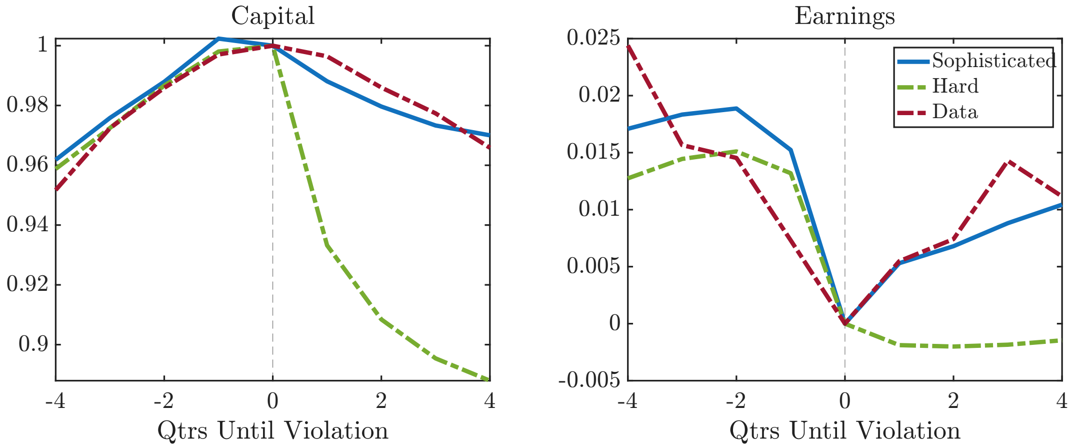
Appendix Figure D.9 also plots the difference in total shareholder and creditor values $V_s(n,\cdot) + V_c(n,\cdot) - (V_s(v,\cdot) + V_c(v,\cdot))$, under manager and creditor control. Similarly, total value is higher under manager control when the earnings-to-debt ratio is high, but higher under creditor control when the earnings-to-debt ratio is low.

4.4 Comparison with the Hard-constraint Model

Can the hard-constraint models described in Section 3.3 explain the dynamics of capital and earnings around covenant violations in Figure 1? We recalibrate this model using the same procedure as in our baseline model, matching both the unconditional empirical moments in Table 3 and the firm outcomes around covenant violations in Figure 1, where we interpret firms hitting the hard borrowing constraint (21) as “violations.” Appendices Tables D.4 and D.5 and Figure D.10 report the calibrated parameters in the hard-constraint model, its fit to the unconditional moments, and the dynamics of capital and earnings around covenant violations.

Despite directly targeting them, the hard-constraint model has difficulty matching both the unconditional moments and the firm outcomes around covenant violations. For unconditional moments, almost all firms with a binding hard borrowing constraint (9.9%, compared with 15% in the data) newly hit the constraint in that quarter (9.4%, compared with 2.5% in the data, despite being directly targeted). In other words, firms in the hard-constraint model counterfactually do not want

Figure 7: Targeted Dynamics Around Covenant Violations and Model Fit: Sophisticated vs Hard



Notes: The left panel plots the median firm's capital in period $t+h$ normalized by its capital at t, i.e., k_{t+h}/k_t in the model. The right panel plots the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The blue lines are based on all firms violating the sophisticated borrowing constraint in the steady state of our baseline model, where t captures when they initially violate the sophisticated borrowing constraint. The green lines are based on all firms hitting the hard borrowing constraint in the steady state of our hard-constraint model, where t denotes the initial period in which the hard constraint binds, recalibrated using the same procedure as in our baseline model, matching both the unconditional empirical moments and the firm outcomes around covenant violations. The red lines correspond to their empirical counterparts, i.e., the median lines in Figure 1.

to remain at the binding constraint and have a strong precautionary motive to maintain some distance from it: being exactly at the hard constraint is costly for firms, since temporarily violating it (as in the sophisticated-constraint model) is not an option. Such a precautionary motive in the hard-constraint model is even stronger than the manager's precautionary motive in the sophisticated constraint model, as further discussed below.

Figure 7 shows that firms' capital and earnings drop significantly after they hit the hard borrowing constraint (the dotted green lines), much more so than in the sophisticated-constraint model (solid blue lines) and in the data (dotted red lines). This is because firms that hit the hard constraint are forced to sell capital to ensure that the hard constraint in (21) is satisfied. Furthermore, in subsequent periods after hitting the hard constraint (even when it no longer binds), firms have a further precautionary motive to maintain some distance from it: exactly binding at the hard constraint is costly, since temporarily violating it, as in the sophisticated-constraint model, is not an option. This further contributes to the large declines in capital and earnings in the periods after hitting the hard constraint. As further studied in the next section, such large declines in firms' outcomes after hitting the hard constraint contribute to the large financial acceleration in hard-constraint models.

In addition, we show that the previous conclusions about the hard-constraint model (e.g., counterfactually large declines in firms' capital and earnings after hitting the constraint) also apply to alternative versions of it: (i) the hard-constraint model with ϕ^{hard} recalibrated such that its median leverage matches that of the data, while keeping all other parameters the same (Appendix Table D.6 and Appendix Figure D.11); (ii) the hard-constraint model with the asset-based borrowing constraint (22), calibrated using the same procedure (Appendix Tables D.7 and D.8, as well as Appendix Figure D.12); (iii) the hard-constraint model with agency frictions, calibrated using the same procedure (Appendix Tables D.9 and D.10, as well as Appendix Figure D.13).

5 Macro Implications of Sophisticated Borrowing Constraints

We now study the macroeconomic implications of sophisticated borrowing constraints, particularly the importance of financial acceleration in determining macroeconomic outcomes. We do so by contrasting these implications with those under hard borrowing constraints. By financial acceleration (e.g., Kiyotaki and Moore, 1997 and Bernanke, Gertler, and Gilchrist, 1999), we refer to the general equilibrium feedback mechanism in which (i) adverse economic outcomes tighten borrowing constraints, and (ii) the tightening of borrowing constraints further worsens economic outcomes.[20]

Section 5.1 studies the macroeconomic effects of a constraint-tightening shock, while Section 5.2 studies the effects of monetary policy shocks. Section 5.3 examines the robustness of these results to alternative model variants and calibrations.

5.1 Constraint-Tightening Shocks

We begin by studying the macroeconomic impact of a constraint-tightening shock that lowers ϕ, a "financial shock" frequently examined in the macro-finance literature (e.g., Jermann and Quadrini, 2012; Khan and Thomas, 2013). This type of shock directly informs the second step of the financial-acceleration feedback mechanism (how tightening borrowing constraints further worsens economic outcomes) and is thus useful for understanding the role of financial acceleration in shaping the macroeconomic impact of other aggregate shocks (e.g., monetary policy shocks), which we analyze later.

In this experiment, we allow ϕ_t to be time varying. We then study the responses of key aggregate variables to an unexpected tightening of borrowing constraints at period 0, with an initial size of 5% of steady-state borrowing capacity and with a persistence of 0.5.[21] We compute the perfect foresight transition path of the economy as it converges back to steady state. We compare the sophisticated-constraint model and the hard-constraint model based on their respective calibrations discussed in the previous section.

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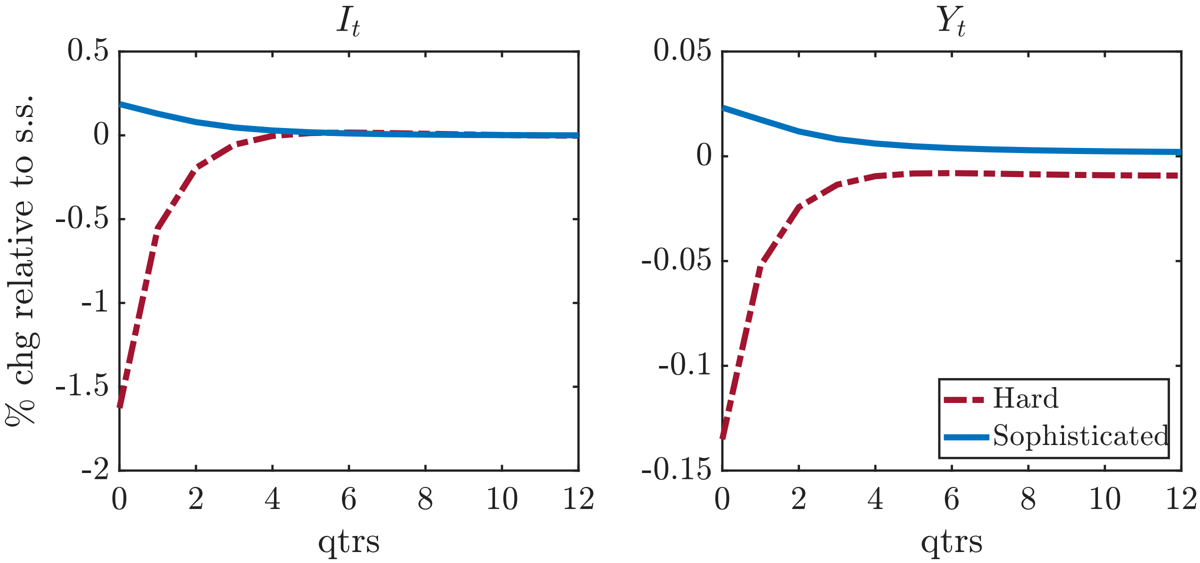
[20] Similarly, the mechanism also applies to positive economic outcomes, which relax borrowing constraints.

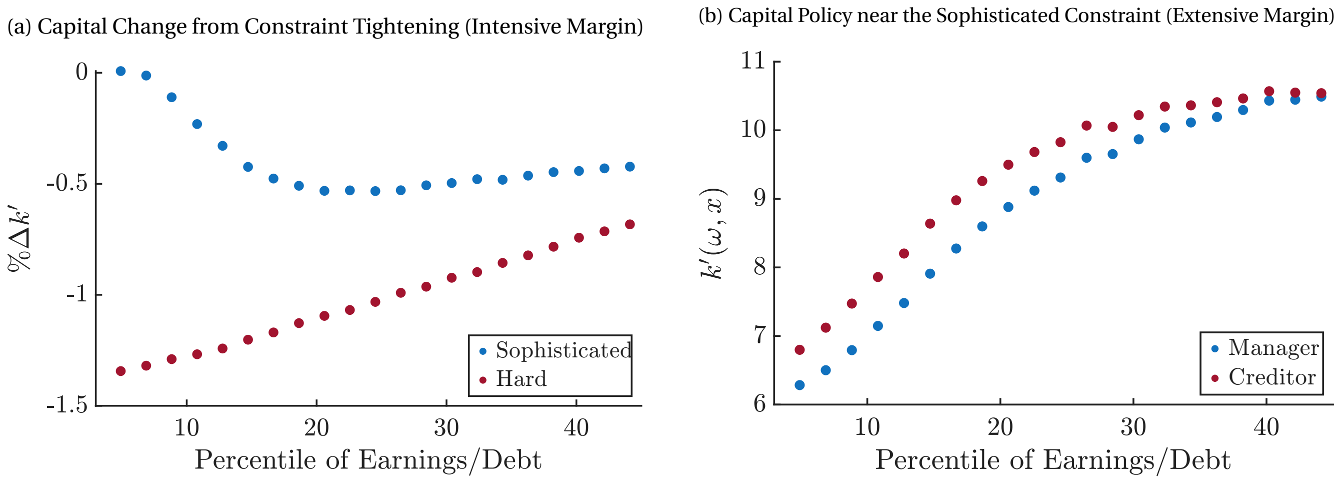
[21] Specifically, in the sophisticated-constraint model, we consider a tightening with initial magnitude $\Delta\phi_0 = -5\% \times \phi$, and in the hard-constraint model, a tightening with initial magnitude $\Delta\phi_0^{\text{hard}} = -5\% \times \phi^{\text{hard}}$.

Figure 8: Macroeconomic Effects of a Constraint-Tightening Shock



Notes: This figure plots the responses of aggregate investment and output to a tightening of sophisticated borrowing constraints with initial magnitude $\Delta\phi_0 = -5\% \times \phi$ in the sophisticated-constraint model (solid blue lines), and the counterpart responses to a tightening of hard borrowing constraints with initial magnitude $\Delta\phi_0^{\text{hard}} = -5\% \times \phi^{\text{hard}}$ in the hard-constraint model (dashed red lines). The constraint-tightening shock has a persistence of 0.5. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure 9: Unpacking the Macroeconomic Effects of a Constraint-Tightening Shock


(a) Capital Change from Constraint Tightening (Intensive Margin)

(b) Capital Policy near the Sophisticated Constraint (Extensive Margin)

Notes: Panel A plots the mean percentage change in next-period capital k' when ϕ is reduced to $0.95 \times \phi$ (and ϕ^{hard} to $0.95 \times \phi^{\text{hard}}$ in the hard-constraint model) as a function of the percentile of firms' earnings-to-debt ratio ($\pi / \left(\tilde{q}_b^{\text{face}} b\right)$ in the model). In both cases, the policy functions under the original and tightened constraint are evaluated at states drawn from the steady-state distribution associated with the benchmark economy, and percentiles are computed with respect to the same distribution. Blue dots correspond to the sophisticated-constraint model; red dots to the hard-constraint model. Panel B plots next-period capital k' as a function of firms' earnings-to-debt ratios ($\pi / \left(\tilde{q}_b^{\text{face}} b\right)$ in the model) under creditor control (red dots) and manager control (blue dots) around the sophisticated borrowing constraint.

Effects under hard borrowing constraints. Figure 8 plots the responses of aggregate investment I_t and output Y_t to the constraint-tightening shock under both hard and sophisticated borrowing constraints. Under hard borrowing constraints, we observe a significant decline in aggregate investment and output. To better understand the large decline in investment, the red dots in panel (a) of Figure 9 show the partial-equilibrium effect of a constraint-tightening on firms' capital as a function of a firm's earnings-to-debt percentile. Firms for which the borrowing constraint is binding, which tend to have low percentiles of earnings-to-debt, adjust debt one-for-one in response to the tightening, resulting in a severe decline in capital as they sell assets to pay down debt to satisfy the constraint. Additionally, firms whose borrowing constraints are not currently binding but are close to the constraint also lower their capital significantly, driven by a precautionary motive to avoid hitting the hard constraint in the future.[22] The decline in aggregate investment then depresses aggregate demand and lowers aggregate output and earnings (see Appendix Figure D.15 for impulse responses of earnings). The resulting decline in earnings further tightens hard borrowing constraints, generating financial acceleration that

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[22] The percentage change in capital in panel (a) of Figure 9 increases smoothly with the earnings-to-debt percentile, without a visible kink separating binding and non-binding firms. This reflects the choice of x-axis variable: the panel plots capital changes against the percentile of the *current* earnings-to-debt ratio ($\pi / \left(\tilde{q}_b^{\text{face}} b\right)$), which maps smoothly into the probability of binding hard constraints that depend on *next period's* ratio as in (21). Appendix Figure D.14 instead plots capital changes against the percentile of firms' *expected* earnings-to-debt ratio ($\mathbb{E}[\pi'] / \left(\tilde{q}_b^{\text{face}} b'\right)$), where firms in lower percentiles exhibit the sharpest declines and a clear kink appears at the binding threshold.

further worsens economic outcomes.

Effects under sophisticated borrowing constraints. Under sophisticated borrowing constraints, in contrast, Figure 8 shows that the constraint-tightening shock generates a small increase in aggregate investment and output. To better understand this, it is useful to decompose the effects of a tightening of the sophisticated borrowing constraint on firms' investment into three channels (formally defined in Appendix C.3): (i) an *intensive-margin channel*, defined as the partial-equilibrium effect of the shock without reallocating control rights; (ii) an *extensive-margin channel*, defined as the partial-equilibrium effect arising from changes in the allocation of control rights driven by violations of the sophisticated constraint induced by the shock; and (iii) *indirect channels*, given by the effects of general-equilibrium movements in prices induced by the shock.

To illustrate the intensive-margin channel, the blue dots in panel (a) of Figure 9 show the partial-equilibrium effect of a constraint-tightening on firms' capital as a function of a firm's earnings-to-debt percentile, without reallocating control rights. Firms in the violation state governed by creditors, which tend to have low percentiles of earnings-to-debt, are only weakly affected by the shock, because ϕ does not directly affect creditors' decisions in violation, as the transition back to the normal state is exogenous. This is in sharp contrast to the hard-constraint model, in which firms at the bottom of the earnings-to-debt distribution are the most responsive to a constraint-tightening shock. Firms in the normal state governed by managers are more strongly affected by the shock, because managers respond precautionarily by cutting debt and investment to mitigate the risk of violating the constraint and losing control. However, this contraction is substantially smaller than in the hard-constraint model because, as shown in Figure 7, the decline in firms' capital after violating the sophisticated constraint is much milder than after hitting hard borrowing constraints.

Turning to the extensive-margin channel, the constraint-tightening shock induces some firms to violate the sophisticated constraint and transition from manager to creditor control. The transfer of control rights leads to a change in capital approximately given by $k^{'}(v,x) - k^{'}(n,x)$. Panel (b) of Figure 9 compares $k^{'}(v,x)$ and $k^{'}(n,x)$ across percentiles of firms' earnings-to-debt distribution, showing that the creditor chooses a higher level of capital $k^{'}(v,x)$ than the manager $k^{'}(n,x)$ near the sophisticated constraint, because managers precautionarily adjust capital to avoid losing control, while creditors lack such a precautionary motive. This implies that the extensive margin leads to an increase in investment in response to the constraint-tightening shock.

As illustrated in Appendix Figure D.16, which decomposes the aggregate effects on investment into the three channels, the extensive margin quantitatively dominates the intensive margin, implying an expansionary partial-equilibrium response of investment to the constraint-tightening shock. Finally, the price of capital increases in response to the partial-equilibrium increase in investment (see Appendix Figure D.15), generating a contractionary indirect effect that moderately attenuates this response.

The responses of aggregate output and earnings (also shown in Appendix Figure D.15) under sophisticated borrowing constraints follow mechanisms similar to those of investment, with an additional channel operating through the extensive margin: creditor control after violation alleviates managerial agency frictions (by setting τ' to zero) and improves firms' production and earnings for a given level of capital. This translates into a lower fraction of capital allocated to unproductive use, as depicted in Appendix Figure D.15. Together, the positive extensive-margin responses break the second step of the financial-acceleration feedback mechanism: tightening borrowing constraints no longer worsens economic outcomes.

The role of agency frictions in macroeconomic effects. To assess how our macroeconomic results depend on the presence of agency frictions, we analyze the effects of the constraint-tightening shock in a variant of the sophisticated-constraint model without managerial agency frictions, described in Section 4 (see also panel (b) of Appendix Figure D.7). Appendix Figure D.17 shows that the aggregate effects of this shock remain muted relative to those in the hard-constraint model, indicating that the lack of amplification does not rely on agency frictions. The sign of the effects, however, reverses relative to the baseline model: the constraint-tightening becomes mildly contractionary on both investment and output. This reversal reflects the fact that, without agency frictions, the transfer of control rights from managers to creditors is no longer associated with an improvement in the productive use of capital. The transfer therefore neither increases output nor raises earnings available to finance investment, weakening the extensive-margin channel that drives the expansionary responses of investment and output in the baseline.

5.2 Monetary Policy Shocks

We now turn to studying the macroeconomic effects of monetary policy shocks. To gauge the importance of financial acceleration in determining the aggregate responses to a monetary policy shock and to connect it to the previously studied responses to the constraint-tightening shock, we compare the benchmark aggregate responses, incorporating the full general-equilibrium feedback of financial acceleration, to counterfactual aggregate responses where the feedback is shut down. In this counterfactual, borrowing constraints are governed by the steady-state earnings function $\pi(\cdots)$ instead of the actual earnings function $\pi_t(\cdots)$. For example, in the sophisticated-constraint model, the evolution of firm states is governed by the counterpart of (2):

$$\tilde{q}_b^{\text{face}} b_{jt+1} \leq \phi\pi\left(z_{jt+1}, \left(1-\tau_{jt+1}\right) k_{jt+1}\right). \tag{24}$$

The counterfactual shuts down financial acceleration by shutting down the general-equilibrium loop, whereby adverse economic outcomes (which lead to lower relative prices of production firms' goods, p_t) tighten borrowing constraints through a lower earnings function, $\pi_t(\cdots)$, and further worsen economic outcomes. Hence, the differences between the aggregate responses that incorporate general equilibrium feedback and the counterfactual responses can be interpreted as the contribution of financial acceleration. These differences are also closely linked to the responses to constraint-tightening shocks studied above.

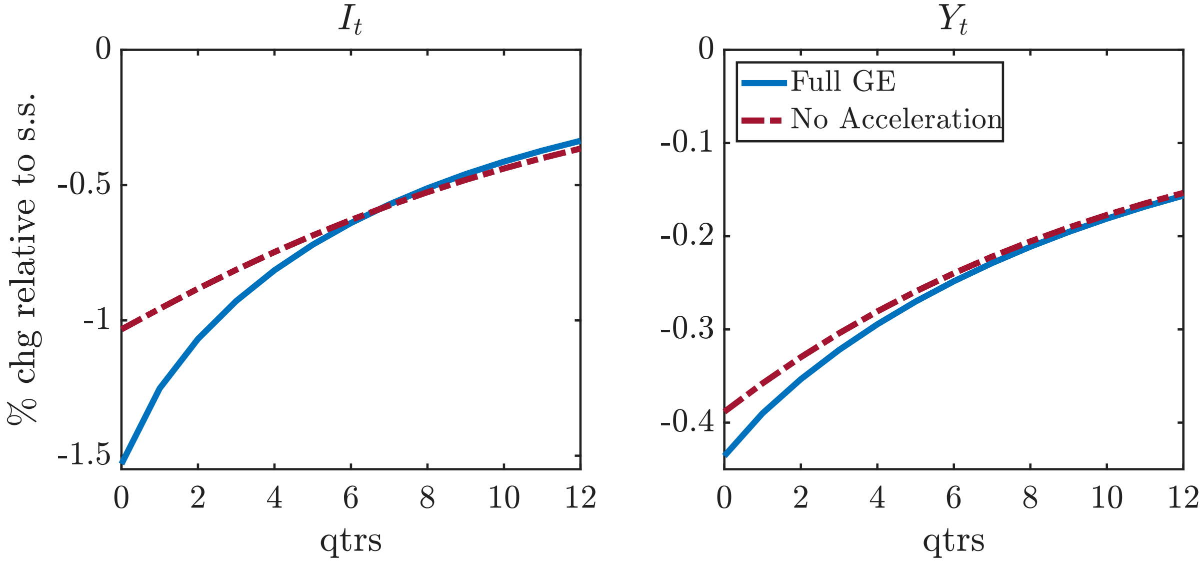
We consider a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.90. The persistence is designed to roughly capture the fact that the magnitude of nominal interest rate changes following a monetary policy shock halves over a one-year horizon, as documented in Caravello, McKay, and Wolf (2024). We compute the perfect foresight transition path of the economy as it converges back to steady state.

Panel (a) of Figure 10 depicts the responses of aggregate investment and output to the monetary tightening in the hard-constraint model, both for the full general-equilibrium responses, which incorporate financial acceleration, and for the counterfactual responses that shut down financial acceleration. The counterfactual responses are significantly dampened relative to the full general-equilibrium responses, indicating that financial acceleration plays a nontrivial role in worsening aggregate outcomes. The hard-constraint model produces considerable acceleration for two reasons. First, the monetary policy shock depresses aggregate demand and lowers the relative price of goods produced by financially constrained firms, p_t, thereby reducing their earnings (see Appendix Figure D.18) and further tightening borrowing constraints. Second, a tightening of the constraint has a significant macroeconomic impact, as emphasized in the previous subsection.

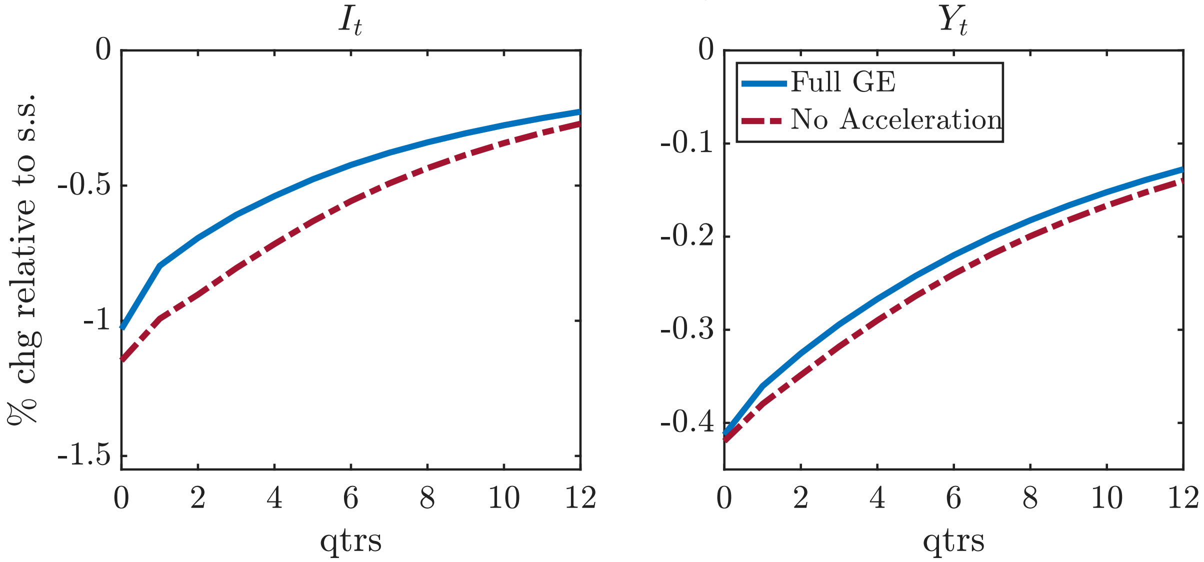
Panel (b) of Figure 10 plots the aggregate responses to the monetary policy shock in the sophisticated-constraint model. Overall, the responses of aggregate variables are less negative than those in the hard-constraint model and are broadly aligned with those in New Keynesian models with heterogeneous firms and default risk (Ottonello and Winberry, 2020). The figure further shows that the full general-equilibrium responses are similar to, and slightly less negative than, the counterfactual responses that shut down financial acceleration. As in the hard-constraint model, the monetary policy shock lowers relative prices p_t and earnings (see Appendix Figure D.19), tightening and triggering violations of the sophisticated borrowing constraint. However, unlike in the hard-constraint model, a tightening of the sophisticated constraint does not have a contractionary macroeconomic impact for the reasons discussed above. Together, these results indicate muted financial acceleration under sophisticated borrowing constraints, in contrast to the significant financial acceleration under hard borrowing constraints.

Figure 10: Macroeconomic Effects of a Contractionary Monetary Policy Shock

(a): Hard Borrowing Constraints



(b): Sophisticated Borrowing Constraints



Notes: The figure plots the responses of aggregate investment and output to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with persistence of 0.9 under hard borrowing constraints in panel (a) and sophisticated borrowing constraints in panel (b). The solid lines show the full general equilibrium responses incorporating financial acceleration, while the dashed lines show the counterfactual responses in which financial acceleration is shut down, i.e., borrowing constraints are governed by steady-state earnings $\pi(\cdots)$ instead of the actual earnings function $\pi_t(\cdots)$. Responses are computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations from steady state.

5.3 Robustness and Additional Results

The main finding that financial acceleration is more significant under hard borrowing constraints than under sophisticated borrowing constraints is robust to alternative model variants and calibrations of hard-constraint and sophisticated-constraint models. In Appendix Figures D.20–D.21, we show that this is indeed the case when comparing the macroeconomic responses of our sophisticated-constraint model with those of the hard-constraint model with asset-based lending described in Section 3.3. In Appendix Figures D.22–D.23, we show that the same finding applies when comparing with a variant of the hard-constraint model where we recalibrate ϕ^{hard} such that its median leverage matches that of the data, while keeping all other parameters the same. In Appendix Figures D.24–D.25, we show that the same finding also applies when comparing with a variant of the hard-constraint model with agency frictions, recalibrated using the same procedure as in our baseline model. We also analyze a variant of the sophisticated-constraint model without agency frictions. Appendix Figure D.17, discussed above, and Appendix Figure D.26 show that financial acceleration is also muted in this variant, though the constraint-tightening becomes mildly contractionary rather than mildly expansionary as in the baseline sophisticated-constraint model with agency frictions.

Finally, while our analysis focuses on the effects of aggregate shocks in economies with different types of borrowing constraints, we also provide complementary evidence by analyzing how firms facing different types of borrowing constraints respond to monetary policy shocks. To do so, we study the investment response to monetary policy shocks (identified in Bauer and Swanson, 2023) in our sample of Compustat firms (Section 2), examining heterogeneous responses across firms with different shares of cash flow-based debt in total debt (using debt classifications from Lian and Ma (2021)). Appendix Figure B.6 shows that, in line with our macro-level findings, firms with an above-median share of debt against cash flows, and therefore primarily subject to sophisticated borrowing constraints from financial covenants,[23] are less sensitive to monetary policy shocks. While this evidence is suggestive, it points to the value of additional empirical work on how different types of debt contracts shape the response to aggregate shocks.[24]

6 Conclusion

For decades, many macroeconomic models have focused on financial frictions faced by nonfinancial firms as the key mechanism for the amplification of aggregate fluctuations. A centerpiece of these models is the presence of hard borrowing constraints, which require indiscriminate reductions of credit when firms hit such limits. These responses lead to declines in prices and quantities that further

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[23] Debt against cash flows places stronger emphasis on monitoring firms' operations for which sophisticated borrowing constraints are useful, whereas debt against physical assets emphasizes the intrinsic liquidation value of the asset regardless of the firm's performance (Diamond, 2023; Kermani and Ma, 2025).

[24] See Caglio, Darst, and Kalemli-Ozcan (2021) for additional evidence in this area. Complementing these findings, Appendix Figure B.7 shows that nonresidential investment by firms is less sensitive than residential investment by households. Although sophisticated borrowing constraints are important among nonfinancial firms, they do not apply to households, for which implementing analogous creditor control is not feasible, and the household sector can therefore be more sensitive to shocks.

tighten borrowing limits, setting off the financial accelerator.

In this paper, we reassess this view by studying the implications of sophisticated borrowing constraints, akin to financial covenants commonly used in the debt contracts of large U.S. nonfinancial firms. We show that the active role played by creditors following covenant violations can dampen financial acceleration. This result reflects the fact that it is not in the interest of any stakeholder to downscale the firm as sharply as hard borrowing constraint models would impose. In recent years, there has been a rise of covenant light (“cov-lite") loans, which have financial covenants that only require compliance when firms take major actions instead of enforcing covenants on a regular quarterly basis, possibly due to an increase in institutional investors (e.g., loan mutual funds) in the syndicated loan market who are more dispersed and face higher coordination costs for implementing creditor control (Becker and Ivashina, 2016). Our analyses suggest that this development could heighten macroeconomic risk, as weaker creditor control means less dampening of financial acceleration after adverse shocks.

More broadly, our findings suggest that the emergence of sophisticated borrowing constraints in firms' debt contracts is important for understanding macroeconomic dynamics. These contract features are first-order in practice but have yet to be incorporated into mainstream macroeconomic analyses. Our work in this paper examines how they shape the transmission of macroeconomic shocks. In addition, they are relevant for research on a number of other questions, such as the assessment of credit booms and financial stability risks, the application of macroprudential policies, as well as the long-run effects of financial development and creditor sophistication. These questions can open new avenues for future work.

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Appendices for: Sophisticated Borrowing Constraints and Macroeconomic Dynamics

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This Appendix contains further material for the article "Sophisticated Borrowing Constraints and Macroeconomic Dynamics". Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."–"D." refer to the main article.

A Violations of Financial Covenants and Creditor Control

In this appendix, we provide further elaboration and examples about creditor control during financial covenant violations. As mentioned in Section 2, violations of financial covenants trigger "technical default," which gives creditors the legal right to demand immediate repayment. Creditors rarely do so, and instead they ask firms to implement their demands in exchange for a waiver for the covenant violation. The waiver will suspend or relax the financial covenant (to a looser ratio) for some period of time (ranging from one quarter to several quarters).[30] Accordingly, in our model, firms spend some amount of time in the violation state where the old covenant no longer binds (but they need to implement creditors' requests).

In exchange for the waiver, creditors commonly require that firms implement several types of changes. First, they often impose restrictions on additional borrowing, and it is well-documented that net debt issuance falls to zero on average following covenant violations (Nini, Smith, and Sufi, 2012). This is consistent with our model. Second, firms substantially reduce capital expenditures, acquisitions, and operating expenses. Third, firms may also replace management. These features are widely viewed by corporate finance and legal studies as evidence that creditors play a role in corporate governance and curb agency problems that led to overexpansion and poor managerial decisions leading up to covenant violations. Finally, firms may pay lenders a fee for the waiver, which we abstract from since the average amount is around 40 basis points (Jiang and Xu, 2019). In some cases,

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[30] As the examples show, sometimes creditors suspend the covenant entirely for a short period of time, and sometimes they relax the covenant to looser thresholds for a longer period of time. These different changes are difficult to measure in a uniform way. But overall it maps reasonably well to our model specification that firms return to the normal state after some period of time which they do not fully decide.

the loans are amended to charge higher interest rates or require additional collateral, and we abstract away from these additional features since they do not always occur (Nini, Smith, and Sufi, 2012; Jiang and Xu, 2019). Some of these concessions are formalized by amending loan contracts, such as adding covenants that restrict capital expenditures and other payments. Some are changes in operations that firms agree to, and as legal scholars write, "lenders may need to do no more than make it understood that they will look more kindly on future waivers of loan covenants" (Baird and Rasmussen, 2006).

Below we present three examples to illustrate further, including the original example of covenant violation discussed in Nini, Smith, and Sufi (2012) and two other well-known companies.

Digital generation systems. This is the original example used in (Nini, Smith, and Sufi, 2012). The company's quarterly report (10-Q) filing on November 9, 2005 states:

> As of September 30, 2005, the Company was not in compliance with the covenant related to its leverage ratio. On November 9, 2005, the Company received a waiver from its lenders as of September 30, 2005. In connection with securing this waiver, certain other changes were made to the credit facility which, among other things, reduced the amount that can be borrowed under the Company's revolving line of credit from $15.0 million to $4.5 million.

The "leverage ratio" covenant in loan contracts refers to debt to EBITDA (not debt to assets or debt to equity). The loan amendment is appended to the 10-Q filing, which states that the covenant is waived for one quarter: "Lenders have agreed to waive the violation of the Senior Leverage Ratio, set forth in Section 9.02 of the Agreement for the fiscal quarter ending September 30, 2005." In exchange for the waiver, the amendment added restrictions on capital expenditures:

> The Borrower will not permit the aggregate Capital Expenditures of the Loan Parties to exceed (a) $400,000 during the Borrower's fiscal quarter ending December 31, 2005, or (b) $300,000 during each fiscal quarter of Borrower ending on March 31, 2006 or any fiscal quarter of Borrower thereafter."[31]

There was also a $10.5 million reduction in the company's line of credit and a 100-basis-point increase in the interest rate spread of the loan.

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[31] The prior restriction on capital expenditures put in place in April 2005 had a more generous limit: "The Borrower will not permit the aggregate Capital Expenditures of the Loan Parties to exceed (a) $1,200,000 during the Borrower's fiscal quarter ending March 31, 2005, or (b) $2,500,000 during the period of four consecutive fiscal quarters of Borrower ending on March 31, 2006 or any period of four consecutive fiscal quarters of Borrower thereafter."

MGM Resorts. MGM Resorts is a major American hospitality and entertainment company. The company's quarterly report (10-Q) filing on May 11, 2009 states:

> As of March 31, 2009, the Company was not in compliance with its financial covenants under its senior credit facility. On March 16,2009, the Company entered into an amendment and waiver to its senior credit facility, which provided for, among other conditions, a waiver of the requirement that the Company comply with such financial covenants through May 15, 2009.

The loan amendment in the current report (8-K) filing states that the covenants are waived for two quarters: "the requirements of Sections 6.5 and 6.6 of the Loan Agreement, but solely with respect to the Fiscal Quarter ending March 31, 2009, are hereby waived, but only for and through the period ending on May 15, 2009" (where Sections 6.5 and 6.6 specify the leverage ratio and interest coverage ratio covenants respectively). In exchange for the waiver, the loan amendment includes restrictions on additional indebtedness and investments: the company cannot borrow new money beyond very limited and defined categories, or make new investments unless they fall into a list of specifically provided categories.[32] Interest rates also increased by around 100 basis points.

Tyson Foods. Tyson Foods is one of the largest meat companies in the world. The company's 10-Q filing on August 10, 2006 states:

> On July 27, 2006, the Company entered into a third amendment to its five-year revolving credit facility and the three-year term loan facility of its subsidiary, Lakeside Farm Industries, Ltd. These amendments modified the minimum required interest coverage ratio, temporarily suspended the maximum allowed leverage ratios and implemented temporary minimum consolidated EBITDA requirements. At the time the Company completed the initial draft of its third quarter interim financial statements, the Company determined it would not have been in compliance with the maximum allowed leverage ratios with respect to the revolving credit facility and term loan as of July 1, 2006. The Company obtained a waiver for such covenants it would not have been in compliance with and negotiated less restrictive debt covenants during the fourth quarter of fiscal 2006. The Company is in compliance with the new covenant requirements as of July 1, 2006.

In this case, the company violated the leverage ratio (i.e., maximum debt to EBITDA) covenant. In the waiver, creditors relaxed the leverage ratio and interest coverage ratio (i.e., minimum EBITDA

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[32] See the amendment for details about the allowed categories: `https://www.sec.gov/Archives/edgar/data/789570/000095013409005579/p14437exv10.htm`.

to interest expense) covenants, and added a minimum EBITDA covenant. The loan amendment is included in a current report (8-K) filing on July 31, 2006, which states:

> (i) Section 7.13 of the Credit Agreement is hereby amended to read in its entirety as follows:
>
> "SECTION 7.13. *Leverage Ratio.* The Borrower shall not permit the Leverage Ratio at any time during any of the periods set forth below to exceed the ratio set forth below opposite such period:

Period	**Ratio**
Second Quarter of Fiscal Year 2007	4.75:1.00
Third Quarter of Fiscal Year 2007	4.50:1.00
Fourth Quarter of Fiscal Year 2007	4.00:1.00
Fiscal Year 2008	3.75:1.00
Fiscal Year 2009 and thereafter	3.25:1.00

> (j) Section 7.14 of the Credit Agreement is hereby amended to read in its entirety as follows:
>
> "SECTION 7.14. *Interest Expense Coverage Ratio.* The Borrower shall not permit the ratio of Consolidated EBITDA to Consolidated Interest Expense for any period of four consecutive fiscal quarters ending with any fiscal quarter referred to below to be less the ratio set forth below with respect to such fiscal quarter:

Fiscal Quarter	**Ratio**
Third Quarter of Fiscal Year 2006 through First Quarter of Fiscal Year 2007	2.00:1.00
Second Quarter of Fiscal Year 2007	2.50:1.00
Each Quarter thereafter	3.00:1.00

> (k) Article VII of the Credit Agreement is hereby amended by adding the following new Sections:
>
> "SECTION 7.15. *Minimum Consolidated EBITDA.* The Borrower shall not permit Consolidated EBITDA for any period of four consecutive fiscal quarters ending with any fiscal quarter referred to below to be less than the amount set forth below with respect to such quarter:

Fiscal Quarter	**Amount**
Third Quarter of Fiscal Year 2006	$625,000,000
Fourth Quarter of Fiscal Year 2006	$490,000,000
First Quarter of Fiscal Year 2007	$550,000,000

Even though in this case lenders did not explicitly restrict investment through loan amendments, the company's capital expenditures fell sharply in the next 4 quarters (from $646 million in the 4 quarters leading up to the covenant violation to only $225 million in the following 4 quarters), and capital stock fell by close to 10%. They also had no issuance of new debt, compared to around $1 billion new debt issuance in the year before. These sudden reductions in capital expenditures and borrowing likely reflect creditors' influence on the company's operations (even without explicitly mandating caps in the amendment).

B Additional Empirical Results

Table B.1: Firm Outcomes after Financial Covenant Violation

	Change in EBITDA/capital		Capital growth	
	(1)	(2)	(3)	(4)
New covenant violation	0.034***	0.052***	-0.039***	-0.025**
	(0.011)	(0.012)	(0.011)	(0.011)
Beginning of period log capital	-0.006***	-0.078***	-0.001	-0.194***
	(0.001)	(0.008)	(0.001)	(0.017)
Beginning of period book leverage	0.004	-0.134**	-0.268***	-0.268***
	(0.061)	(0.053)	(0.077)	(0.072)
L.EBITDA/capital	-0.007	-0.097**	0.074***	0.086***
	(0.030)	(0.041)	(0.010)	(0.015)
L.Capital growth	-2.749*	-1.678	-9.519***	-4.971**
	(1.638)	(1.178)	(1.661)	(2.084)
Firm FE	No	Yes	No	Yes
Higher-order covenant controls	Yes	Yes	Yes	Yes
Lagged covenant controls	Yes	Yes	Yes	Yes
Observations	46713	46594	49057	48939
R-squared	0.039	0.260	0.140	0.394

Notes: This table presents regressions at the firm-quarter level:

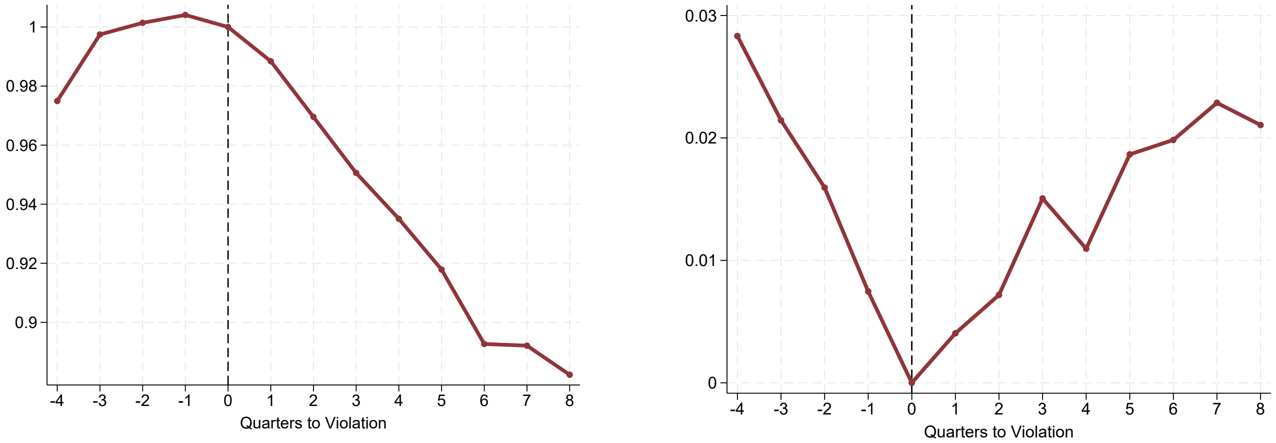
$$\Delta Y_{i,t\to t+4} = \alpha_t + \beta \text{Viol}_{i,t} + \sigma \log k_{i,t} + \sum_{k=1}^{4} \gamma_k z_{i,t-k} + \varepsilon_{i,t}.$$

The dependent variable is $(\text{EBITDA}_{i,t+4} - \text{EBITDA}_{i,t})/k_{i,t}$ in columns (1) and (2), and $k_{i,t+4}/k_{i,t}$ in columns (3) and (4). The key independent variable is an indicator variable, $\text{Viol}_{i,t}$, which takes value one if firm i reports an initial financial covenant violation in quarter t. Control variables $z_{i,t-k}$ for k from one to four include book leverage (debt/assets), EBITDA (normalized by beginning-of-period capital), and capital growth during quarter $t-k$, as well as their squares and cubes following Nini, Smith, and Sufi (2012). We include quarter fixed effects in columns (1) and (3), and additional firm fixed effects in columns (2) and (4). We double cluster standard errors by firm and quarter. Asterisks denote significance levels (*** 1%, ** 5%, * 10%). Sample years are 2002 to 2016.

Figure B.1: Firm Outcomes around Covenant Violations: Macro Recessions

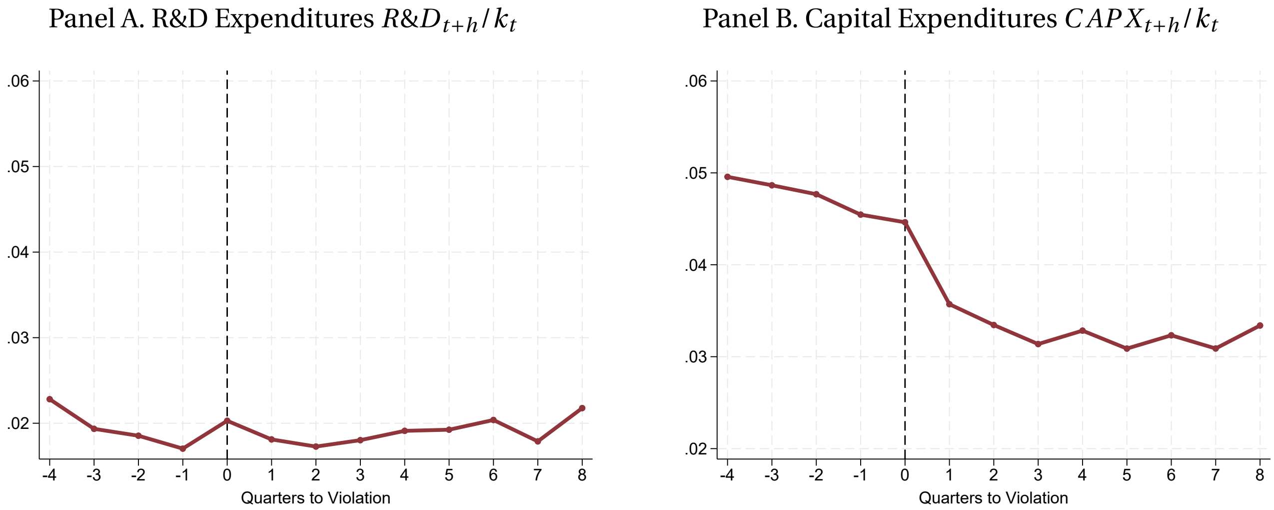
Panel A. Dynamics of Capital k_{t+h}/k_t

Panel B. Dynamics of Earnings Change $(EBITDA_{t+h} - EBITDA_t)/k_t$



Notes: This figure shows the same plot of capital and earnings around covenant violation as Figure 1, restricted to covenant violations during macro recessions (2001 to 2002 and 2007 to 2009).

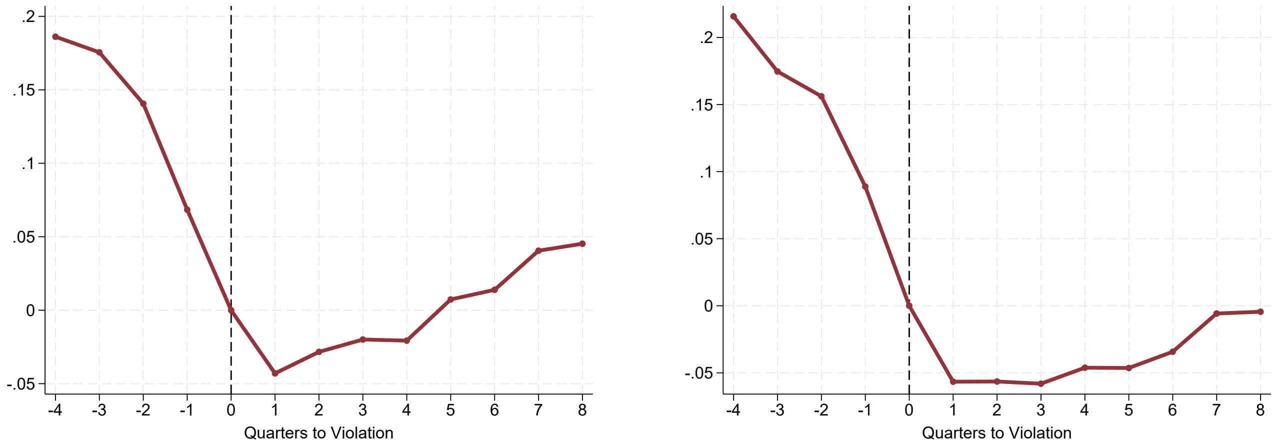
Figure B.2: R&D Expenditures vs Capital Expenditures around Covenant Violations


Panel A. R&D Expenditures $R\&D_{t+h}/k_t$

Panel B. Capital Expenditures $CAPX_{t+h}/k_t$

Notes: Panel A shows firms' quarterly research and development (R&D) expenditures in quarter $t+h$ (normalized by capital at the beginning of quarter t). Panel B shows firms' quarterly capital expenditures (CAPX) in quarter $t+h$ (normalized by capital at the beginning of quarter t). The line shows the median value for each quarter $t+h$. The sample includes US Compustat firms that have a debt-to-EBITDA covenant in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.

Figure B.3: Cumulative Stock Returns around Covenant Violations



Panel A. Cumulative Stock Returns

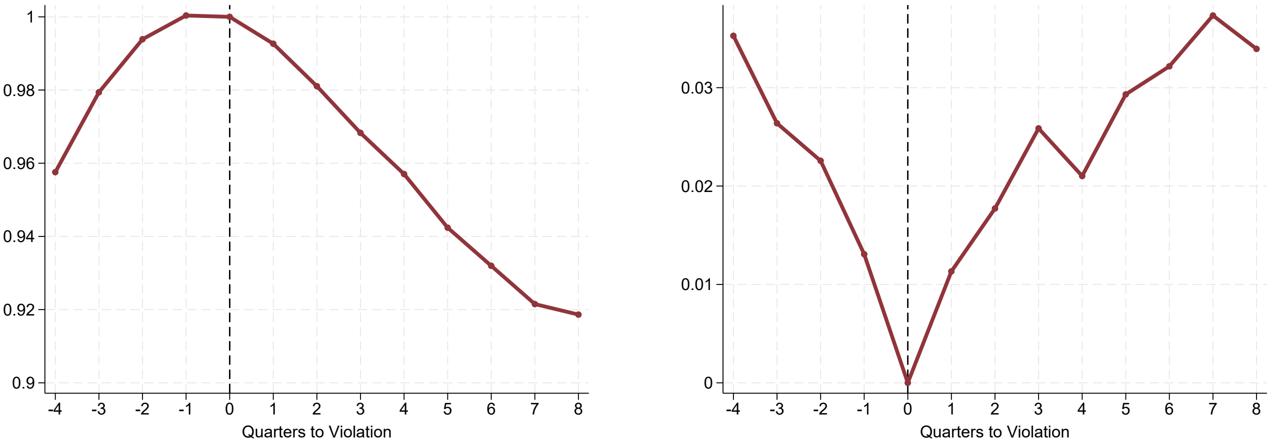
Panel B. Cumulative Abnormal Returns (Net of Beta)

Notes: Panel A shows firms' cumulative stock returns from quarter t (when they initially report a covenant violation) to quarter $t+h$. Specifically, we use each quarter's returns R_{t+s} (including dividends) from Center for Research in Security Prices (CRSP) and then calculate the cumulative sum of log returns as $\sum_{s=1}^{h} \log R_{t+s}$ for $h \geq 1$, $-\sum_{s=h+1}^{0} \log R_{t+s}$ for $h \leq -1$, and 0 for $h = 0$. Panel B shows firms' cumulative abnormal stock returns (raw returns net of beta times market returns) from quarter t (when they initially report a covenant violation) to quarter $t+h$. The line shows the median value for each quarter $t+h$. The sample includes US Compustat firms that have all types of financial covenants in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.

Figure B.4: Firm Outcomes around Covenant Violations: All Firms with Financial Covenants

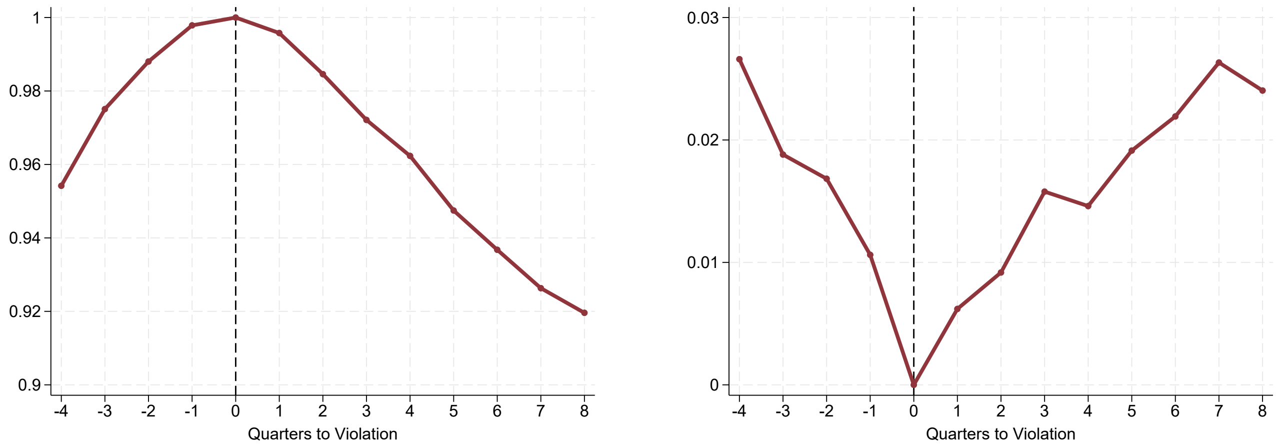
Panel A. Cumulative Capital Changes k_{t+h}/k_t

Panel B. Earnings Changes $(EBITDA_{t+h} - EBITDA_t)/k_t$



Notes: Panel A shows firms' capital in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by capital at the beginning of quarter t so that the differences over time come from the numerator rather than the denominator). Panel B shows firms' quarterly earnings in quarter $t+h$ relative to quarter t (normalized by capital at the beginning of quarter t). The line shows the median value for each quarter $t+h$. The sample includes US Compustat firms that have all types of financial covenants in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.

Figure B.5: Firm Outcomes around Covenant Violations: Firms with Earnings-Based Financial Covenants

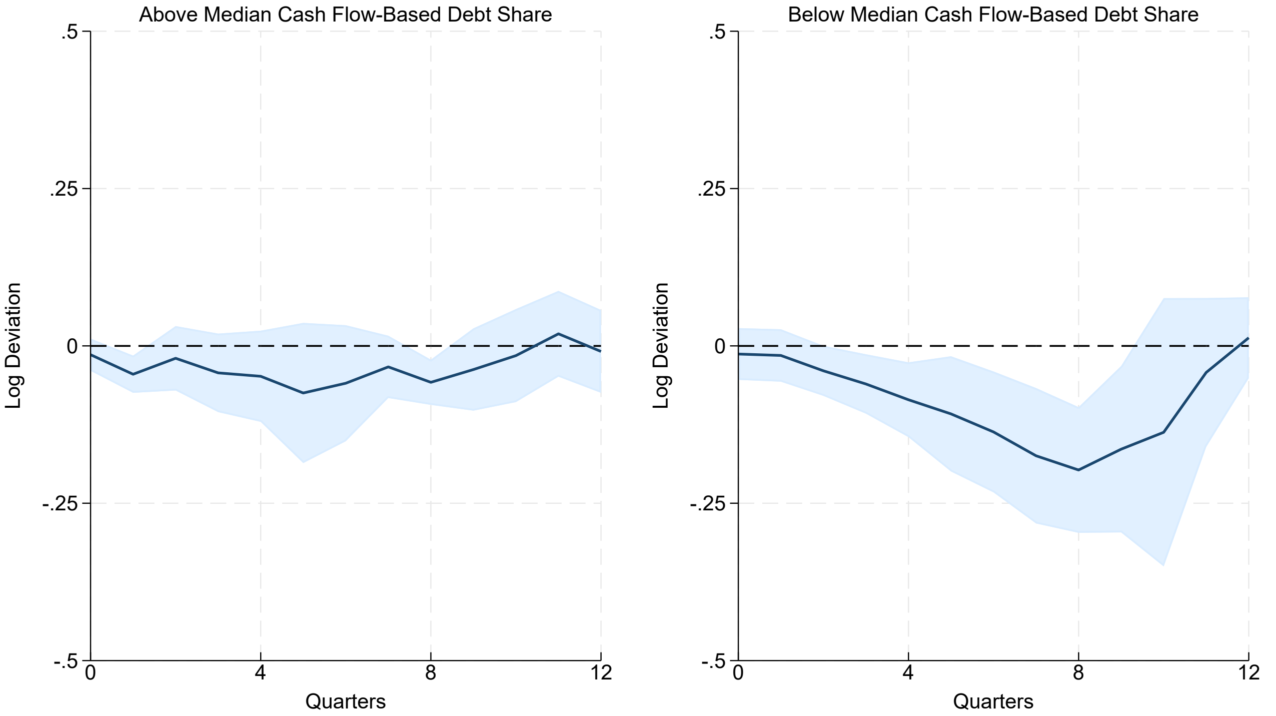


Panel A. Dynamics of Capital $k_{i,t+h}/k_{i,t}$

Panel B. Dynamics of Earnings Changes $\left(EBITDA_{i,t+h} - EBITDA_{i,t}\right)/k_{i,t}$

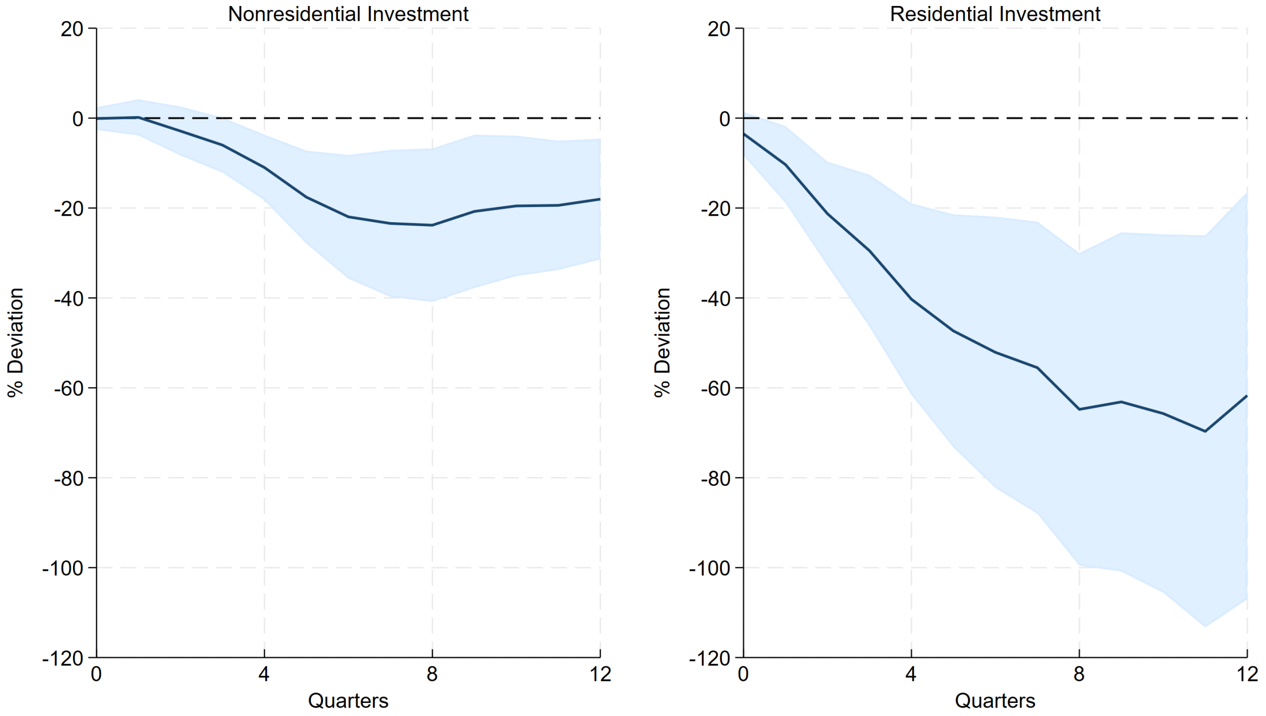
Notes: Panel A shows firms' capital in quarter $t+h$ relative to quarter t when they initially report a covenant violation (normalized by capital at the beginning of quarter t so that the differences over time come from the numerator rather than the denominator). Panel B shows firms' quarterly earnings in quarter $t+h$ relative to quarter t (normalized by capital at the beginning of quarter t). The line shows the median value for each quarter $t+h$. The sample includes US Compustat firms that have earnings-based covenants (including debt-to-EBITDA covenants and interest coverage ratio covenants) in quarter t (according to DealScan data). Covenant violation data are kindly shared by Greg Nini and cover 1996 to 2016.

Figure B.6: Impulse Response of Firm Capital to Monetary Policy Shock



Notes: This figure shows the impulse response of firm-level deflated capital (property, plant, and equipment) to orthogonalized monetary policy shocks (Bauer and Swanson, 2023). We estimate Jordà (2005) local projections $y_{i,t+h} = \alpha_i + \beta mps_t + \gamma z_{i,t} + \varepsilon_{i,t}$. The outcome variable $y_{i,t+h}$ is log deflated capital of firm i in quarter $t+h$. The left panel plots the coefficient β among firms with above median share of cash flow-based debt in total debt, and therefore more subject to sophisticated borrowing constraints. The right panel plots the coefficient β among firms with below median share of cash flow-based debt in total debt, and therefore less subject to sophisticated borrowing constraints. The control variables $z_{i,t}$ include 4 lags of the outcome variable, 4 lags of the monetary policy shock, and 4 lags of the Fed Funds rate, log real GDP, log CPI, unemployment rate, the excess bond premium (Gilchrist and Zakrajšek, 2012), book leverage, EBITDA (normalized by capital), and Tobin's Q. The sample covers Compustat firms from 2002 (beginning of firm debt composition data) to 2016. Standard errors are Driscoll and Kraay (1998) with a bandwidth of 8 quarters. The shaded area represents the 90% confidence interval.

Figure B.7: Impulse Response of Nonresidential vs Residential Investment to Monetary Policy Shock



Notes: This figure shows the impulse response of aggregate nonresidential and residential investment to orthogonalized monetary policy shocks (Bauer and Swanson, 2023). We estimate quarterly Jordà (2005) local projections $y_{t+h} = \alpha + \beta mps_t + \gamma z_t + \varepsilon_t$, and plot the regression coefficient β on the monetary policy shock. The outcome variable y_{t+h} is log deflated nonresidential investment in the left panel, and log deflated residential investment in the right panel. The control variables z_t include 4 lags of the outcome variable, 4 lags of the monetary policy shock, and 4 lags of the Fed Funds rate, log real GDP, log CPI, unemployment rate, and the excess bond premium (Gilchrist and Zakrajšek, 2012). The sample covers 1988 (the beginning of Bauer and Swanson (2023) monetary policy shocks) to 2016. Standard errors are Eicker-Huber-White following Montiel Olea and Plagborg-Møller (2021). The shaded area represents the 90% confidence interval.

C Supplementary Theoretical Details

C.1 Distribution of Firms

Let $F_t(\omega, x)$ denote the measure of firms at time t (after the allocation of control rights and liquidation), where $x = (b, k, \tau, z, a)$ and $\omega \in \{n, v\}$, and let $x' = (b', k', \tau', z', a')$. For computation, we work with a discretized approximation to this distribution. We construct a tensor grid over the state vector $x = (b, k, \tau, z, a)$: b, k, and τ are discretized on uniformly spaced grids, while the idiosyncratic shocks z and a are discretized using the Rouwenhorst (1995) method, with $Q_z(z'|z)$ and $Q_a(a'|a)$ denoting the Markov transitions. Let $\mathbf{F}_{\omega,t}$ denote the $n_x \times 1$ vector of firm measures (i.e., discrete histogram weights) under control rights $\omega \in \{n, v\}$ at time t, evaluated at each grid point x. The total number of grid points is $n_x = n_b n_k n_\tau n_z n_a$, where n_b is the number of grid points for b and the other dimensions are defined analogously.

The law of motion is

$$\mathbf{F}_{n,t+1} = \left(\mathbf{I} - \boldsymbol{\chi}_{n,t+1}\right)\left(\boldsymbol{\Gamma}_{n,t+1}\mathbf{Q}'_{n,t}\mathbf{F}_{n,t} + \lambda \mathbf{Q}'_{v,t}\mathbf{F}_{v,t}\right) + F^0_{t+1}\mathbf{F}^{\text{ent}}_{t+1} \tag{C.1}$$

$$\mathbf{F}_{v,t+1} = \left(\mathbf{I} - \boldsymbol{\chi}_{v,t+1}\right)\left(\boldsymbol{\Gamma}_{v,t+1}\mathbf{Q}'_{n,t}\mathbf{F}_{n,t} + (1-\lambda)\,\mathbf{Q}'_{v,t}\mathbf{F}_{v,t}\right) \tag{C.2}$$

$$F^0_{t+1} = \mathbf{1}'\Big(\boldsymbol{\chi}_{n,t+1}\big(\boldsymbol{\Gamma}_{n,t+1}\mathbf{Q}'_{n,t}\mathbf{F}_{n,t} + \lambda \mathbf{Q}'_{v,t}\mathbf{F}_{v,t}\big) + \boldsymbol{\chi}_{v,t+1}\big(\boldsymbol{\Gamma}_{v,t+1}\mathbf{Q}'_{n,t}\mathbf{F}_{n,t} + (1-\lambda)\,\mathbf{Q}'_{v,t}\mathbf{F}_{v,t}\big)\Big) \tag{C.3}$$

$$\mathbf{F}^{\text{ent}}_{t+1} = \frac{\left(\mathbf{I} - \boldsymbol{\chi}_{n,t+1}\right)\mathbf{F}_{n,t}}{\mathbf{1}'\left(\left(\mathbf{I} - \boldsymbol{\chi}_{n,t+1}\right)\mathbf{F}_{n,t}\right)} \tag{C.4}$$

where the diagonal matrices $\boldsymbol{\chi}_{n,t+1}$ and $\boldsymbol{\chi}_{v,t+1}$ collect, along the diagonal, the values of $\chi^l_{t+1}(n, x)$ and $\chi^l_{t+1}(v, x)$ evaluated on the discretized state space, and indicate whether a firm is liquidated. Similarly, the diagonal matrices $\boldsymbol{\Gamma}_{n,t+1}$ and $\boldsymbol{\Gamma}_{v,t+1}$ collect $\mathbb{I}\left(n = \Gamma_b\left(b_{t+1}, \bar{b}_{t+1}(x')\right)\right)$ and $\mathbb{I}\left(v = \Gamma_b\left(b_{t+1}, \bar{b}_{t+1}(x')\right)\right)$, and indicate whether firms that were in the normal state in the previous period remain there or transition to the violation state. The matrices $\mathbf{Q}_{n,t}$ and $\mathbf{Q}_{v,t}$ map the state $(\omega, b, k, \tau, z, a)$ at time t into the post-decision state $(\omega, b', k', \tau', z', a')$ for $\omega = n$ and $\omega = v$, respectively, combining the Markov transition kernels for shocks with manager and creditor policies; see the computational appendix for details on their construction. Finally, F^0_{t+1} is the implied mass of entrants (equal to the mass liquidated) at $t+1$, $\mathbf{F}^{\text{ent}}_{t+1}$ gives the entrants' initial distribution over (n, x'), and $\mathbf{1}$ is an $n_x \times 1$ vector of ones.

The first two equations track surviving incumbents, while the last term in $\mathbf{F}_{n,t+1}$ adds a mass F^0_{t+1} of entrants that replace liquidated firms. The expression for $\mathbf{F}^{\text{ent}}_{t+1}$ specifies the entrants' distribution: each entrant starts in the normal state, with initial borrowing, capital, fraction of capital in unproductive use, and technology and agency shocks drawn from the distribution of x among normal-state firms in the previous period. If a drawn x would imply liquidation at $t+1$, we immediately redraw.

Combining these equations gives a more compact expression for the evolution of the distribution:

$$\underbrace{\begin{bmatrix} \mathbf{F}_{n,t+1} \\ \mathbf{F}_{v,t+1} \end{bmatrix}}_{\mathbf{F}_{t+1}} = \begin{bmatrix} \mathbf{I}-\boldsymbol{\chi}_{n,t+1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}-\boldsymbol{\chi}_{v,t+1} \end{bmatrix} \underbrace{\begin{bmatrix} \boldsymbol{\Gamma}_{n,t+1} & \lambda\mathbf{I} \\ \boldsymbol{\Gamma}_{v,t+1} & (1-\lambda)\mathbf{I} \end{bmatrix}}_{\boldsymbol{\Gamma}_{t+1}} \underbrace{\begin{bmatrix} \mathbf{Q}'_{n,t} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}'_{v,t} \end{bmatrix}}_{\mathbf{Q}_t} \begin{bmatrix} \mathbf{F}_{n,t} \\ \mathbf{F}_{v,t} \end{bmatrix} + \begin{bmatrix} F^0_{t+1}\mathbf{F}^{\text{ent}}_{t+1} \\ \mathbf{0} \end{bmatrix}, \tag{C.5}$$

which simplifies to

$$\mathbf{F}_{t+1} = \left(\mathbf{I}-\boldsymbol{\chi}_{t+1}\right)\boldsymbol{\Gamma}_{t+1}\mathbf{Q}_t\mathbf{F}_t + \begin{bmatrix} F^0_{t+1}\mathbf{F}^{\text{ent}}_{t+1} \\ \mathbf{0} \end{bmatrix}, \tag{C.6}$$

where $\boldsymbol{\chi}_{t+1} = \text{diag}\left(\boldsymbol{\chi}_{n,t+1}, \boldsymbol{\chi}_{v,t+1}\right)$ and the identity matrix is conformable with the new dimensions.

C.2 Calibration Details

We target the dynamics of capital and earnings in a one-year window around covenant violations. To construct the model counterparts, we simulate a panel of firms from the model's steady state. For firm i, let $h = 0$ denote the quarter in which the firm first violates the sophisticated borrowing constraint, and compute $\frac{k_{i,h}}{k_{i,0}}$ and $\frac{\pi_{i,h}-\pi_{i,0}}{k_{i,0}}$ for $h \in \{-4,-3,\ldots,-1,1,\ldots,3,4\}$ (we omit $h = 0$, which is mechanical). From the simulated panel, we then compute the median capital ratio and median normalized earnings change h quarters from violation:

$$m^k_h = \text{median}\left(\frac{k_{i,h}}{k_{i,0}}\right) \qquad m^\pi_h = \text{median}\left(\frac{\pi_{i,h}-\pi_{i,0}}{k_{i,0}}\right)$$

We also target a set of unconditional cross-sectional moments. From the same simulation, we compute median leverage (m^{lev}), the fraction of firms in violation (m^v), the new violation rate ($m^{n\to v}$), the median dividend-to-capital ratio (m^d), the median equity-issuance-to-capital ratio (m^e), the standard deviation of earnings-to-capital (m^π), and the standard deviation of the investment rate (m^{inv}). This gives a total of 23 moments. We index them by m_i for $i = 1,\ldots,23$, and denote their empirical counterparts by $\hat{m}_i$. We calibrate the model parameters to minimize the distance between the model moments and their empirical counterparts, using the objective

$$\sum_{i=1}^{23} w_i\left(\frac{m_i-\hat{m}_i}{1+\hat{m}_i}\right)^2 \tag{C.7}$$

The weights w_i are allowed to differ across moments and are chosen to capture salient features of the targeted dynamics around covenant violations in Figure 1. Specifically, we set $w_i = 1$ for m^e, m^d, m^π, and m^{inv}; $w_i = 2$ for m^v, $m^{n\to v}$, and m^{lev}; $w_i = 4$ for m^k_h and m^π_h with $h \in \{1,2,3,4\}$; and $w_i = 8$ for m^k_h and m^π_h with $h \in \{-4,-3,-2,-1\}$. We minimize (C.7) in several steps. We first conduct a broad parameter search, generating 5,000 candidate parameter combinations using Sobol sequences and retaining the 30 best according to (C.7). We then search locally around each of these calibrations: for each candidate, we draw 100 additional parameter combinations via Sobol sequences and form convex combinations with the candidate calibration. We evaluate the objective at these new points, compare across all candidate parameters, and repeat until convergence.

C.3 Computation Details

C.3.1 Preliminaries

We discretize the state space on a tensor grid over $x = (b, k, \tau, z, a)$. We discretize b, k, and τ on uniformly spaced grids, and use the Rouwenhorst (1995) method to discretize a and z. Let $n_x = n_b n_k n_\tau n_z n_a$ denote the total number of grid points, where n_b is the number of nodes in the b-dimension and other variables are defined analogously. We set $n_a = 3$ and $n_z = 5$ for the shocks, and $n_b = n_k = 17$ and $n_\tau = 15$ for the policy variables.

C.3.2 Computing Transition Matrices

Following Young (2010), let the nodes in the b-dimension be $\{b_1, \ldots, b_{n_b}\}$, and define the following function for $\omega \in \{n, v\}$:

$$Q^b_{\omega,t}\left(x, j\right) = \left[\mathbb{1}\left(b'_t(\omega,x) \in \left[b_{j-1}, b_j\right)\right) \frac{b'_t(\omega,x) - b_{j-1}}{b_j - b_{j-1}} + \mathbb{1}\left(b'_t(\omega,x) \in \left[b_j, b_{j+1}\right]\right) \frac{b_{j+1} - b'_t(\omega,x)}{b_{j+1} - b_j}\right].$$

Evaluating this for every $j \in \{1, \ldots, n_b\}$ and node x generates a matrix $\mathbf{Q}^b_{\omega,t}$ of dimension $n_x \times n_b$, where rows index nodes x and columns index nodes of b.[33] Repeat this with k and τ, generating $\mathbf{Q}^k_{\omega,t}$ and $\mathbf{Q}^\tau_{\omega,t}$. Each matrix records the probability of reaching the j-th grid point in the corresponding dimension. Similarly, for z and a, define

$$Q_z\left(x, j\right) = Q_z(z_j | z) \qquad Q_a\left(x, \ell\right) = Q_a(a_\ell | a)$$

where $Q_z(z'|z)$ and $Q_a(a'|a)$ are transition matrices from the Rouwenhorst method. Evaluate this for every $j \in \{1, \ldots, n_z\}$ and $\ell \in \{1, \ldots, n_a\}$ and node x, generating matrices $\mathbf{Q}_z$ and $\mathbf{Q}_a$. Finally, $\mathbf{Q}_{\omega,t} = \mathbf{Q}_a \otimes \mathbf{Q}_z \otimes \mathbf{Q}^\tau_{\omega,t} \otimes \mathbf{Q}^k_{\omega,t} \otimes \mathbf{Q}^b_{\omega,t}$, where $\otimes$ denotes the row-wise Kronecker product, and hence $\mathbf{Q}_{\omega,t}$ is $n_x \times n_x$.

C.3.3 Solving Bellman Equations

We solve the manager and creditor problems using value function iteration. The solution algorithm proceeds as follows.

---

[33] For the boundary cases, $Q^b_{\omega,t}(x,1) = \mathbb{1}\left(b'_t(\omega,x) \in [b_1, b_2]\right) \frac{b_2 - b'_t(\omega,x)}{b_2 - b_1}$ and $Q^b_{\omega,t}(x,n_b) = \mathbb{1}\left(b'_t(\omega,x) \in \left[b_{n_b-1}, b_{n_b}\right]\right) \frac{b'_t(\omega,x) - b_{n_b-1}}{b_{n_b} - b_{n_b-1}}$.

Step 1. Specify initial guesses for $V_{c,t}(n,x)$, $V_{c,t}(v,x)$, $V_{m,t}(n,x)$, and $V_{m,t}(v,x)$ on the tensor product grid of x. Following the literature on quantitative models of long-term debt (Hatchondo and Martinez, 2009), we use the terminal problem from a finite-horizon approximation as the initial guess.

Step 2. We precompute expected continuation values for each agent. To evaluate $\mathbb{E}\left[\Lambda_{t+1}V^0_{m,t+1}(\omega',x')|z,a\right]$ and $\mathbb{E}\left[\Lambda_{t+1}V^0_{c,t+1}(\omega',x')|z,a\right]$ when $\omega=n$, we interpolate all value functions using cubic interpolation. Since policies (b',k',τ') affect ω' through the sophisticated borrowing constraints, we compute the integral carefully around the cut-off innovation in the law of motion for z. To do so, we split the integral into two parts around $\bar{\epsilon}_{t+1}(z,b',k',\tau')$, the cut-off innovation in TFP:

$$\begin{aligned}\mathbb{E}\left[\Lambda_{t+1}V^0_{m,t+1}(\omega',x')|z,a\right] = &\int_0^\infty\int_{-\infty}^{\bar{\epsilon}_{t+1}(z,b',k',\tau')}\Lambda_{t+1}V^0_{m,t+1}(v,b',k',\tau',z^{\rho_z}\exp(\epsilon),a')f(\epsilon|\sigma_z)d\epsilon dQ(a'|a)\\ &+\int_0^\infty\int_{\bar{\epsilon}_{t+1}(z,b',k',\tau')}^{\infty}\Lambda_{t+1}V^0_{m,t+1}(n,b',k',\tau',z^{\rho_z}\exp(\epsilon),a')f(\epsilon|\sigma_z)d\epsilon dQ(a'|a),\end{aligned}$$

where $f(\epsilon|\sigma_z)$ is the density of the normal distribution with mean zero and standard deviation σ_z, and $\bar{\epsilon}_{t+1}(z,b',k',\tau')$ is defined as the solution to

$$\tilde{q}_b^{\text{face}}b' = \phi\pi_{t+1}(z^{\rho_z}\exp(\bar{\epsilon}_{t+1}(z,b',k',\tau')),(1-\tau')k').$$

We apply quadrature to each integral separately, performing the computation over a dense grid for (b',k',τ') and then using cubic interpolation on the resulting expectations. We construct the price of debt $q_{b,t}(n,b',k',\tau',z,a)$ using analogous methods. Objects for $\omega=v$ are precomputed in the same way, but without the need to account for a cut-off.

Step 3. We plug the precomputed continuation values into each Bellman equation and use a nonlinear solver to find the optimal b', k', and τ' at each state. Given the high dimensionality of the problem and the large number of policy variables, good initial guesses are essential. To obtain them, we first solve the manager and creditor problems on the policy grid without a nonlinear solver. Because of the fixed cost C_H paid when firms choose $\tau'>0$, we approximate the indicator function $\mathbb{I}(\tau'>0)$ by a smooth function of τ'; otherwise computation time becomes prohibitive. Specifically, we use $\mathbb{I}(\tau'>0)\approx\frac{1-\exp(-\kappa\tau')}{1-\exp(-\kappa)}$, where $\kappa=750$, and restrict the minimum feasible choice of τ' to 0.0001 for managers.

Step 4. We iterate until $V_c(n,x)$, $V_c(v,x)$, $V_m(n,x)$, and $V_m(v,x)$ converge within a specified tolerance.

Step 5. Given the policy functions, we compute the distribution $\mathbf{F}_t$. We normalize $w_t=1$ in steady state, recover C_t from the resource constraint, and set Ψ so that household optimality is consistent with this wage normalization.

C.3.4 IRFs

We compute IRFs to ϕ-shocks and monetary policy shocks using Sequence Space Jacobian methods following Auclert et al. (2021). Our equilibrium conditions are

$$
\begin{aligned}
0 &= \log R_t^{nom} - \log\frac{1}{\beta} - \varphi_\pi \log\Pi_t - \epsilon_t^m \\
0 &= \log\Pi_t - \frac{\gamma_{CES}-1}{\varphi}\log\frac{p_t}{p^*} - \beta\log\Pi_{t+1} \\
0 &= \Lambda_{t+1} - \frac{\Pi_{t+1}}{R_t^{nom}} \\
0 &= w_t - \Psi C_t \\
0 &= C_t - \left(Y_t - I_t - AC_t - f_c\right) \\
0 &= q_{k,t}^{\eta}\delta K_t - I_t \\
0 &= \beta\frac{C_t}{C_{t+1}} - \Lambda_{t+1}
\end{aligned}
$$

We require the vector of unknowns $\{R_t^{nom}, \Pi_t, \Lambda_{t+1}, C_t, w_t, p_t, q_{k,t}\}_{t=0}^{\infty}$ to satisfy the equations above given an exogenous sequence of shocks $\{\phi_t, \epsilon_t^m\}_{t=0}^{\infty}$. For computation, we truncate the infinite horizon to $T = 500$ quarters. The economy begins in steady state at $t = -1$; at $t = 0$, the path of shocks is realized and is known with perfect foresight by all agents.

We first introduce notation to summarize the equilibrium. For any variable X, stack its aggregates X_t into a $(T+1)\times 1$ vector X. Define a solution $\mathbf{X} = \begin{bmatrix} R^{nom} & \Pi & \Lambda & C & w & p & q_k \end{bmatrix}$ associated with the sequence of shocks $\{\phi_t, \epsilon_t^m\}_{t=0}^{\infty}$. Let $\mathbf{H}_k$ be the $(T+1)\times 1$ vector obtained by evaluating the k-th equilibrium condition from $t=0$ to $t=T$ at a candidate $\mathbf{X}$, and stack $\mathbf{H}_k$ into a $7(T+1)\times 1$ vector $\mathbf{H}$ in the order they appear above. For a monetary policy shock, the equilibrium path $\mathbf{X}$ given ϵ^m solves $\mathbf{H}(\mathbf{X}, \epsilon^m) = \mathbf{0}$, so the first-order response is

$$
d\mathbf{X} = -\mathbf{H}_{\mathbf{X}}^{-1}\mathbf{H}_{\epsilon^m} d\epsilon^m
$$

Evaluating this expression requires the Jacobian of $\mathbf{H}$ with respect to $\mathbf{X}$ and ϵ^m, evaluated at steady state. To construct it, we need to compute $\left\{\frac{\partial Y_t}{\partial x_s}, \frac{\partial AC_t}{\partial x_s}, \frac{\partial K_{t+1}}{\partial x_s}\right\}_{s,t}$ for $x = \{p, w, q_k, \Lambda\}$, which we obtain using the Fake News Algorithm of Auclert et al. (2021). The remaining elements of $\mathbf{H}_{\mathbf{X}}$ can be determined analytically.

Fake News Matrix. We illustrate how we compute the necessary Jacobians by explaining how to compute the partial derivative of K_{t+1} with respect to a shock or variable x_s. We work with the discretized law of motion for the distribution. For exposition, we ignore the entry and exit of new firms, which is unimportant for first-order impacts. Recall that the distribution evolves as

$$
\mathbf{F}_{t+1} = \mathbf{\Gamma}_{t+1}\mathbf{Q}_t\mathbf{F}_t.
$$

The matrix $\mathbf{Q}_t$ is measurable with respect to information available at time t, since policy choices made at t determine the evolution of debt, capital, and the diversion share τ into period $t+1$. By contrast, $\mathbf{\Gamma}_{t+1}$ is measurable with respect to information at time $t+1$, since changes in aggregate prices (p_{t+1}, w_{t+1}) affect realized earnings and therefore the allocation of control rights. Thus, $\mathbf{Q}_t$ captures endogenous transitions due to agents' decisions, while $\mathbf{\Gamma}_{t+1}$ captures ex post transitions arising from realized outcomes and covenant enforcement. Our approach largely follows Auclert et al. (2021), but emphasizes the role of transitions between the normal and violation states, which are specific to our model, in computing first-order responses to aggregate shocks.

Given this notation, aggregate capital chosen for period $t+1$ is

$$K_{t+1} = \mathbf{F}_t' \mathbf{k}_{t+1},$$

where $\mathbf{k}_{t+1} = \left[k_t'(\omega, x)\right]_{(\omega,x)}$. The partial derivative of K_{t+1} with respect to a shock x_s is

$$\frac{\partial K_{t+1}}{\partial x_s} = \mathbf{F}' \frac{\partial \mathbf{k}_{t+1}}{\partial x_s} + (\mathbf{k})' \frac{\partial \mathbf{F}_t}{\partial x_s}$$

where variables without time subscripts denote steady-state values. The first observation: $\frac{\partial \mathbf{k}_{t+1}}{\partial x_s}, \frac{\partial \mathbf{Q}_t}{\partial x_s} \neq 0$ if $s \geq t$, and $\frac{\partial \mathbf{k}_{t+1}}{\partial x_s}, \frac{\partial \mathbf{Q}_t}{\partial x_s} = 0$ if $s < t$. Since policy variables are forward-looking, transition probabilities between states and the policies themselves respond to contemporaneous and future changes in x_s. Next, consider $\frac{\partial \mathbf{F}_{t+1}}{\partial x_s}$:

$$\frac{\partial \mathbf{F}_{t+1}}{\partial x_s} = \mathbf{\Gamma}\mathbf{Q}\frac{\partial \mathbf{F}_t}{\partial x_s} + \mathbf{\Gamma}\frac{\partial \mathbf{Q}_t}{\partial x_s}\mathbf{F} + \frac{\partial \mathbf{\Gamma}_{t+1}}{\partial x_s}\mathbf{Q}\mathbf{F}$$

The second observation: $\frac{\partial \mathbf{\Gamma}_{t+1}}{\partial x_s} = 0$ if $s \neq t+1$, since the extensive margin only moves as a function of p_{t+1} and w_{t+1}. The third observation: $\frac{\partial \mathbf{k}_{t+1}}{\partial x_s} = \frac{\partial \mathbf{k}_t}{\partial x_{s-1}}$, $\frac{\partial \mathbf{Q}_t}{\partial x_s} = \frac{\partial \mathbf{Q}_{t-1}}{\partial x_{s-1}}$, and $\frac{\partial \mathbf{\Gamma}_{t+1}}{\partial x_s} = \frac{\partial \mathbf{\Gamma}_t}{\partial x_{s-1}}$; that is, calendar time is irrelevant. Using these three observations, we construct the Fake News Matrix. We first evaluate changes in the distribution, for $t \geq 1, s \geq 1$:

$$\begin{aligned}
\frac{\partial \mathbf{F}_t}{\partial x_s} - \frac{\partial \mathbf{F}_{t-1}}{\partial x_{s-1}} &= \mathbf{\Gamma}\mathbf{Q}\frac{\partial \mathbf{F}_{t-1}}{\partial x_s} + \mathbf{\Gamma}\frac{\partial \mathbf{Q}_{t-1}}{\partial x_s}\mathbf{F} + \frac{\partial \mathbf{\Gamma}_t}{\partial x_s}\mathbf{Q}\mathbf{F} - \left(\mathbf{\Gamma}\mathbf{Q}\frac{\partial \mathbf{F}_{t-2}}{\partial x_{s-1}} + \mathbf{\Gamma}\frac{\partial \mathbf{Q}_{t-2}}{\partial x_{s-1}}\mathbf{F} + \frac{\partial \mathbf{\Gamma}_{t-1}}{\partial x_{s-1}}\mathbf{Q}\mathbf{F}\right) \\
&= (\mathbf{\Gamma}\mathbf{Q})^{t-1}\left(\mathbf{\Gamma}\mathbf{Q}\frac{\partial \mathbf{F}_0}{\partial x_s} + \mathbf{\Gamma}\frac{\partial \mathbf{Q}_0}{\partial x_s}\mathbf{F}\right) \qquad \text{(apply the third observation many times)} \\
&= (\mathbf{\Gamma}\mathbf{Q})^{t}\frac{\partial \mathbf{\Gamma}_0}{\partial x_s}\mathbf{Q}\mathbf{F} + (\mathbf{\Gamma}\mathbf{Q})^{t-1}\mathbf{\Gamma}\frac{\partial \mathbf{Q}_0}{\partial x_s}\mathbf{F}
\end{aligned}$$

Let $\mathscr{F}^K_{t,s}$ denote the $(t+1, s+1)$ entry of the Fake News Matrix for capital $\mathscr{F}^K$. For $t \geq 1, s \geq 1$, we have

$$\begin{aligned}
\mathscr{F}^K_{t,s} &= \frac{\partial K_{t+1}}{\partial x_s} - \frac{\partial K_t}{\partial x_{s-1}} \\
&= (\mathbf{k})'\left(\frac{\partial \mathbf{F}_t}{\partial x_s} - \frac{\partial \mathbf{F}_{t-1}}{\partial x_{s-1}}\right) \\
&= (\mathbf{k})'\left((\mathbf{\Gamma}\mathbf{Q})^{t}\frac{\partial \mathbf{\Gamma}_0}{\partial x_s}\mathbf{Q}\mathbf{F} + (\mathbf{\Gamma}\mathbf{Q})^{t-1}\mathbf{\Gamma}\frac{\partial \mathbf{Q}_0}{\partial x_s}\mathbf{F}\right)
\end{aligned}$$

To fill in the entries of the Fake News Matrix, we need to compute $\frac{\partial \mathbf{\Gamma}_0}{\partial x_s}$ and $\frac{\partial \mathbf{Q}_0}{\partial x_s}$ for every s. Constructing $\mathscr{F}^K$ is useful because, for $t \geq 1, s \geq 1$,

$$\frac{\partial K_{t+1}}{\partial x_s} = \mathscr{F}^K_{t,s} + \frac{\partial K_t}{\partial x_{s-1}}$$

What remains is to compute $\frac{\partial K_{t+1}}{\partial x_s}$ for $s = 0, t \geq 1$; for $t = 0, s \geq 1$; and for $s = t = 0$. We express these directly and assign them as the entries of the first column and first row of $\mathscr{F}^K$. First, for $s = 0, t \geq 1$,

$$\begin{aligned}\frac{\partial K_{t+1}}{\partial x_0} &= \mathbf{F}'\frac{\partial \mathbf{k}_{t+1}}{\partial x_0} + (\mathbf{k})'\frac{\partial \mathbf{F}_t}{\partial x_0} \\ &= (\mathbf{k})'\left\{(\mathbf{\Gamma Q})^t \frac{\partial \mathbf{\Gamma}_0}{\partial x_0}\mathbf{QF} + (\mathbf{\Gamma Q})^{t-1}\mathbf{\Gamma}\frac{\partial \mathbf{Q}_0}{\partial x_0}\mathbf{F}\right\}\end{aligned}$$

This defines the first column of $\mathscr{F}^K$, except for the first entry. We use $\frac{\partial \mathbf{k}_{t+1}}{\partial x_0} = 0$, since policies chosen at $t \geq 1$ only respond to changes in the aggregate object x_s for $s \geq t$. Next, we evaluate the first row. For $t = 0, s \geq 1$,

$$\begin{aligned}\frac{\partial K_1}{\partial x_s} &= \mathbf{F}'\frac{\partial \mathbf{k}_1}{\partial x_s} + (\mathbf{k})'\frac{\partial \mathbf{F}_0}{\partial x_s} \\ &= \mathbf{F}'\frac{\partial \mathbf{k}_1}{\partial x_s}\end{aligned}$$

We use $\frac{\partial \mathbf{F}_0}{\partial x_s} = 0$, since the distribution at $t = 0$ does not respond to shocks at $s \geq 1$. Lastly, we are left with the first entry, $s = t = 0$,

$$\begin{aligned}\frac{\partial K_1}{\partial x_0} &= \mathbf{F}'\frac{\partial \mathbf{k}_1}{\partial x_0} + (\mathbf{k})'\frac{\partial \mathbf{F}_0}{\partial x_0} \\ &= \mathbf{F}'\frac{\partial \mathbf{k}_1}{\partial x_0} + (\mathbf{k})'\left\{\frac{\partial \mathbf{\Gamma}_0}{\partial x_0}\mathbf{QF} + \mathbf{\Gamma}\frac{\partial \mathbf{Q}_{-1}}{\partial x_0}\mathbf{F}\right\} \\ &= \mathbf{F}'\frac{\partial \mathbf{k}_1}{\partial x_0} + (\mathbf{k})'\frac{\partial \mathbf{\Gamma}_0}{\partial x_0}\mathbf{QF}\end{aligned}$$

Since $\mathbf{Q}_{-1} = \mathbf{Q}$ is a steady-state object, $\frac{\partial \mathbf{Q}_{-1}}{\partial x_0} = 0$. In sum, we need to compute $\frac{\partial \mathbf{k}_1}{\partial x_s}$ and $\frac{\partial \mathbf{Q}_0}{\partial x_s}$ for $s \in \{0, \ldots, T\}$, as well as $\frac{\partial \mathbf{\Gamma}_0}{\partial x_0}$. These can be computed via finite differences from a single perturbation at time $T-1$, as suggested by Auclert et al. (2021).

To obtain a well-defined derivative $\frac{\partial \mathbf{\Gamma}_0}{\partial x_0}$, we approximate $\mathbf{\Gamma}$ by a smooth function. This is only necessary for $\mathbf{\Gamma}_{n,0}$, whose diagonal entries are $\mathbb{I}\left(n = \Gamma_b\left(b_{t+1}, \bar{b}_{t+1}(x')\right)\right)$. Following Guerreiro et al. (2025), we approximate $\mathbb{I}\left(n = \Gamma_b\left(b_{t+1}, \bar{b}_{t+1}(x')\right)\right) \approx \frac{1}{1+\exp\left(\kappa_n\left(b_{t+1} - \bar{b}_{t+1}(x')\right)\right)}$, with $\kappa_n = 5000$.

Computing Outcomes. Given the Jacobians above, we can compute $\mathbf{H_X}$. We next describe how to compute the equilibrium path of investment and output in response to monetary policy shocks and ϕ-shocks.

Define $\mathbf{M}^I_x = \left\{\frac{\partial I_t}{\partial x_s}\right\}_{t,s}$, which collects the Jacobians of investment with respect to aggregate variable x. Similar matrices can be defined for output Y_t. Let $d\mathbf{I}$ be a $(T+1) \times 1$ vector for the difference in the path of investment relative to its steady-state level in response to an aggregate shock. Given solutions for p_t, w_t, Λ_t, and $q_{k,t}$ in response to a monetary policy shock, we can write

$$d\mathbf{I} = \mathbf{M}^I_p d\mathbf{p} + \mathbf{M}^I_w d\mathbf{w} + \mathbf{M}^I_q d\mathbf{q} + \mathbf{M}^I_\Lambda d\boldsymbol{\Lambda}$$

For a ϕ-shock, we also include the direct effect of the shock:

$$d\mathbf{I} = \mathbf{M}^I_\phi d\boldsymbol{\phi} + \mathbf{M}^I_p d\mathbf{p} + \mathbf{M}^I_w d\mathbf{w} + \mathbf{M}^I_q d\mathbf{q} + \mathbf{M}^I_\Lambda d\boldsymbol{\Lambda}$$

Decomposition. We now explain the extensive-intensive margin decomposition for a ϕ-shock. Consider the partial-equilibrium response to ϕ, holding other aggregates fixed. Differentiating K_{t+1} with respect to a shock in ϕ at s and iterating backwards on $\mathbf{F}_t$, we obtain

$$\frac{\partial K_{t+1}}{\partial \phi_s} = \underbrace{\sum_{j=0}^{t} \left[(\boldsymbol{\Gamma}\mathbf{Q})^j \frac{\partial \boldsymbol{\Gamma}_{t-j}}{\partial \phi_s} \mathbf{Q}\mathbf{F} \right]' \mathbf{k}}_{\text{extensive margin}} + \underbrace{\sum_{j=0}^{t-1} \left[(\boldsymbol{\Gamma}\mathbf{Q})^j \boldsymbol{\Gamma} \frac{\partial \mathbf{Q}_{t-1-j}}{\partial \phi_s} \mathbf{F} \right]' \mathbf{k} + \mathbf{F}' \frac{\partial \mathbf{k}_{t+1}}{\partial \phi_s}}_{\text{intensive margin}}$$

The extensive-margin term captures changes in aggregate capital driven by shifts in $\boldsymbol{\Gamma}$, i.e., changes in the ex post mapping that reallocates firms across control states when sophisticated borrowing constraints are evaluated following a change in ϕ in period s. This corresponds to the extensive margin in the decomposition of impulse response functions. The intensive-margin terms capture changes in aggregate capital driven by policy adjustments, holding the constraint-evaluation mapping fixed: the term involving $\frac{\partial \mathbf{Q}_t}{\partial \phi_s}$ reflects how past changes in decision rules reshape the distribution over states, while $\mathbf{F}' \frac{\partial \mathbf{k}_{t+1}}{\partial \phi_s}$ captures the direct effect of contemporaneous changes in capital policy. An analogous decomposition can be performed for other variables.

Conveniently, the construction of the Jacobians for aggregate shocks decomposes in the same way. Note that the inclusion of $\frac{\partial \boldsymbol{\Gamma}_0}{\partial x_0}$ in the Fake News Matrix reflects the extensive margin as defined above. The intensive-margin response is obtained by excluding this term from all Jacobians. The extensive-margin response is obtained by keeping only this term and setting $\frac{\partial \mathbf{k_1}}{\partial x_s}$ and $\frac{\partial \mathbf{Q_0}}{\partial x_s}$ to zero. Define $\frac{\partial K_{t+1}}{\partial x_s}|_{\text{intensive}}$ as the Jacobian constructed by setting $\frac{\partial \boldsymbol{\Gamma}_0}{\partial x_0}$ to zero, and, analogously, $\frac{\partial K_{t+1}}{\partial x_s}|_{\text{extensive}} = \frac{\partial K_{t+1}}{\partial x_s} - \frac{\partial K_{t+1}}{\partial x_s}|_{\text{intensive}}$. To summarize,

$$\frac{\partial K_{t+1}}{\partial \phi_s}|_{\text{intensive}} = \sum_{j=0}^{t-1} \left[(\boldsymbol{\Gamma}\mathbf{Q})^j \boldsymbol{\Gamma} \frac{\partial \mathbf{Q}_{t-1-j}}{\partial \phi_s} \mathbf{F} \right]' \mathbf{k} + \mathbf{F}' \frac{\partial \mathbf{k}_{t+1}}{\partial \phi_s}$$

and

$$\frac{\partial K_{t+1}}{\partial \phi_s}|_{\text{extensive}} = \sum_{j=0}^{t} \left[(\boldsymbol{\Gamma}\mathbf{Q})^j \frac{\partial \boldsymbol{\Gamma}_{t-j}}{\partial \phi_s} \mathbf{Q}\mathbf{F} \right]' \mathbf{k}$$

As a result, we can decompose the paths of aggregate outcomes using the same Jacobians used to recover them. To decompose the path of investment in response to a ϕ-shock, define $\mathbf{M}^{I,i}_x = \left\{ \frac{\partial I_t}{\partial x_s}|_{\text{intensive}} \right\}_{t,s}$.

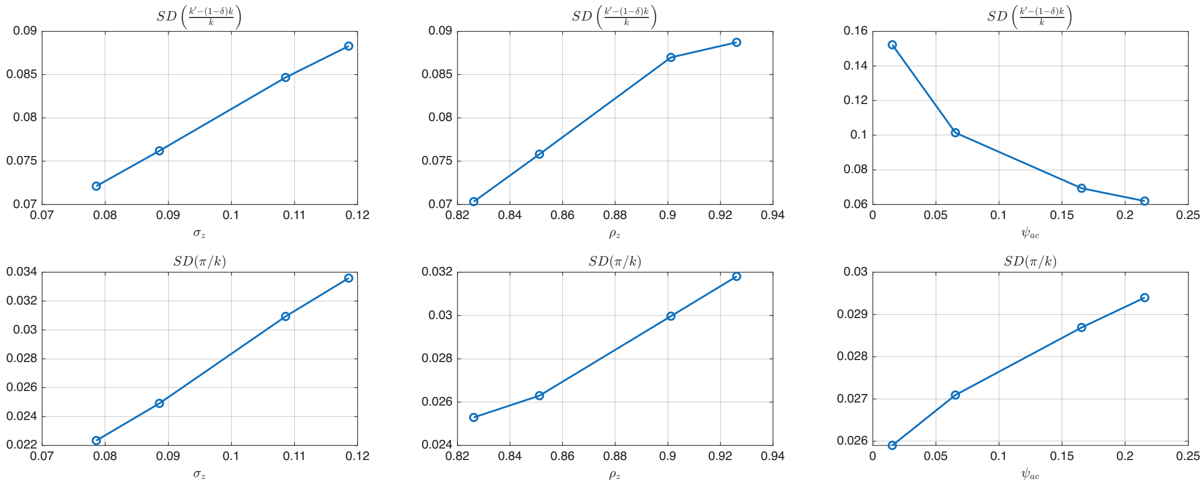
Using this matrix, consider the following decomposition:

$$d\mathbf{I} = \underbrace{\left[\mathbf{M}^{I}_{\phi} - \mathbf{M}^{I,i}_{\phi}\right] d\boldsymbol{\phi}}_{\text{extensive}} + \underbrace{\mathbf{M}^{I,i}_{\phi} d\boldsymbol{\phi}}_{\text{intensive}} + \underbrace{\mathbf{M}^{I}_{p} d\mathbf{p} + \mathbf{M}^{I}_{w} d\mathbf{w} + \mathbf{M}^{I}_{q} d\mathbf{q} + \mathbf{M}^{I}_{\Lambda} d\boldsymbol{\Lambda}}_{\text{G.E}}$$

The first term is the extensive margin, which reflects the direct effect of the ϕ-shock on the transfer of control rights with policies held fixed at steady state. The second term is the intensive margin, which reflects how policy adjustments contribute to the equilibrium path of investment, shutting down the direct effect of the ϕ-shock on the transfer of control rights. The remaining terms reflect general-equilibrium feedback through the prices of the intermediate good, wages, capital, and interest rates.

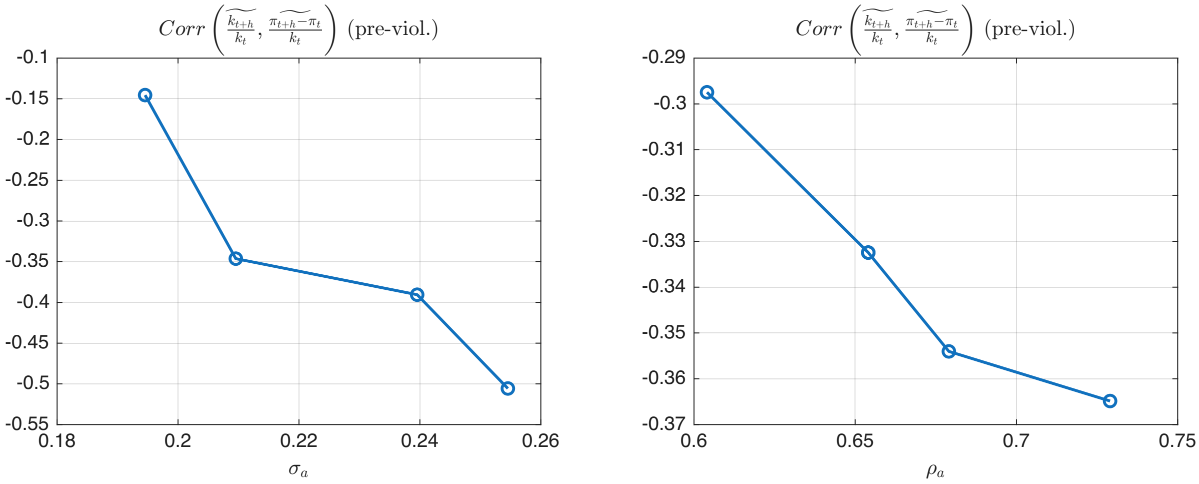
D Additional Results for Quantitative Analysis

Figure D.1: Identification: Standard Deviation of Investment Rate and Earnings-to-Capital Ratio



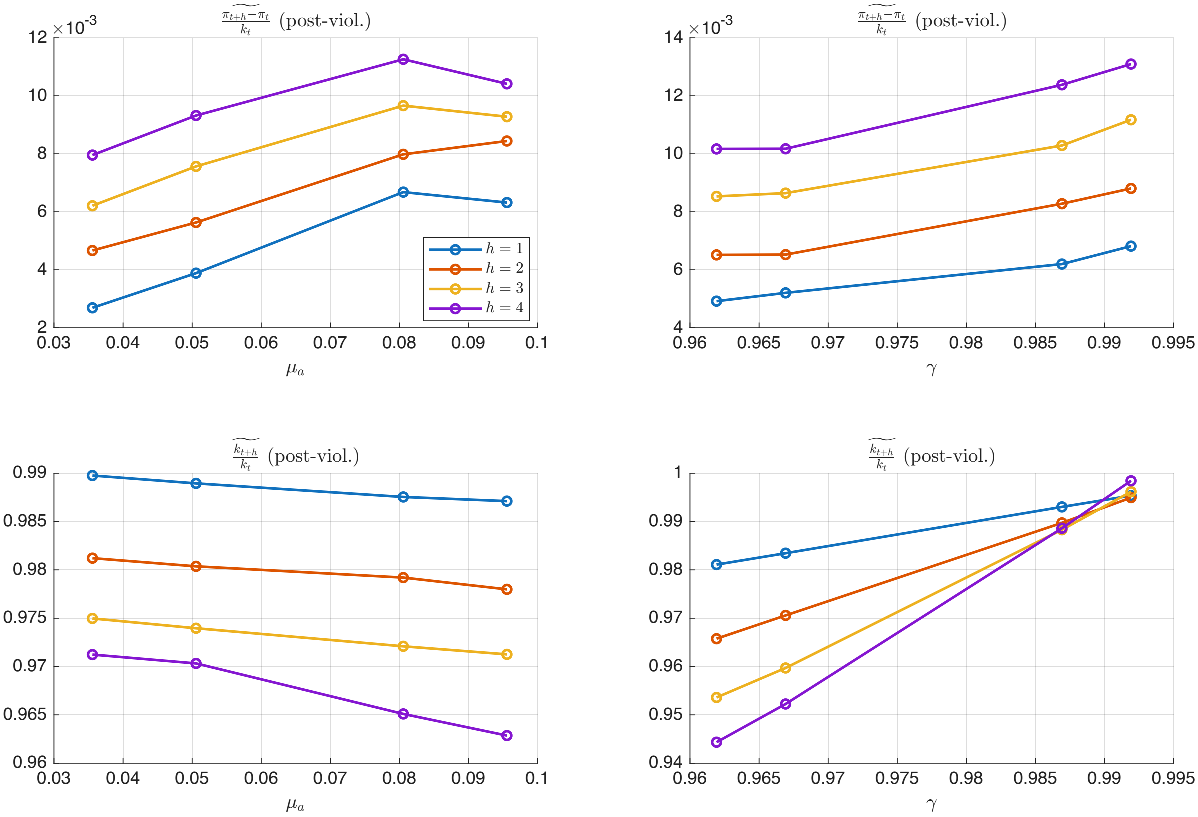
Notes: The figure displays the steady-state standard deviation of investment rate and earnings-to-capital ratio in the steady states as the calibration changes. The x-axis in every panel denotes the value a particular parameter is evaluated at to compute the steady state, keeping all other parameters fixed at the benchmark calibration. Investment rate is defined as $\frac{k'-(1-\delta)k}{k}$, earnings-to-capital ratio is $\frac{\pi}{k}$.

Figure D.2: Identification: Pre-Covenant-Violation Dynamics



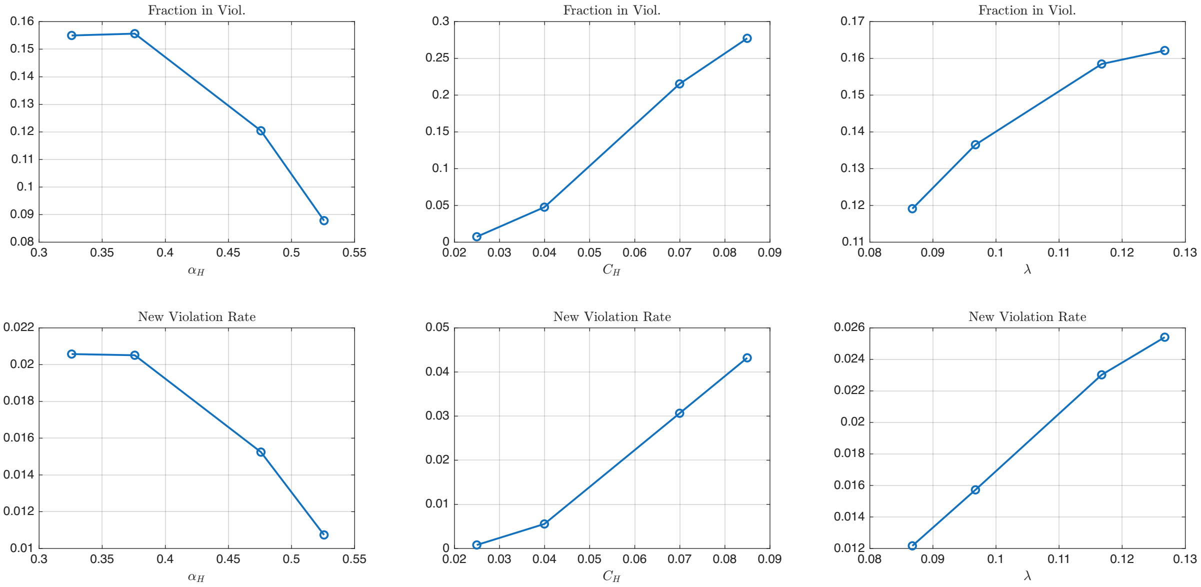
Notes: The figure plots the correlation between median capital ratio, $\widetilde{\frac{k_{t+h}}{k_t}}$, and median normalized earnings change, $\widetilde{\frac{\pi_{t+h}-\pi_t}{k_t}}$, for violating firms over four quarters before violation as the calibration changes. A tilde over a variable corresponds to the median value of that variable across all firms violating the sophisticated borrowing constraint in the steady state of the model, where t denotes the period when they initially violate the constraint. The correlation is computed over $h \in \{-4,-3,-2,-1\}$. The x-axis in every panel denotes the value a particular parameter is evaluated at to compute the steady state, keeping all other parameters fixed at the benchmark calibration.

Figure D.3: Identification: Post-Covenant-Violation Dynamics



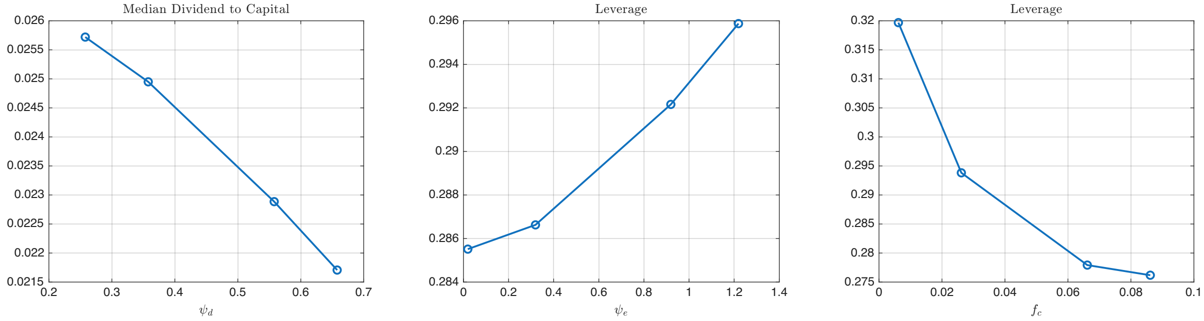
Notes: The figure plots median normalized earnings change, $\widetilde{\frac{\pi_{t+h}-\pi_t}{k_t}}$, and median capital ratio, $\widetilde{\frac{k_{t+h}}{k_t}}$, for the first four quarters after violating the financial covenant as the calibration changes. A tilde over a variable corresponds to the median value of that variable across all firms violating the sophisticated borrowing constraint in the steady state of the model, where t denotes the period when they initially violate the constraint. The x-axis in every panel denotes the value a particular parameter is evaluated at to compute the steady state, keeping all other parameters fixed at the benchmark calibration.

Figure D.4: Identification: Fraction of Firms in Violation and the New Violation Rate



Notes: The figure plots the steady-state fraction of firms in violation and the new violation rate as the calibration changes. The x-axis in every panel denotes the value a particular parameter is evaluated at to compute the steady state, keeping all other parameters fixed at the benchmark calibration.

Figure D.5: Identification: Dividends and Leverage



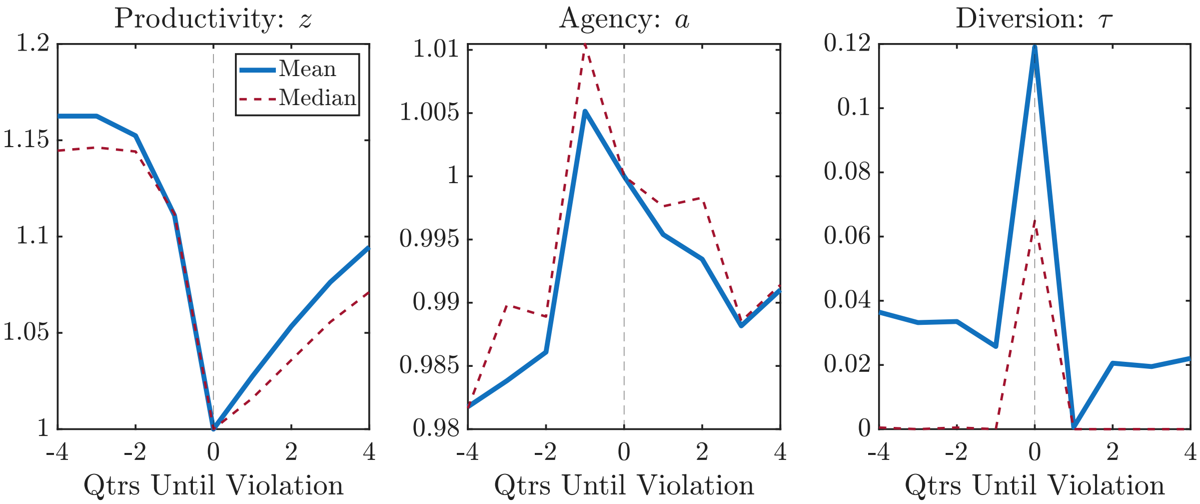
Notes: The figure plots the steady-state median dividend-to-capital ratio ($\max\{d,0\}/k$) and median leverage ($\tilde{q}_b^{\text{face}} b/k$) as the calibration changes. The x-axis in every panel denotes the value a particular parameter is evaluated at to compute the steady state, keeping all other parameters fixed at the benchmark calibration.

Table D.1: Non-Targeted Moments

	Model	Data
Correlation with Earnings-to-Capital		
Investment Rate	0.518	0.095
Violation	-0.069	-0.163
Autocorrelation		
Investment Rate	0.290	0.340
Earnings-to-Capital	0.782	0.350
Capital Distribution		
Median to 25th pct	1.182	1.320
75th pct to Median	1.162	1.230

Notes: This table displays the empirical moments not included in the calibration targets and the corresponding values generated by the steady state of our model. Investment rate is defined as $\frac{k'-(1-\delta)k}{k}$, earnings-to-capital ratio is $\frac{\pi}{k}$, and violation is an indicator for whether the firm has violated the sophisticated borrowing constraint. Autocorrelation and correlation are averaged both across firms and over time (in the steady state without aggregate shocks).

Figure D.6: Model Fit: Shocks around Covenant Violation



Notes: The left panel plots the average and median firm's idiosyncratic productivity shocks z in period $t+h$ normalized by its productivity shocks at t, i.e., z_{t+h}/z_t in the model. The middle panel plots the average and median firm's idiosyncratic agency shock a in period $t+h$ normalized by its agency shock at t when they initially violate the sophisticated borrowing constraint, i.e., a_{t+h}/a_t in the model. The right panel plots the average and median firm's fraction of unproductive capital use τ in period $t+h$, i.e., τ_{t+h} in the model. The lines are based on all firms violating the sophisticated borrowing constraint in the steady state of the model.

Table D.2: Fitted Parameters: No Agency Frictions

Parameter	Value	Description
Panel 1: Technology		
ρ_z	0.853	Persistence of TFP Shocks
σ_z	0.116	SD of TFP Shocks
ψ_{ac}	0.260	Capital Adj Cost
Panel 2: Financial Frictions		
γ	0.968	Coupon Decay Rate
λ	0.098	Prob. of Transition to Normal from Violation
ψ_d	0.199	Dividend Adj Cost
ψ_e	0.698	Equity Issuance Cost
f_c	0.031	Fixed Operating Cost

Notes: This table displays parameters that are chosen to match moments in Table 3 and Figure 1 in a variant of our model without agency frictions.

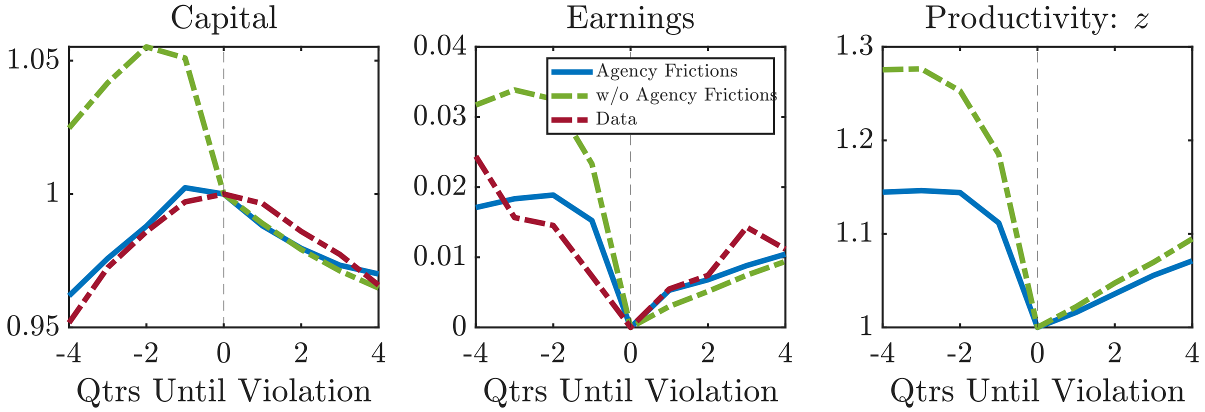
Table D.3: Unconditional Targeted Moments and Model Fit: No Agency Frictions

	Model	Data
Median Leverage	0.323	0.320
New Violation Rate	0.025	0.025
Frac. in Violation	0.195	0.150
Median Dividend/Capital	0.034	0.000
Median Equity/Capital	0.000	0.001
SD of Earnings/Capital	0.035	0.028
SD of Investment Rate	0.062	0.056

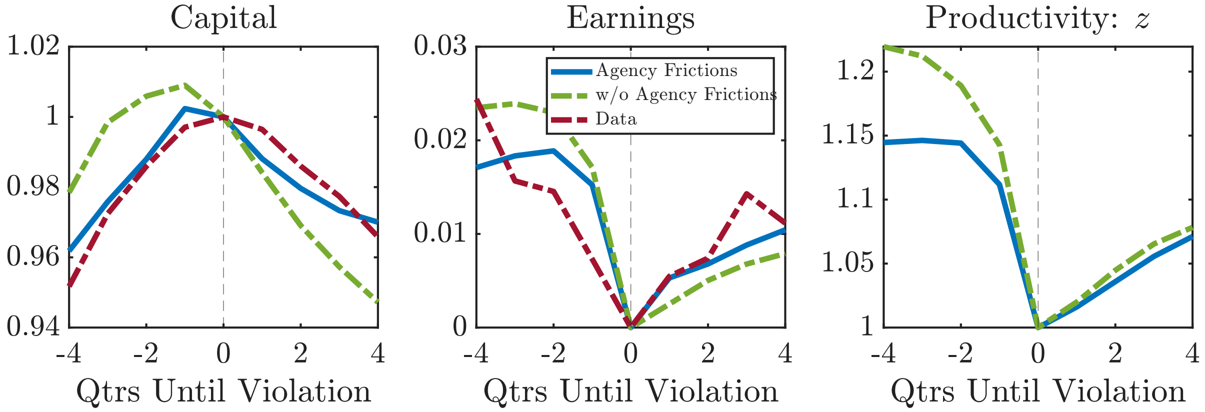
Notes: This table displays the empirical moments targeted in our calibration and the corresponding values generated by the steady state of a variant of our model without agency frictions. Leverage, dividend-to-capital ratio, equity-issuance-to-capital ratio, earnings-to-capital ratio, and investment rate correspond to $\tilde{q}_b^{\text{face}} b/k$, $\max\{d,0\}/k$, $\max\{-d,0\}/k$, $\frac{\pi}{k}$, and $\frac{k'-(1-\delta)k}{k}$ in the model, respectively. We use assets as a proxy for capital when calculating unconditional moments in this table.

Figure D.7: Targeted Dynamics Around Covenant Violations and Model Fit: No Agency Frictions

(a) No Recalibrating

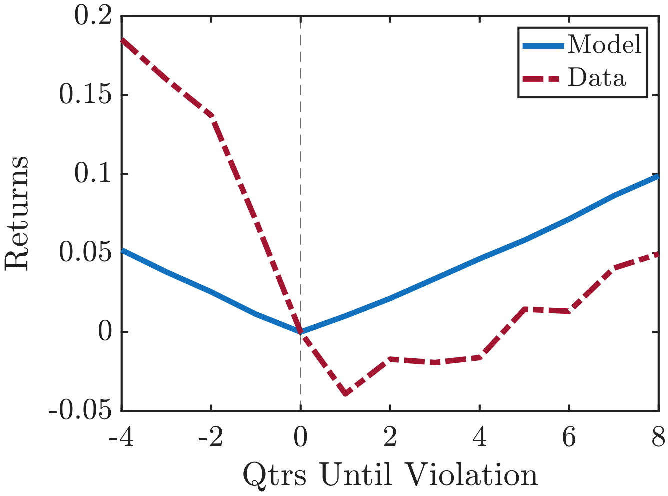


(b) Recalibrating



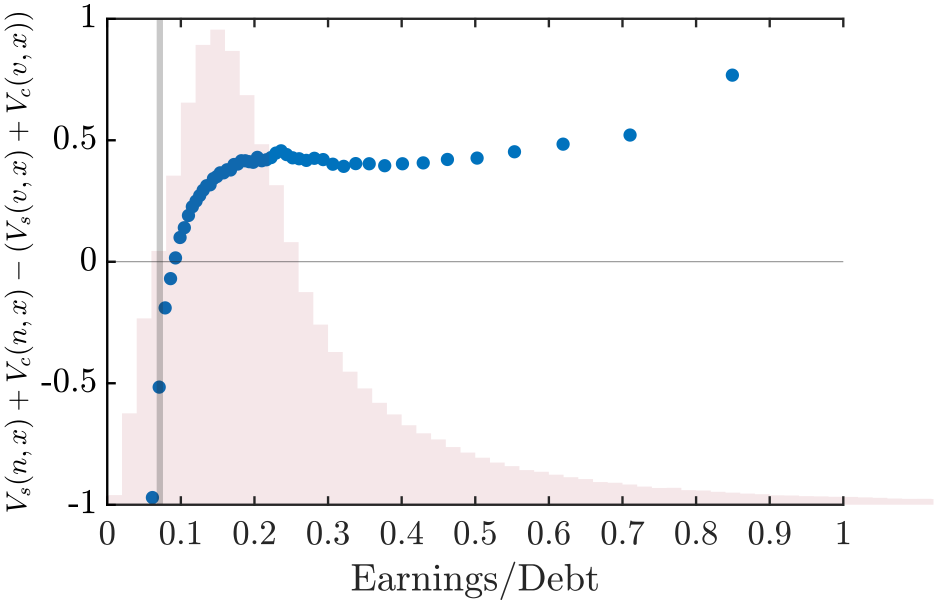
Notes: The left panel plots the median firm's capital in period $t+h$ normalized by its capital at t, i.e., k_{t+h}/k_t in the model. The middle panel plots the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h} - \pi_t)/k_t$ in the model. The right panel plots the median firm's idiosyncratic productivity shocks z in period $t+h$ normalized by its productivity shocks at t, i.e., z_{t+h}/z_t in the model. t captures when firms initially violate the sophisticated borrowing constraint. The blue lines are based on all firms violating the sophisticated borrowing constraint in the steady state of our baseline model. The green lines are based on all firms violating the sophisticated borrowing constraint in the steady state of a variant of our model without agency frictions. The green line in Panel A keeps all parameter values the same as in the baseline model, except for those governing agency frictions. The green line in Panel B recalibrates this model using the same procedure as in our baseline model, matching both the unconditional empirical moments in Table 3 and the firm outcomes around covenant violations in Figure 1. The red lines correspond to their empirical counterparts, i.e., the median lines in Figure 1.

Figure D.8: Cumulative Shareholder Returns around the Violation of Sophisticated Borrowing Constraints



Notes: The solid blue line plots the median firm's cumulative shareholder returns in each period $t+h$ relative to period t when it initially violates the sophisticated borrowing constraint. Specifically, we first calculate each period t's dividend-inclusive log return: $\log R_t^e \equiv \log \frac{d_t + V_{s,t+1}}{V_{s,t}}$, and then calculate the cumulative log return: $\sum_{s=1}^{h} \log R_{t+s}^e$ for $h \geq 1$, $-\sum_{s=h+1}^{0} \log R_{t+s}^e$ for $h \leq -1$, and 0 for $h = 0$. The blue line is based on all firms violating the sophisticated borrowing constraint in the steady state of the model. The red dashed line corresponds to the empirical counterparts, i.e., the median lines in Panel A of Figure B.3.

Figure D.9: Differences in Total Value between Manager and Creditor Control



Notes: The figure plots the difference in total shareholder and creditor values, $V_s(n,\cdot)+V_c(n,\cdot)-(V_s(v,\cdot)+V_c(v,\cdot))$, under manager and creditor control, as a function of firms' earnings-to-debt ratios ($\pi/\left(\tilde{q}_b^{\text{face}}b\right)$ in the model). Each dot represents a percentile of the earnings-to-debt ratio in the ergodic distribution of our model. The pink bar plot shows the probability density of the earnings-to-debt ratio in the ergodic distribution of our model. The grey line indicates the earnings-to-debt ratio corresponding to $1/\phi$.

Table D.4: Fitted Parameters: The Hard-Constraint Model with Full Recalibration

Parameter	Value	Description
Panel 1: Technology		
ρ_z	0.888	Persistence of TFP Shocks
σ_z	0.051	SD of TFP Shocks
ψ_{ac}	0.211	Capital Adj Cost
Panel 2: Financial Frictions		
γ	0.954	Coupon Decay Factor
ψ_d	0.181	Dividend Adj Cost
ψ_e	0.640	Equity Issuance Cost
f_c	0.040	Fixed Operating Cost
Panel 3: Debt to Earnings Constraint		
ϕ	6.334	Debt to Earnings Limit

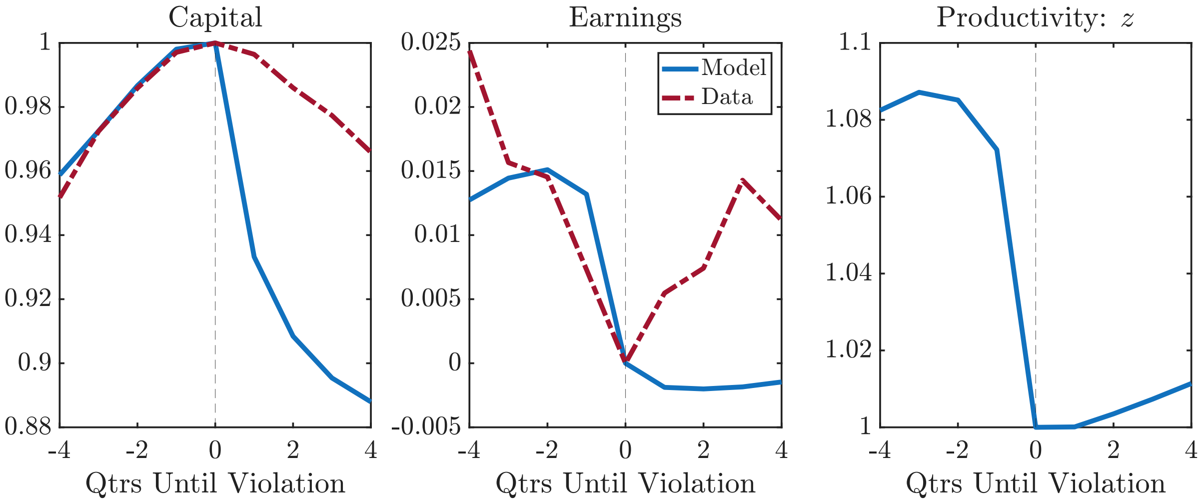
Notes: This table displays parameters that are chosen to match moments in Table 3 and Figure 1 in the hard-constraint model described in Section 3.3.

Table D.5: Unconditional Targeted Moments and Model Fit: The Hard-Constraint Model with Full Recalibration

	Model	Data
Median Leverage	0.375	0.320
New Violation Rate	0.093	0.025
Frac. in Violation	0.098	0.150
Median Dividend/Capital	0.026	0.000
Median Equity Issuance	0.000	0.001
SD of Earnings/Capital	0.016	0.028
SD of Investment Rate	0.046	0.056

Notes: This table displays the empirical moments targeted in our calibration and the corresponding values generated by the steady state of the hard-constraint model described in Section 3.3. Leverage, dividend-to-capital ratio, equity-issuance-to-capital ratio, earnings-to-capital ratio, and investment rate correspond to $\tilde{q}_b^{\text{face}} b/k$, $\max\{d,0\}/k$, $\max\{-d,0\}/k$, $\frac{\pi}{k}$, and $\frac{k'-(1-\delta)k}{k}$ in the model, respectively. We interpret firms hitting the hard borrowing constraint as "violations." We use assets as a proxy for capital when calculating unconditional moments in this table.

Figure D.10: Targeted Dynamics Around Covenant Violations and Model Fit: The Hard-Constraint Model with Full Recalibration



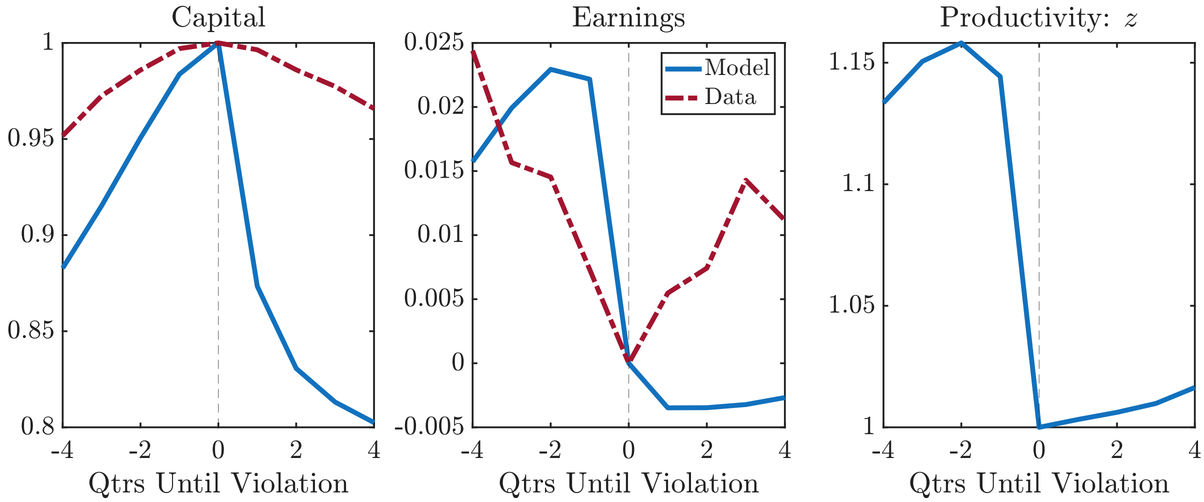
Notes: The blue solid line in the left panel shows the median firm's capital in period $t+h$ normalized by its capital at t (the initial period in which the hard constraint binds), i.e., k_{t+h}/k_t in the model. The blue solid line in the middle panel shows the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h} - \pi_t)/k_t$ in the model. The right panel plots the median firm's idiosyncratic productivity shocks z (solid line) in period $t+h$ normalized by its productivity shocks at t, i.e., z_{t+h}/z_t in the model. The lines are based on all firms hitting the hard borrowing constraint in the steady state of the hard-constraint model recalibrated using the same procedure as in our baseline model, matching both the unconditional empirical moments in Table 3 and the firm outcomes around covenant violations in Figure 1. The red dashed lines in the first two panels correspond to their empirical counterparts, i.e., the median lines in Figure 1.

Table D.6: Model Fit: The Hard-Constraint Model Recalibrating ϕ^{hard} only

	Model	Data
Median Leverage	0.322	0.320
New Violation Rate	0.077	0.025
Frac. in Violation	0.091	0.150
Median Dividend/Capital	0.026	0.000
Median Equity Issuance	0.000	0.001
SD of Earnings/Capital	0.030	0.028
SD of Investment Rate	0.091	0.056

Notes: This table displays the empirical moments and the corresponding values generated by the steady state of the hard-constraint model where we recalibrate ϕ^{hard} such that its median leverage matches that of the data, while keeping all other parameters the same. Leverage, dividend-to-capital ratio, equity-issuance-to-capital ratio, earnings-to-capital ratio, and investment rate correspond to $\tilde{q}_b^{\text{face}} b/k$, $\max\{d,0\}/k$, $\max\{-d,0\}/k$, $\frac{\pi}{k}$, and $\frac{k'-(1-\delta)k}{k}$ in the model, respectively. We interpret firms hitting the hard borrowing constraint as "violations." We use assets as a proxy for capital when calculating unconditional moments in this table.

Figure D.11: Targeted Dynamics Around Covenant Violations and Model Fit: The Hard-Constraint Model Recalibrating ϕ^{hard} only



Notes: The blue solid line in the left panel shows the median firm's capital in period $t+h$ normalized by its capital at t (the initial period in which the hard constraint binds), i.e., k_{t+h}/k_t in the model. The blue solid line in the middle panel shows the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The right panel plots the median firm's idiosyncratic productivity shocks z (solid line) in period $t+h$ normalized by its productivity shocks at t, i.e., z_{t+h}/z_t in the model. The lines are based on all firms hitting the hard borrowing constraint in the steady state of the hard-constraint model where we recalibrate ϕ^{hard} such that its median leverage matches that of the data, while keeping all other parameters the same. The red dashed lines in the first two panels correspond to their empirical counterparts, i.e., the median lines in Figure 1.

Table D.7: Fitted Parameters: The Hard-Constraint Model with Asset-based Lending

Parameter	Value	Description
Panel 1: Technology		
ρ_z	0.858	Persistence of TFP Shocks
σ_z	0.056	SD of TFP Shocks
ψ_{ac}	0.098	Capital Adj Cost
Panel 3: Financial Frictions		
γ	0.955	Coupon Decay Factor
ψ_d	0.533	Dividend Adj Cost
ψ_e	0.411	Equity Issuance Cost
f_c	0.028	Fixed Operating Cost
ϕ	0.391	Debt to Capital Limit

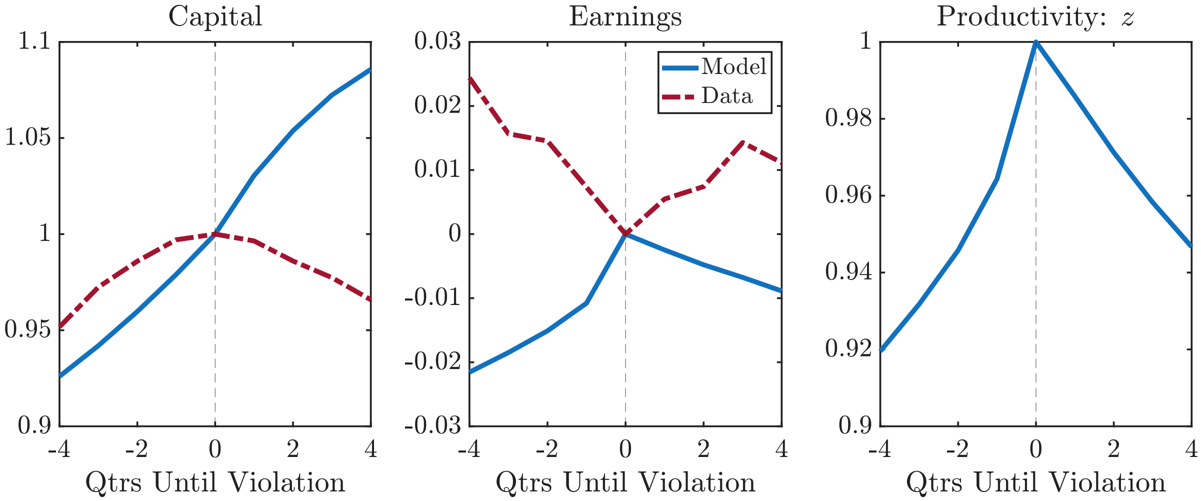
Notes: This table displays parameters that are chosen to match moments in Table 3 and Figure 1 in the hard-constraint model with asset-based lending described in Section 3.3.

Table D.8: Unconditional Targeted Moments and Model Fit: The Hard-Constraint Model with Asset-based Lending

	Model	Data
Median Leverage	0.378	0.320
New Violation Rate	0.061	0.025
Frac. in Violation	0.106	0.150
Median Dividend/Capital	0.025	0.000
Median Equity Issuance	0.000	0.001
SD of Earnings/Capital	0.016	0.028
SD of Investment Rate	0.040	0.056

Notes: This table displays the empirical moments targeted in our calibration and the corresponding values generated by the steady state of the hard-constraint model with asset-based lending described in Section 3.3. Leverage, dividend-to-capital ratio, equity-issuance-to-capital ratio, earnings-to-capital ratio, and investment rate correspond to $\tilde{q}_b^{\text{face}} b/k$, $\max\{d,0\}/k$, $\max\{-d,0\}/k$, $\frac{\pi}{k}$, and $\frac{k'-(1-\delta)k}{k}$ in the model, respectively. We interpret firms hitting the hard borrowing constraint as "violations." We use assets as a proxy for capital when calculating unconditional moments in this table.

Figure D.12: Targeted Dynamics Around Covenant Violations and Model Fit: The Hard-Constraint Model with Asset-based Lending



Notes: The blue solid line in the left panel shows the median firm's capital in period $t+h$ normalized by its capital at t, i.e., k_{t+h}/k_t in the model. The blue solid line in the middle panel shows the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The right panel plots the median firm's idiosyncratic productivity shocks z (solid line) in period $t+h$ normalized by its productivity shocks at t, i.e., z_{t+h}/z_t in the model. The lines are based on all firms hitting the hard borrowing constraint in the steady state of the hard-constraint model with asset-based lending described in Section 3.3. The red dashed lines in the first two panels correspond to their empirical counterparts, i.e., the median lines in Figure 1.

Table D.9: Fitted Parameters: The Hard-Constraint Model with Agency Frictions

Parameter	Value	Description
Panel 1: Technology		
ρ_z	0.839	Persistence of TFP Shocks
σ_z	0.050	SD of TFP Shocks
ψ_{ac}	0.106	Capital Adj Cost
Panel 2: Agency Frictions		
ρ_a	0.738	Persistence of Agency Shocks
σ_a	0.050	SD of Agency Shocks
μ_a	0.099	Mean of Agency Shocks
α_H	0.477	Agency Curvature
C_H	0.018	Agency Fixed Cost
Panel 3: Financial Frictions		
γ	0.930	Coupon Decay Rate
ψ_d	0.251	Dividend Adj Cost
ψ_e	0.565	Equity Issuance Cost
f_c	0.045	Fixed Operating Cost
ϕ	8.500	Debt to Earnings Limit

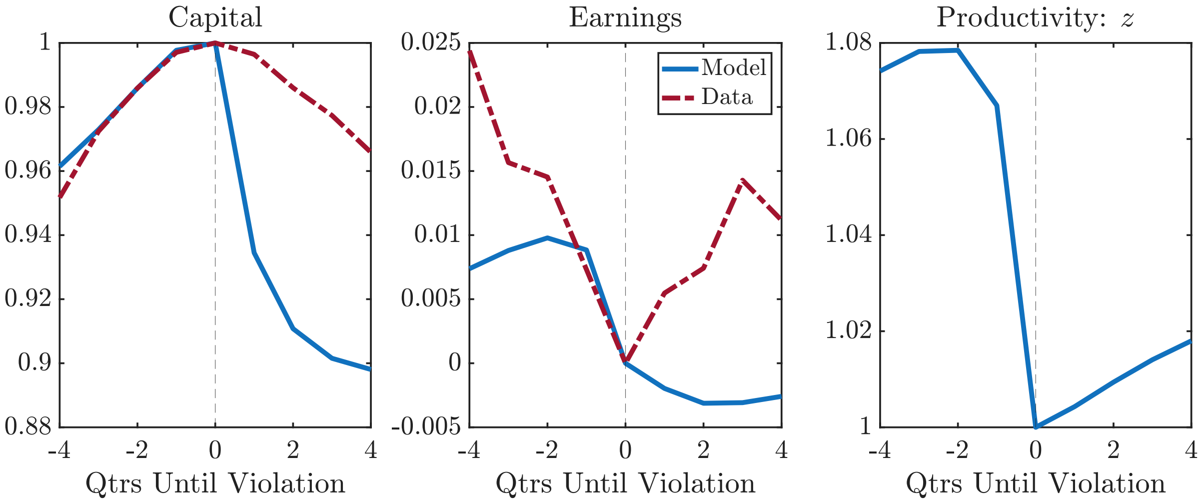
Notes: This table displays parameters that are chosen to match moments in Table 3 and Figure 1 in a variant of the hard-constraint model with agency frictions recalibrated using the same procedure as in our baseline model.

Table D.10: Unconditional Targeted Moments and Model Fit: The Hard-Constraint Model with Agency Frictions

	Model	Data
Median Leverage	0.355	0.320
New Violation Rate	0.114	0.025
Frac. in Violation	0.124	0.150
Median Dividend/Capital	0.011	0.000
Median Equity Issuance	0.000	0.001
SD of Earnings/Capital	0.012	0.028
SD of Investment Rate	0.047	0.056

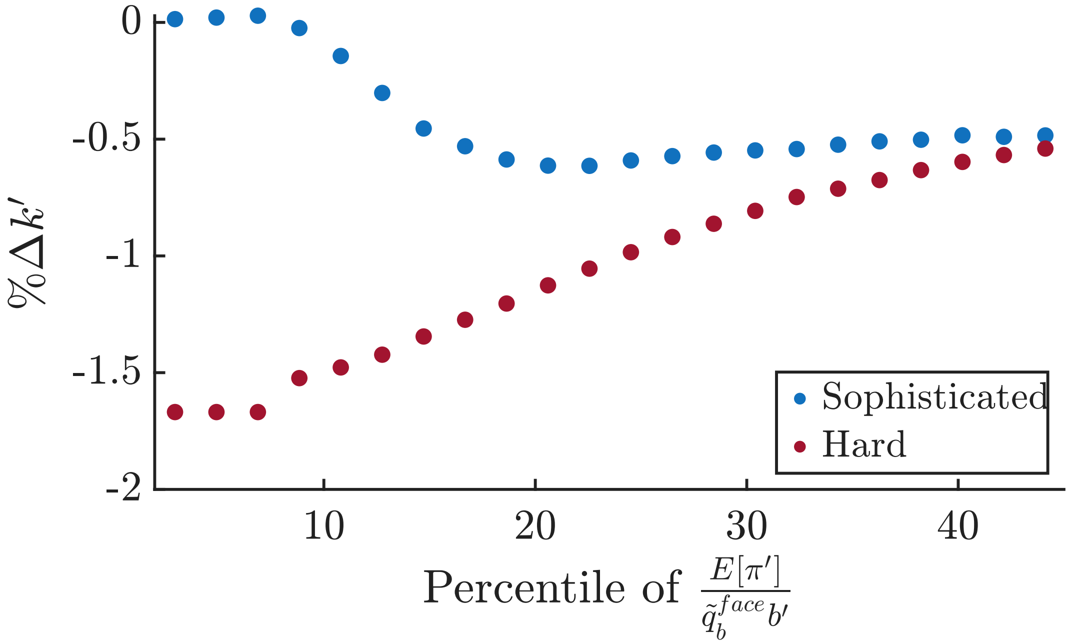
Notes: This table displays the empirical moments targeted in our calibration and the corresponding values generated by the steady state in a variant of the hard-constraint model with agency frictions recalibrated using the same procedure as in our baseline model. Leverage, dividend-to-capital ratio, equity-issuance-to-capital ratio, earnings-to-capital ratio, and investment rate correspond to $\tilde{q}_b^{\text{face}} b/k$, $\max\{d,0\}/k$, $\max\{-d,0\}/k$, $\frac{\pi}{k}$, and $\frac{k'-(1-\delta)k}{k}$ in the model, respectively. We interpret firms hitting the hard borrowing constraint as "violations." We use assets as a proxy for capital when calculating unconditional moments in this table.

Figure D.13: Targeted Dynamics Around Covenant Violations and Model Fit: The Hard-Constraint Model with Agency Frictions



Notes: The blue solid line in the left panel shows the median firm's capital in period $t+h$ normalized by its capital at t (the initial period in which the hard constraint binds), i.e., k_{t+h}/k_t in the model. The blue solid line in the middle panel shows the median firm's earnings in period $t+h$ relative to period t, normalized by capital at t, i.e., $(\pi_{t+h}-\pi_t)/k_t$ in the model. The right panel plots the median firm's idiosyncratic productivity shocks z (solid line) in period $t+h$ normalized by its productivity shocks at t, i.e., z_{t+h}/z_t in the model. The lines are based on all firms hitting the hard borrowing constraint in the steady state of a variant of the hard-constraint model with agency frictions recalibrated using the same procedure as in our baseline model. The red dashed lines in the first two panels correspond to their empirical counterparts, i.e., the median lines in Figure 1.

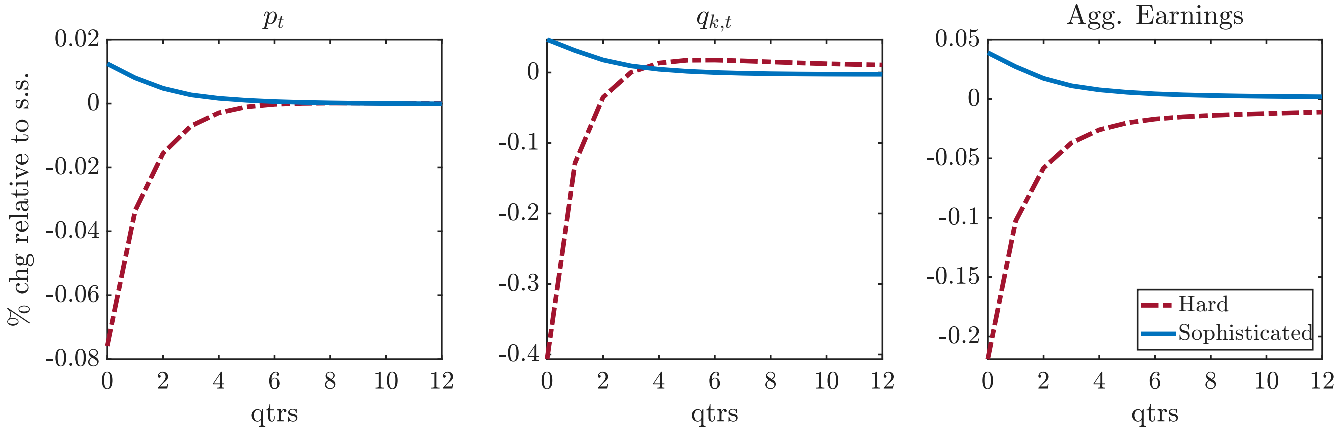
Figure D.14: Capital Change from Constraint-Tightening (Intensive Margin)



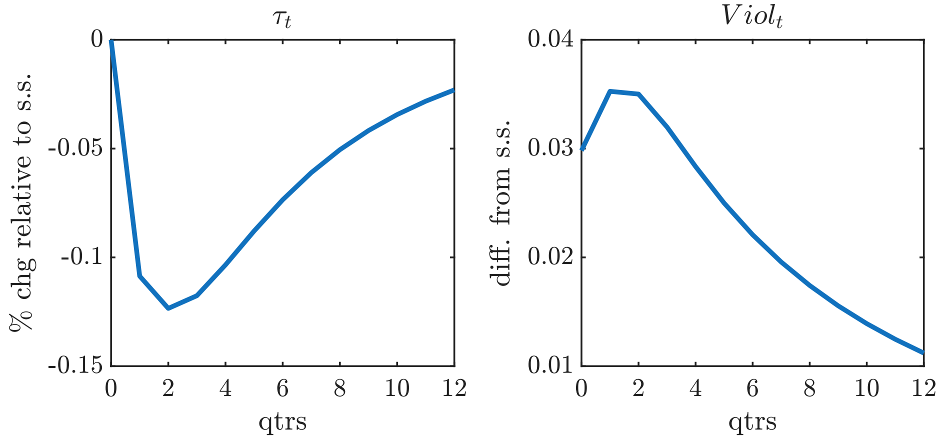
Notes: This figure plots the mean percentage change in next-period capital k' when ϕ is reduced to $0.95 \times \phi$ (and ϕ^{hard} to $0.95 \times \phi^{\text{hard}}$ in the hard-constraint model) as a function of the percentile of firms' expected earnings-to-debt ratio ($\mathbb{E}[\pi'] / \left(\tilde{q}_b^{\text{face}} b'\right)$ in the model). In both cases, the policy functions under the original and tightened constraint are evaluated at states drawn from the steady-state distribution associated with the benchmark economy, and percentiles are computed with respect to the same distribution. Blue dots correspond to the sophisticated-constraint model; red dots to the hard-constraint model.

Figure D.15: Macroeconomic Effects of a Constraint-Tightening Shock

(a) Hard vs Sophisticated Borrowing Constraints

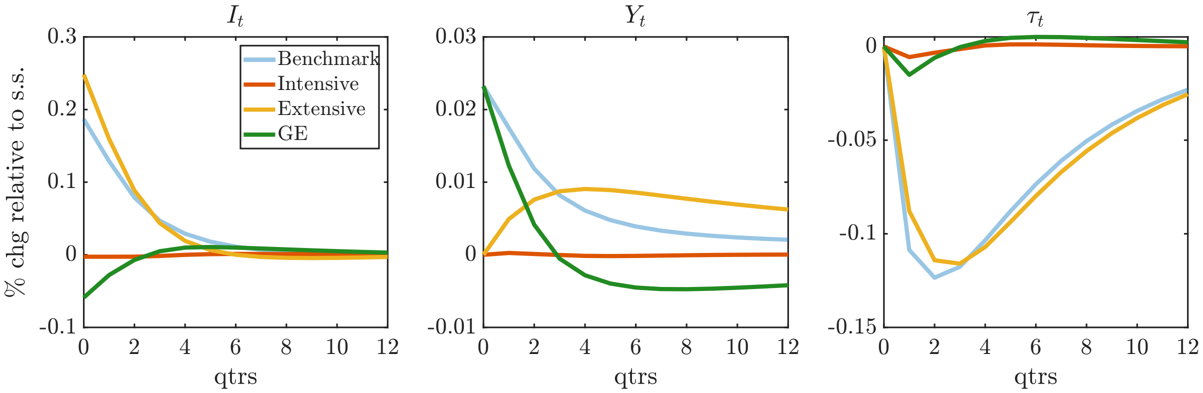


(b) Sophisticated Borrowing Constraints



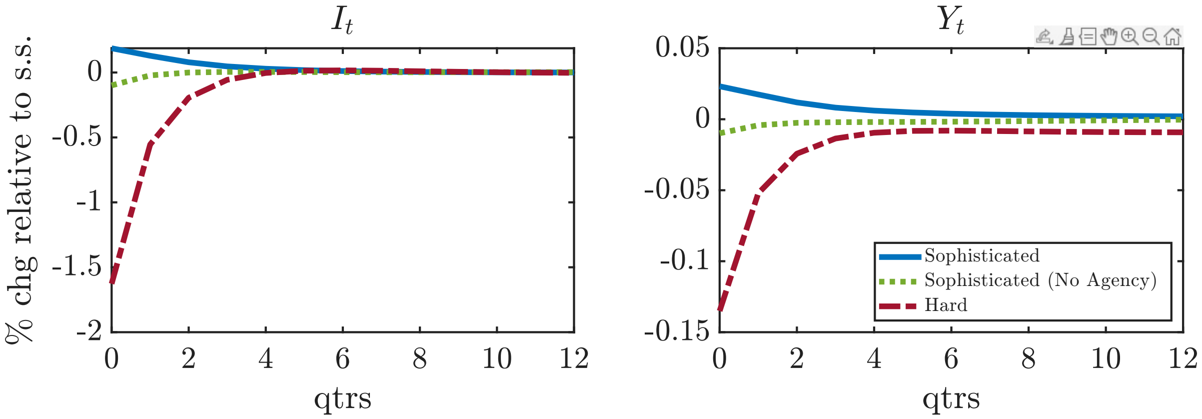
Notes: Panel A plots the responses of the relative price of goods produced by financially constrained firms p_t, the relative price of capital $q_{k,t}$, and aggregate earnings to a tightening of sophisticated borrowing constraints with initial magnitude $\Delta\phi_0 = -5\% \times \phi$ in the sophisticated-constraint model (solid blue lines), and the counterpart responses to a tightening of hard borrowing constraints with initial magnitude $\Delta\phi_0^{\text{hard}} = -5\% \times \phi^{\text{hard}}$ in the hard-constraint model (dashed red lines). The constraint tightening shock has a persistence of 0.5. Panel B focuses on the responses in the sophisticated borrowing constraints model and plots the average fraction of capital in unproductive use $\tau_t \equiv \int \tau_{jt} di$ and the aggregate fraction of firms in the violation state. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.16: Decomposing the Macroeconomic Effects of a Constraint-Tightening Shock: Sophisticated Borrowing Constraints



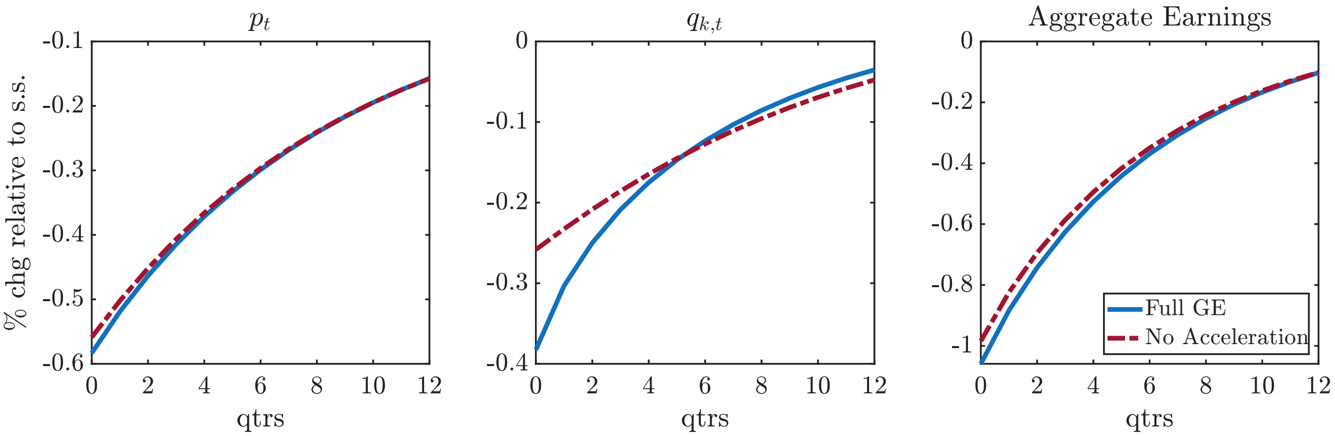
Notes: The figure plots the responses of aggregate investment, output, and fraction of capital for unproductive use to a tightening of borrowing constraints with an initial magnitude $\Delta\phi_0 = -5\% \times \phi$ and a persistence of 0.5 under sophisticated borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The extensive, intensive, and GE margins are defined in Appendix C.3. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.17: Macroeconomic Effects of a Constraint-Tightening Shock: Sophisticated Borrowing Constraints without Agency Frictions



Notes: This figure plots the responses of aggregate investment, output, and earnings to a tightening of borrowing constraints, with initial magnitudes $\Delta\phi_0 = -5\% \times \phi$ and with a persistence of 0.5, under a variant of the sophisticated borrowing constraints model without agency frictions, recalibrated using the same procedure as in our baseline model. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

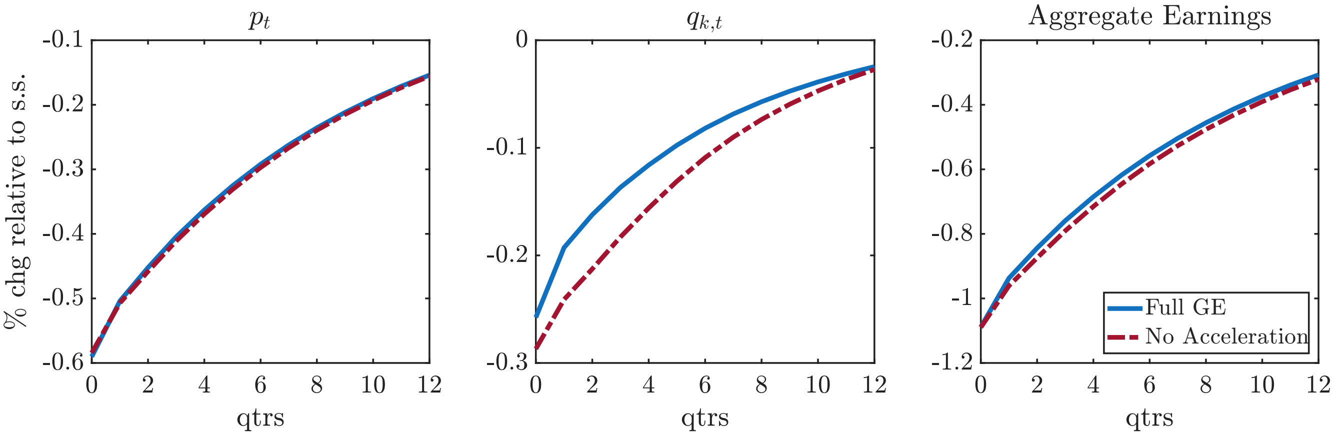
Figure D.18: Macroeconomic Effects of a Contractionary Monetary Policy Shock: Hard Borrowing Constraints



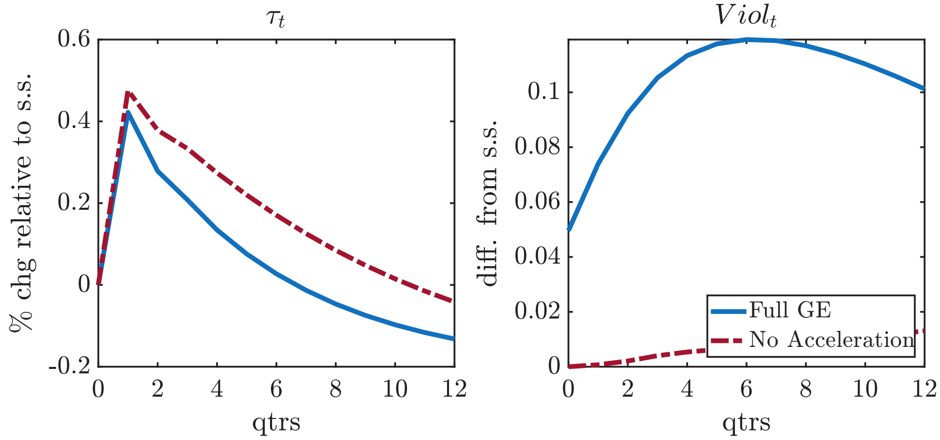
Notes: The figure plots the responses of the relative price of goods produced by financially constrained firms p_t, the relative price of capital $q_{k,t}$, and aggregate earnings to a 30 bps monetary tightening (ϵ_0^m = 0.3%) with a persistence of 0.9 under hard borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.19: Macroeconomic Effects of a Contractionary Monetary Policy Shock: Sophisticated Borrowing Constraints

(a) Prices and Earnings

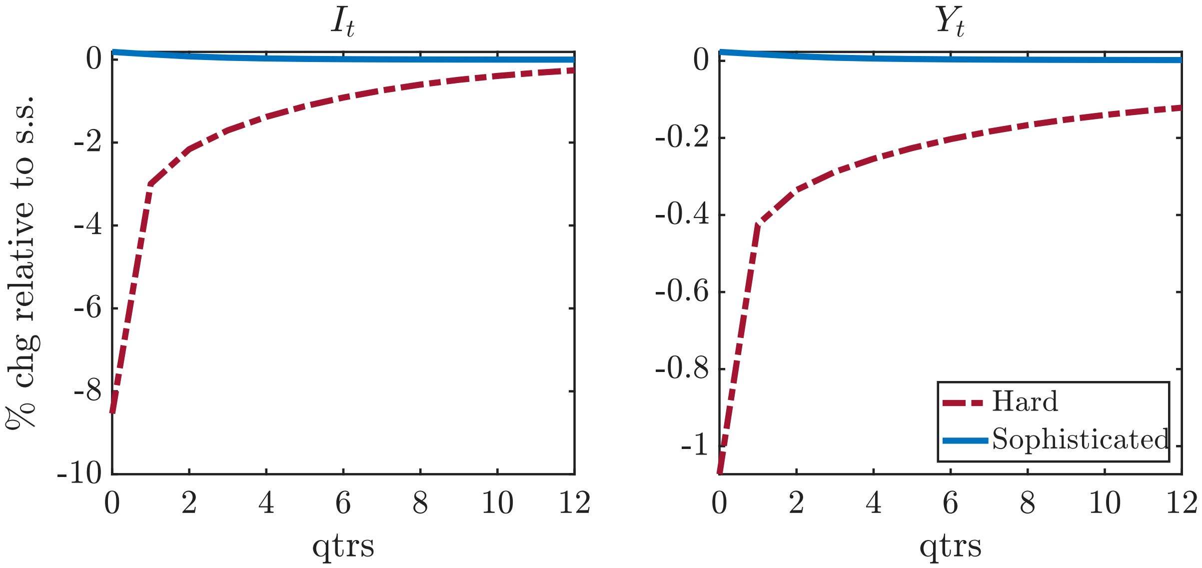


(b) Agency Frictions and Violation Rate



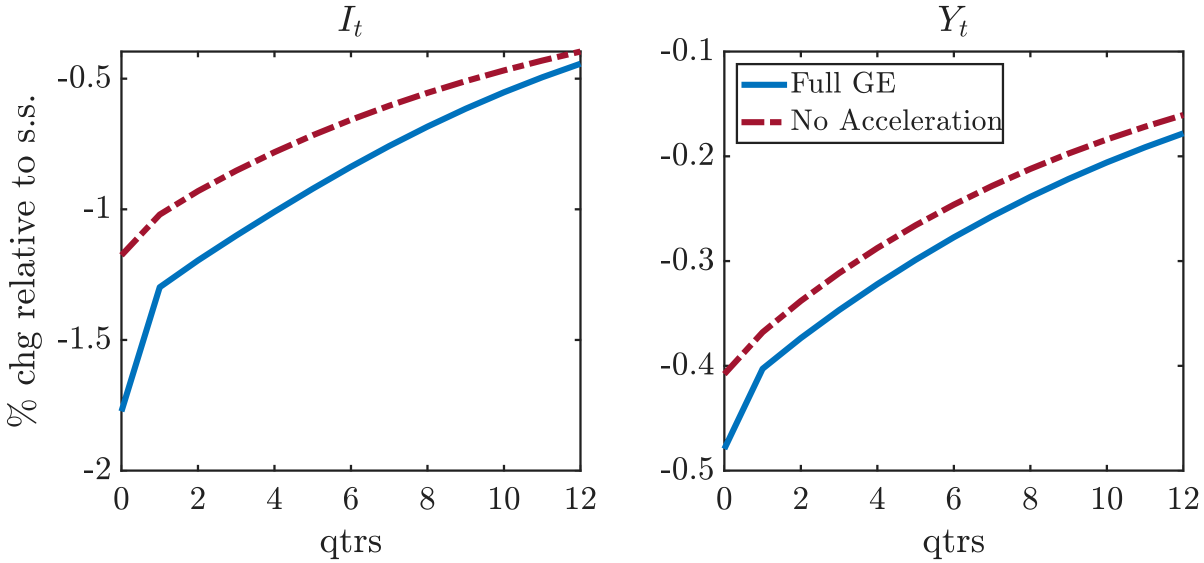
Notes: Panel (a) plots the responses of the relative price of goods produced by financially constrained firms p_t, the relative price of capital $q_{k,t}$, and aggregate earnings to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.9 under sophisticated borrowing constraints. Panel (b) plots the responses of the average fraction of capital in unproductive use $\tau_t \equiv \int \tau_{jt} di$ and the aggregate fraction of firms in the violation state. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.20: Macroeconomic Effects of a Constraint-Tightening Shock: Hard Borrowing Constraints with Asset-based Lending



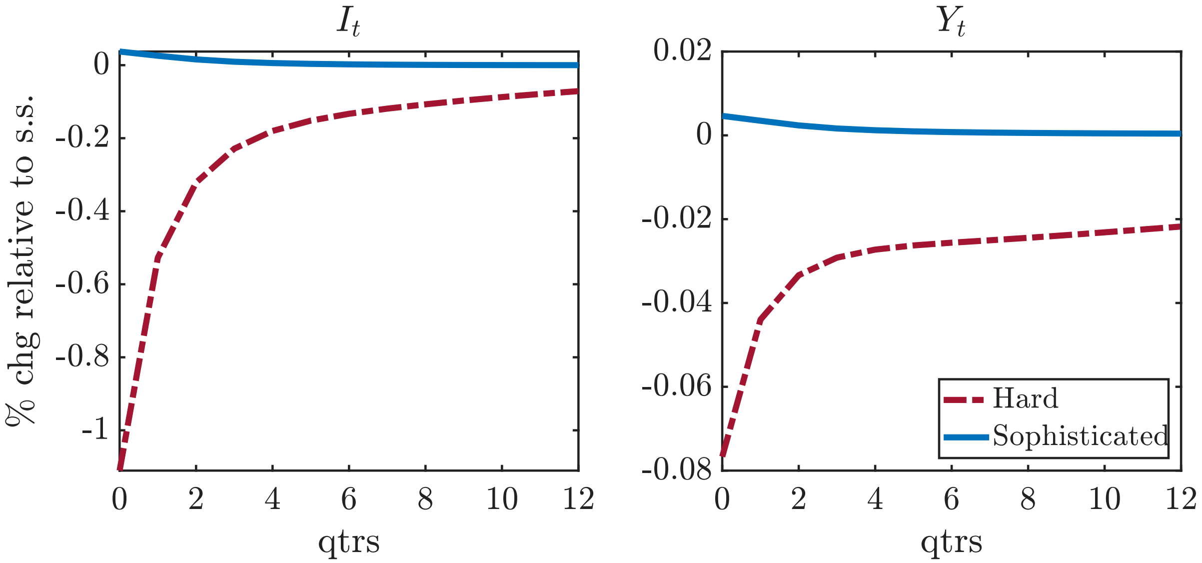
Notes: This figure plots the responses of aggregate investment, output, and earnings to a tightening of borrowing constraints, with initial magnitudes $\Delta\phi_0 = -5\% \times \phi$ in the sophisticated-constraint model and $\Delta\phi_0^{\text{hard}} = -5\% \times \phi^{\text{hard}}$ in the hard-constraint model, and with a persistence of 0.5, under both hard (dashed red lines) and sophisticated borrowing constraints (solid blue lines). The hard-constraint model with asset-based lending is described in Section 3.3. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.21: Macroeconomic Effects of a Contractionary Monetary Policy Shock: Hard Borrowing Constraints with Asset-based Lending



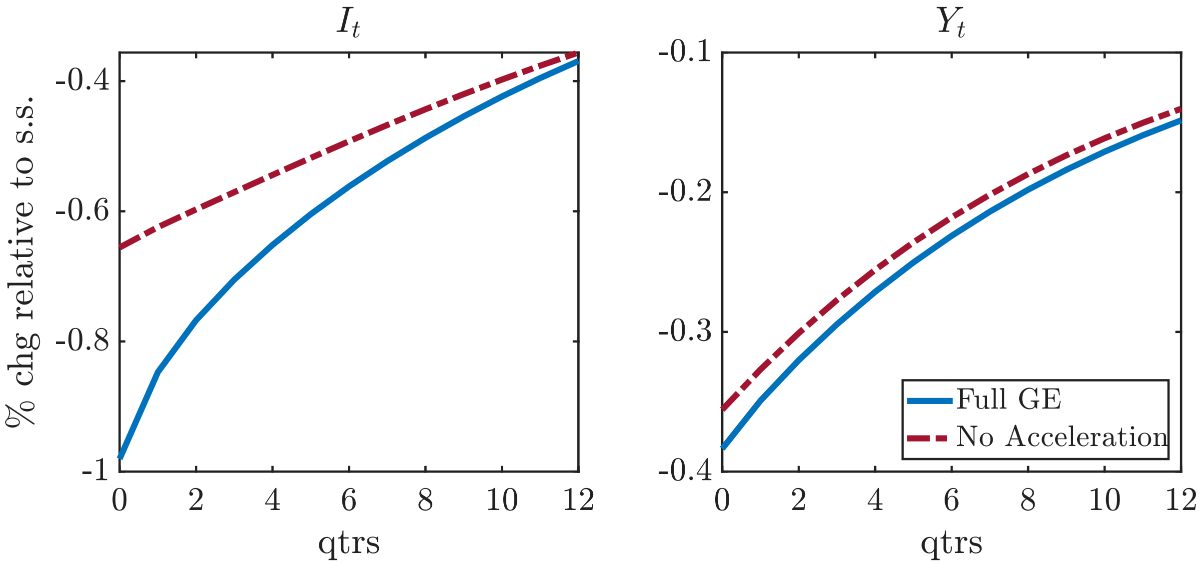
Notes: The figure plots the responses of aggregate investment, output, and earnings to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.9 under both hard (dashed red lines) and sophisticated borrowing constraints (solid blue lines). The hard-constraint model with asset-based lending is described in Section 3.3. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.22: Macroeconomic Effects of a Constraint-Tightening Shock: Hard Borrowing Constraints Recalibrating ϕ^{hard} only



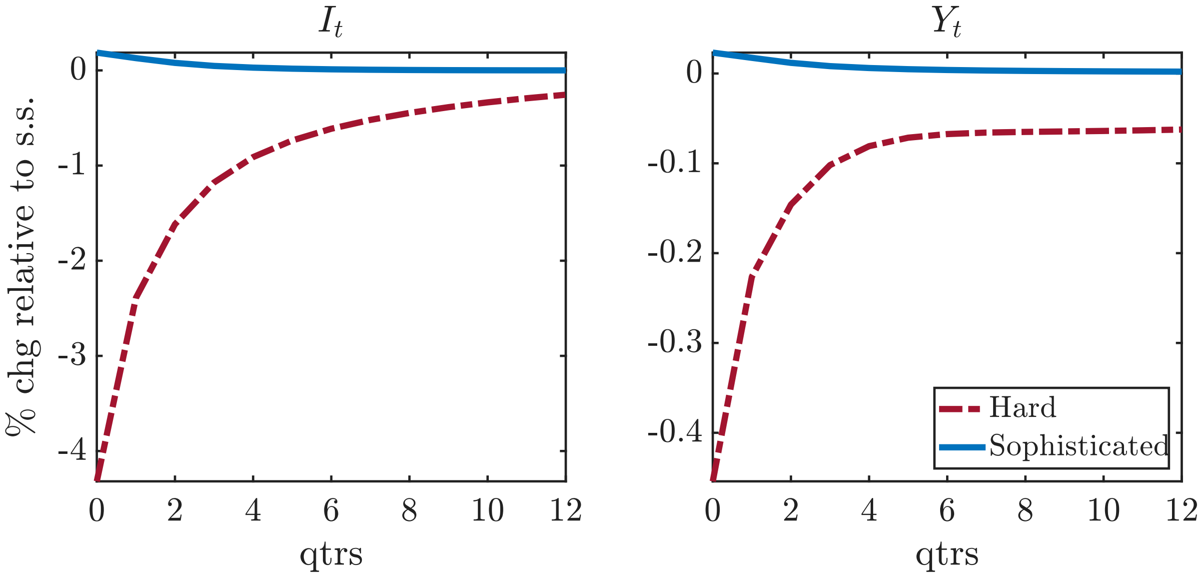
Notes: This figure plots the responses of aggregate investment and output to a tightening of borrowing constraints, with initial magnitudes $\Delta\phi_0 = -5\% \times \phi$ in the sophisticated-constraint model and $\Delta\phi_0^{\text{hard}} = -5\% \times \phi^{\text{hard}}$ in the hard-constraint model, and with a persistence of 0.5, under both hard (dashed red lines) and sophisticated borrowing constraints (solid blue lines). The hard-constraint model is based on a variant where we recalibrate ϕ^{hard} such that its median leverage matches that of the data, while keeping all other parameters the same. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.23: Macroeconomic Effects of a Contractionary Monetary Policy Shock: Hard Borrowing Constraints Recalibrating ϕ^{hard} only



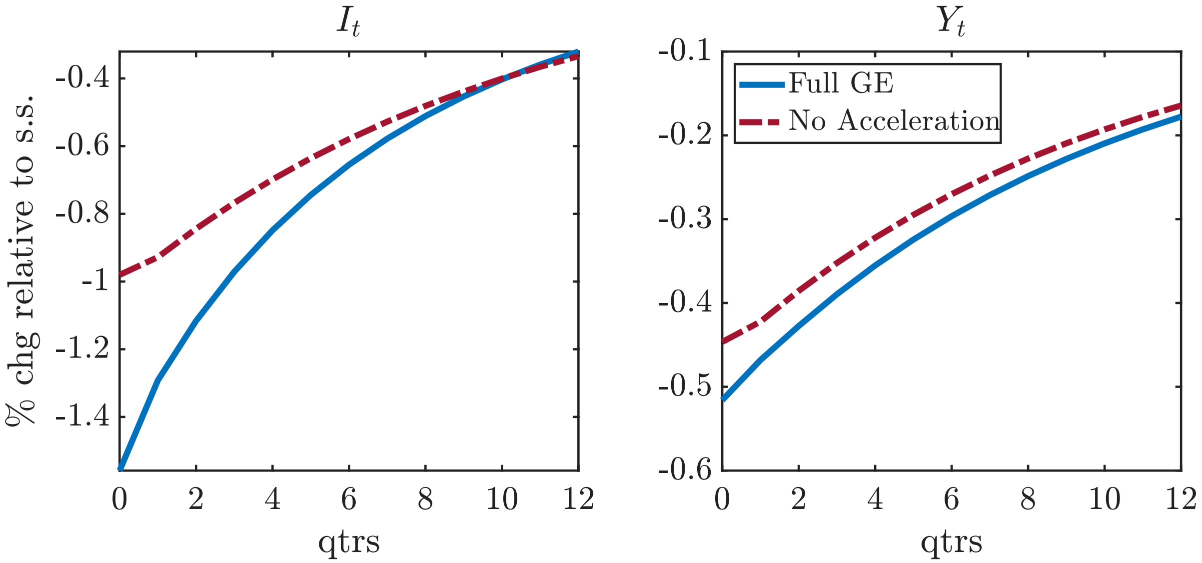
Notes: The figure plots the responses of aggregate investment and output to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.9 under hard borrowing constraints. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The hard-constraint model is based on a variant where we recalibrate ϕ^{hard} such that its median leverage matches that of the data, while keeping all other parameters the same. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.24: Macroeconomic Effects of a Constraint-Tightening Shock: Hard Borrowing Constraints with Agency Frictions



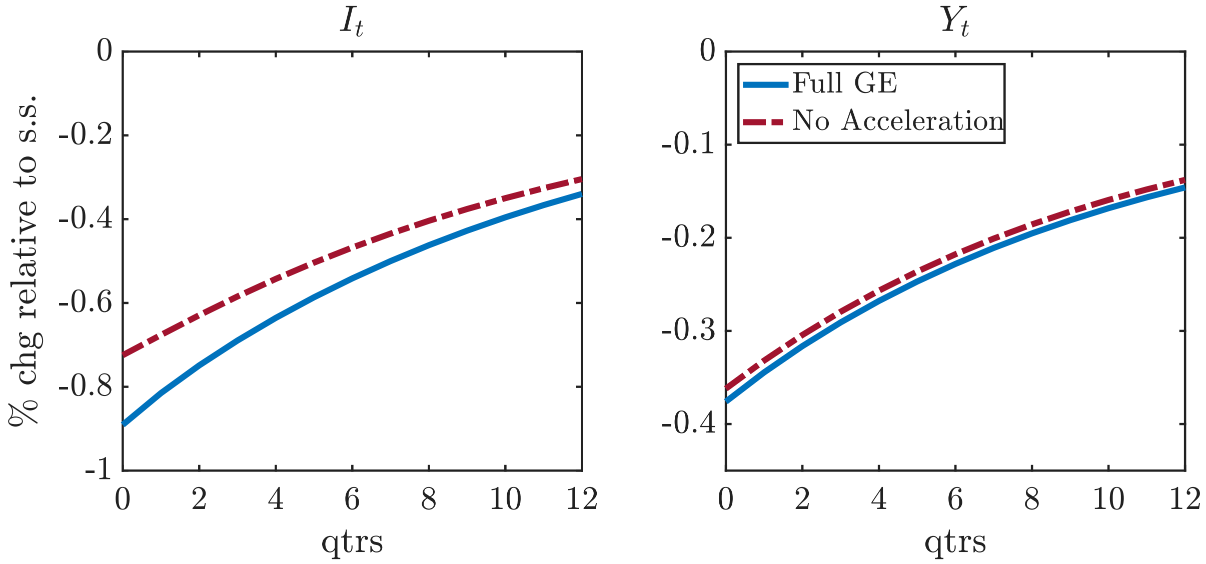
Notes: This figure plots the responses of aggregate investment, output, and earnings to a tightening of borrowing constraints, with initial magnitudes $\Delta\phi_0 = -5\% \times \phi$ in the sophisticated-constraint model and $\Delta\phi_0^{\text{hard}} = -5\% \times \phi^{\text{hard}}$ in the hard-constraint model, and with a persistence of 0.5, under both hard (dashed red lines) and sophisticated borrowing constraints (solid blue lines). The hard-constraint model is based on a variant with agency frictions and is recalibrated using the same procedure as in our baseline model. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.25: Macroeconomic Effects of a Contractionary Monetary Policy Shock: Hard Borrowing Constraints with Agency Frictions



Notes: The figure plots the responses of aggregate investment, output, and earnings to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.9 under both hard (dashed red lines) and sophisticated borrowing constraints (solid blue lines). The hard-constraint model is based on a variant with agency frictions and is recalibrated using the same procedure as in our baseline model. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.

Figure D.26: Macroeconomic Effects of a Contractionary Monetary Policy Shock: Sophisticated Borrowing Constraints without Agency Frictions



Notes: The figure plots the responses of aggregate investment, output, and earnings to a 30 bps monetary tightening ($\epsilon_0^m = 0.3\%$) with a persistence of 0.9 under a variant of the sophisticated borrowing constraints model without agency frictions, recalibrated using the same procedure as in our baseline model. It is computed as the perfect foresight transition in response to an unexpected shock from the steady state. The units of responses are expressed in terms of percentage deviations relative to steady state.